# Deformation Processing Drawing 

ver. 1

Prof. Ramesh Singh, Notes by Dr.
Singh/ Dr. Colton

## Overview

- Description
- Characteristics
- Mechanical Analysis
- Thermal Analysis
- Tube drawing


## Geometry



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## WIRE DRAWING



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## Equipment



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## Cold Drawing



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A. Durer - Vire Drawing Mill (1489)



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## Characteristics

- Product sizes:
-0.0002 " $(5 \mu \mathrm{~m})$ to several inches (100-150 mm)
- Mostly cold ( $\mathrm{T}<0.4 \mathrm{~T}_{\text {melting }}$ )
- below recrystallization point
- Small diameter (wire):
- uses a capstan
- Diameter > 1 inch ( 25 mm ) (rod):
- bull blocks on a draw bench
- length up to 40 feet ( 12 m )


## Characteristics

- Fine wire done through several dies
- Speeds
- large diameter: 30 feet per minute ( $9 \mathrm{~m} / \mathrm{min}$ )
- small diameter: 300 feet per minute ( $90 \mathrm{~m} / \mathrm{min}$ )
- fine wires: 5,000 feet per minute ( 60 mph - $100 \mathrm{~km} / \mathrm{h}$ )


## Die Materials

- Large diameter
- high carbon steel
- high speed steel
- Moderate diameter

- tungsten carbide (WC)
- Small diameter
- diamond inserts


## Characteristics

- Lubrication
- Coatings
- Oil
- Die angle ( $\alpha$ )
- typically small: 4-6º


## Mechanical analysis (round wire / rod)

Reduction in area (RA)


$$
\begin{aligned}
& R A=\frac{D_{b}^{2}-D_{a}^{2}}{D_{b}^{2}}=1-\left(\frac{D_{a}}{D_{b}}\right)^{2} \\
& \varepsilon_{t}=\ln \left(\frac{1}{1-R A}\right)=2 \cdot \ln \left(\frac{D_{b}}{D_{a}}\right)
\end{aligned}
$$

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## Slab analysis



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## Equilibrium

$$
\begin{aligned}
& \left(\sigma_{x}+d \sigma_{x}\right) \frac{\pi}{4}(D+d D)^{2}-\sigma_{x} \frac{\pi}{4} D^{2} \\
& +p \frac{\pi D \cdot d x}{\cos \alpha} \sin \alpha+\mu p \frac{\pi D \cdot d x}{\cos \alpha} \cos \alpha=0
\end{aligned}
$$

## Expanding

$$
\begin{aligned}
& \left(\sigma_{x}+d \sigma_{x}\right) \frac{\pi}{4}\left(D^{2}+2 D d D+d D^{2}\right)-\sigma_{x} \frac{\pi}{4} D^{2} \\
& +p \frac{\pi D \cdot d x}{\cos \alpha} \sin \alpha+\mu p \frac{\pi D \cdot d x}{\cos \alpha} \cos \alpha=0
\end{aligned}
$$

## Equilibrium

$$
\begin{aligned}
& \frac{\pi}{4}\left[\left(\sigma_{x} D^{2}+2 \sigma_{x} D d D+\sigma_{x} d D^{2}\right)+\left(d \sigma_{x} D^{2}+2 d \sigma_{x} p d d D+d \sigma_{x}^{\text {small }} d D^{2}\right)\right] \\
& -\sigma_{x} \frac{\pi}{4} D^{2}+p \frac{\pi D \cdot d x}{\cos \alpha} \sin \alpha+\mu p \frac{\pi D \cdot d x}{\cos \alpha} \cos \alpha=0
\end{aligned}
$$

Eliminating higher order terms, dividing by $\mathrm{D} \& \pi$, multiplying by 4 and canceling

$$
2 \sigma_{x} d D+d \sigma_{x} D+4 p \frac{d x}{\cos \alpha} \sin \alpha+4 \mu p \frac{d x}{\cos \alpha} \cos \alpha=0
$$

## Equilibrium

$$
\begin{aligned}
& \mathrm{dD} / 2 \text { Noting } \\
& \mathrm{dx} \quad \tan \alpha=\frac{d D / 2}{d x} \quad d x=\frac{d D}{2 \tan \alpha} \\
& 2 \sigma_{x} d D+d \sigma_{x} D+4 p \frac{\sin \alpha}{\cos \alpha} \frac{d D}{2 \tan \alpha}+4 \mu p \frac{\cos \alpha}{\cos \alpha} \frac{d D}{2 \tan \alpha}=0
\end{aligned}
$$

or

$$
2 \sigma_{x} d D+d \sigma_{x} D+2 p d D+2 \mu p \frac{d D}{\tan \alpha}=0
$$

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## Equilibrium

Finally

$$
2 \sigma_{x} \cdot d D+D \cdot d \sigma_{x}+2 p \cdot\left(1+\frac{\mu}{\tan \alpha}\right) \cdot d D=0
$$

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# Maximum shear stress (Tresca) criterion 

$$
\sigma_{x}+p=2 \tau_{\text {flow }}=\sigma_{\text {flow }}
$$

$$
\mu
$$

$\tan \alpha$


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## Differential form

$$
\begin{aligned}
\frac{d D}{D} & =\frac{d p}{4 \tau_{\text {flow }}+2 p B} \\
\frac{d D}{D} & =\frac{d \sigma_{x}}{2 B \sigma_{x}-4 \tau_{\text {flow }}(1+B)}
\end{aligned}
$$

## Integrating

$$
\int_{D_{a}}^{D_{b}} \frac{d D}{D}=\int_{\sigma_{x a}}^{\sigma_{x b}} \frac{d \sigma_{x}}{2 B \sigma_{x}-4 \tau_{\text {flow }}(1+B)}
$$

$$
\begin{aligned}
& 10010=\int_{0=0}^{2 x} \frac{1}{D} d D \\
& \text { Outi| } \mathrm{It}\left[\frac{-\mathrm{Im}[\mathrm{DD}] \mathrm{Na}[\mathrm{Da}]+\mathrm{In}[\mathrm{Da}] \mathrm{Ne}[\mathrm{Db}]}{\mathrm{Im}[\mathrm{Da}]-\mathrm{Im}[\mathrm{Db}]} \geqslant 0.66\right. \\
& \left(\left[\mathrm{Da}\left[\frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}}\right] \geqslant 06 \mathrm{Da} \frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}} \neq 0\right)\left|1 \mathrm{Na}\left[\frac{\mathrm{Da}}{\mathrm{Da}-\mathrm{Ib}}\right] \geqslant 21\right| \mathrm{Im}\left[\frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}}\right] \neq 0\right),-\operatorname{seg}[\mathrm{Da}]+\mathrm{Log}[\mathrm{Db}] \text {, } \\
& \text { Inkegrate }\left[\frac{2}{D},(D, D a, D b), \text { harumptiona }+1\left(\frac{-\mathrm{In}[\mathrm{Db}] \mathrm{Na}[\mathrm{Da}]+\mathrm{In}[\mathrm{Da}] \mathrm{Na}[\mathrm{Db}]}{\mathrm{Im}[\mathrm{Da}]-\mathrm{Im}[\mathrm{Db}]} \geqslant 0 \mathrm{Lb}\right.\right. \\
& \left.\left.\left(\left(\mathrm{Na}\left[\frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}}\right] \geqslant 06 a \frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}} \neq 0\right)\left\|\mathrm{Da}\left[\frac{\mathrm{Da}}{\mathrm{Da}-\mathrm{Db}}\right] \geqslant 1\right\| \mathrm{Im}\left[\frac{\mathrm{Da}}{-\mathrm{Da}+\mathrm{Db}}\right] \neq 0\right)\right]\right] \\
& -\log [\mathrm{Da}]+\log [\mathrm{Db}] \\
& \ln p \mathrm{p}+\int_{\operatorname{man}}^{\operatorname{man}} \frac{1}{2 \mathrm{D} \theta-4 \mathrm{x}_{a}(1+\mathrm{D})} d v
\end{aligned}
$$

Integration rearult :
$\frac{-\log \left[2 n \operatorname{axn}-4(1+D) x_{1}\right]+\log \left[2 D \operatorname{cosb}-4(1+D) x_{4}\right]}{2 D}$

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## Drawing stress

$$
\frac{\sigma_{x a}}{2 \tau_{\text {flow }}}=\frac{1+B}{B}\left[1-\left(\frac{D_{a}}{D_{b}}\right)^{2 B}\right]+\frac{\sigma_{x b}}{2 \tau_{\text {flow }}}\left(\frac{D_{a}}{D_{b}}\right)^{2 B}
$$

- where:

$$
\begin{aligned}
& \sigma_{x b}=\text { back stress (tension) } \\
& \sigma_{x a}=\text { pulling stress (tension) }
\end{aligned}
$$



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## Strain hardening

## (cold - below recrystallization

 point)- For round parts - Tresca

$$
2 \tau_{\text {flow }}=\sigma_{\text {flow }}=\frac{K}{\bar{Y}}=\frac{K \varepsilon^{n}}{n+1}
$$

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## Strain rate effect

## (hot - above recrystallization point)

$$
2 \tau_{\text {flow }}=\sigma_{\text {flow }}=\bar{Y}=C \dot{\bar{\varepsilon}}^{m}
$$

- For a round part (derived for extrusion)

$$
\dot{\bar{\varepsilon}}=\frac{6 v_{b} D_{b}^{2} \cdot \tan \alpha}{D_{b}^{3}-D_{a}^{3}} \cdot \ln \left(\frac{A_{b}}{A_{a}}\right)
$$

- average strain rate due to shape of element
$-v_{b}=$ velocity of "b" side
- $\mathrm{A}=$ area



## Value for $p$

$$
p=Y-\sigma
$$



$$
p=2 \tau_{\text {flow }}-\sigma
$$

## maximum at entrance

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## Effect of back tension



## Maximum RA

- Solve previous equations with:

$$
\begin{aligned}
& \alpha=6^{\circ} \text { (typical value) } \\
& \mu=0.1 \\
& \therefore B=1 \\
& \sigma_{x b}=0
\end{aligned}
$$

For failure: draw stress = material flow \{yield\} stress

$$
\sigma_{x a}=\sigma=K \varepsilon^{n} \quad 2 \tau_{\text {flow }}=\sigma_{\text {flow }}=\bar{Y}=\frac{K \varepsilon^{n}}{n+1}
$$

here, say $\mathrm{K}=760 \mathrm{MPa}$, and $\mathrm{n}=0.19$

## Maximum RA

$$
\begin{aligned}
& \frac{K \varepsilon^{n}}{\frac{K \varepsilon^{n}}{n+1}}=\left(\frac{1+B}{B}\right) \cdot\left(1-\left(\frac{D_{a}}{D_{b}}\right)^{2 B}\right) \\
& \frac{0.19+1}{1}=\left(\frac{1+1}{1}\right) \cdot\left(1-\left(\frac{D_{a}}{D_{b}}\right)^{2}\right)
\end{aligned}
$$

- Yields RA = 0.6
- must be solved for each $\mu, \alpha, \sigma_{x b}$

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# Energy / unit volume (u) 

$\mathrm{u}=\mathrm{FV} / \mathrm{A}_{\mathrm{a}} \mathrm{V}=\sigma_{\mathrm{xa}}$<br>(with no back stress)<br>V= volume

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## Rod/Wire Drawing Analysis

- Ideal deformation

External work $=$ Work of ideal plastic deformation

$$
\sigma_{d}\left(A_{f} L\right)=u\left(A_{f} L\right)
$$

$$
\sigma_{t}=K \varepsilon_{t}^{n}
$$

$$
\sigma_{d}=u=\int_{0}^{\varepsilon_{t}} \sigma_{t} d \varepsilon_{t}
$$

for

$$
\sigma_{d}=\frac{K \varepsilon_{t}^{n}}{n+1} \varepsilon_{t}=\bar{Y}_{f} \varepsilon_{t}=\bar{Y}_{f} \ln \left(\frac{A_{0}}{A_{f}}\right)
$$

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## Rod/Wire Drawing Analysis

- Ideal deformation

Drawing force, $F_{d}=\sigma_{d} A_{f}$
Drawing power, $P_{d}=F_{d} V_{f}$


Source: S. Kalpakjian \& S. Schmidt,
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## Drawing Limit

- Ideal deformation of a perfectly plastic material

$$
\begin{aligned}
& \sigma_{d}=Y \cdot \ln \left(\frac{A_{o}}{A_{f}}\right) \\
& \sigma_{\varepsilon}=Y
\end{aligned}
$$

$$
\sigma_{d}=\sigma_{\varepsilon} \Rightarrow \ln \left(\frac{A_{o}}{A_{f}}\right)=1 \Rightarrow \frac{A_{o}}{A_{f}}=e
$$

Maximum reduction per pass

$$
=\frac{A_{o}-A_{f}}{A_{o}}=1-\frac{1}{e}=0.63=63 \%
$$

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## Drawing Limit

- Ideal deformation of a strain hardening material

$$
\begin{aligned}
\sigma_{d} & =\bar{Y} \cdot \ln \left(\frac{A_{o}}{A_{f}}\right)=\frac{K \varepsilon^{n+1}}{n+1} \\
\sigma_{\varepsilon} & =K \varepsilon^{n} \\
\sigma_{d} & =\sigma_{\varepsilon} \Rightarrow \varepsilon=n+1
\end{aligned}
$$



Maximum reduction per pass

$$
=\frac{A_{o}-A_{f}}{A_{o}}=1-e^{-(n+1)}
$$

## Example Problem

Assuming zero redundant work and frictional work to be $20 \%$ of the ideal work, derive an expression for the maximum reduction in area per pass for a wire drawing operation for a material with a true-stress strain curve of $\sigma=K \varepsilon^{n}$

Total work = Ideal work + frictional work + redundant work
Total work $=$ Ideal work $+0.2 \times$ Ideal work $=1.2 \times$ Ideal work
Or, Total work of deformation $=1.2$ [ $u \times$ volume]

In drawing, external work of deformation $=\sigma_{d} \times$ volume
Equating (1) and (2), we get

$$
\begin{gather*}
\sigma_{d}=1.2 u \text { or } \sigma_{d}=1.2 \int_{0}^{\varepsilon_{1}} \sigma_{t} d \varepsilon_{t}=1.2 \int_{0}^{\varepsilon_{1}} K \varepsilon_{t}^{n} d \varepsilon_{t}=1.2 \frac{K \varepsilon_{1}^{n+1}}{n+1} \\
\sigma_{d}=1.2 \bar{Y} \varepsilon_{1} \quad \text { where } \varepsilon_{1}=\ln \left(\frac{A_{0}}{A_{f}}\right) \quad \ldots \text { (3) } \\
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\end{gather*}
$$

## Example Problem

Max reduction occurs when total drawing stress, $\sigma_{d}=$ Flow stress of material at die exit, $Y$
$\sigma_{d}=Y$
$1.2 \bar{Y} \varepsilon_{1}=K \varepsilon_{1}^{n}$
$1.2 \frac{K \varepsilon_{1}^{n+1}}{n+1}=K \varepsilon_{1}^{n}$
$\varepsilon_{1}=\frac{n+1}{1.2} \Rightarrow \ln \frac{A_{0}}{A_{f}}=\frac{n+1}{1.2} \Rightarrow \frac{A_{0}}{A_{f}}=e^{\frac{n+1}{1.2}}$
$\therefore$ max reduction per pass $=\frac{A_{0}-A_{f}}{A_{0}}=1-e^{-\left(\frac{n+1}{1.2}\right)}$

## Drawing - Ex. 1-1

Determine power, and plot $\sigma_{x}$ and $p$ along die length.

- Drawing steel rod from $\phi=13 \mathrm{~mm}$ to $\phi=12 \mathrm{~mm} @ 1.5 \mathrm{~m} / \mathrm{s}$
- $\mathrm{K}=760 \mathrm{MPa}, \mathrm{n}=0.19$
- $\mu=0.1, \alpha=4^{\circ}, \sigma_{x b}=0$


## Drawing - Ex. 1-2

- First, we must see if we can do the process, the limit is

$$
\sigma_{x a}=\sigma_{\max }=K \varepsilon^{n}
$$

- $R A=1-\left(D_{a} / D_{b}\right)^{2}=0.15=15 \%$
- $\varepsilon_{\mathrm{t}}=\ln \{1 /(1-\mathrm{RA})\}$

$$
=\ln \{1 /(1-0.15)\}=0.16
$$

- $B=\mu / \tan \alpha=0.1 / \tan 4^{\circ}=1.43$

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## Drawing - Ex. 1-3

$$
\begin{aligned}
& \frac{\sigma_{x a}}{2 \tau_{\text {flow }}}=\frac{1+B}{B}\left[1-\left(\frac{D_{a}}{D_{b}}\right)^{2 B}\right]+\frac{\sigma_{x b}}{2 \tau_{\text {flow }}}\left(\frac{D_{a}}{D_{b}}\right)^{2 B} \\
& 2 \tau_{\text {flow }}=\bar{Y}=\frac{K \varepsilon^{n}}{n+1} \\
& \sigma_{\max }=K \varepsilon^{n}
\end{aligned}
$$

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## Drawing - Ex. 1-4

- So, equating the equations (with no back stress) yields

$$
\begin{aligned}
& 1=\frac{1}{n+1}\left(\frac{1+B}{B}\right) \cdot\left(1-\left(\frac{D_{a}}{D_{b}}\right)^{2 B}\right) \\
& 1=\frac{1}{0.19+1}\left(\frac{1+1.43}{1.43}\right) \cdot\left(1-\left(\frac{D_{a-\min }}{13}\right)^{2 \times 1.43}\right)
\end{aligned}
$$

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## Drawing - Ex. 1-5

- Solving gives $D_{a-m i n}=8.53 \mathrm{~mm}$, so we can do the process and proceed with the analysis


## Drawing - Ex. 1-6

$$
\begin{gathered}
\frac{\sigma_{x a}}{2 \tau_{\text {flow }}}=\frac{1+1.43}{1.43}\left[1-\left(\frac{12}{13}\right)^{2 \times 1.43}\right]+0=0.35 \\
2 \tau_{\text {flow }}=\bar{Y}=\frac{K \varepsilon^{n}}{n+1}=\frac{760 \cdot(0.16)^{0.19}}{0.19+1}=446 \mathrm{MPa}
\end{gathered}
$$

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## Drawing - Ex. 1-7

$$
\begin{aligned}
& \text { - } \sigma_{\text {xa }}=0.35 \times 2 \tau_{\text {flow }} \\
& =0.35 \times 446 \mathrm{MPa}=156 \mathrm{MPa} \\
& \text { - } F_{\text {draw }}=\sigma_{\mathrm{xa}} \times \text { Area }=156 \times \pi(12 / 2)^{2} \\
& =17.6 \mathrm{kN}=3938 \mathrm{lbf} \\
& \text { - Power }=F_{\text {draw }} \times \text { speed } \\
& =17.6 \mathrm{kN} \text { x } 1.5 \mathrm{~m} / \mathrm{s}=26.4 \mathrm{~kW}=35.4 \mathrm{hp}
\end{aligned}
$$

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## Drawing - Ex. 1-8

Dimensionless pressures (divided by $2 \boldsymbol{\tau}$ flow)


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## Limits on analysis

- Larger die angles
- more redundant work
$-\sigma, p, u$ will be larger than predicted


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## Redundant work

- $\Delta=d_{\mathrm{m}} / \mathrm{L}$
- $\mathrm{d}_{\mathrm{m}}=\left(\mathrm{D}_{\mathrm{a}}+\mathrm{D}_{\mathrm{b}}\right) / 2$
- $p=Q_{r} \sigma_{\text {flow }}$


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## Redundant work factor (Backofen) (frictionless)



Fig. 7-1. The $\Delta$-dependence of yield pressure for the frictionless plane strain-indentation of a nonstrain-hardening material.

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## Temperature rise



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## Temperatures

$$
\theta=\theta_{\mathrm{o}}+\theta_{\mathrm{s}}+\theta_{\mathrm{f}}
$$

$\theta_{\mathrm{o}}=$ ambient (room) temperature
$\theta_{\mathrm{s}}=$ temperature rise in the wire due to plastic shear energy, $u_{s}$
$\theta_{f}=$ interface temperature rise due to frictional energy, $u_{f}$

## Specific energies

$$
\begin{aligned}
& \mathrm{u}=\mathrm{u}_{\mathrm{s}}+\mathrm{u}_{\mathrm{f}} \\
& \mathrm{u}=\sigma_{\mathrm{xa}} \\
& \mathrm{u}_{\mathrm{s}}=2 \tau_{\text {flow }} \varepsilon
\end{aligned}
$$

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## Specific energies

From the example above (steel rod):

$$
\begin{gathered}
u=\sigma_{\mathrm{xa}}=156 \mathrm{MPa} \\
\mathrm{u}_{\mathrm{s}}=2 \tau_{\text {flow }} \varepsilon=446 * 0.16 \\
=71.4 \mathrm{MPa} \\
\therefore \mathrm{u}_{\mathrm{f}}=\mathrm{u}-\mathrm{u}_{\mathrm{s}}=156-71.4 \\
=84.6 \mathrm{MPa}
\end{gathered}
$$

## Shear temperature $\left(\theta_{\mathrm{s}}\right)$

- Since the shear strain is uniform in the wire
- and all the shear energy remains in the rod as heat
- Then, we can obtain the shear temperature in the wire:

$$
\theta_{s}=\frac{u_{s}}{\rho_{w} c_{w}}
$$

## Material properties

- For this material:

$$
\begin{aligned}
& -k_{w}=60 \mathrm{~W} / \mathrm{m}-\mathrm{K} \\
& -\rho_{\mathrm{w}}=7850 \mathrm{~kg} / \mathrm{m}^{3} \\
& -\mathrm{c}_{\mathrm{w}}=500 \mathrm{~J} / \mathrm{kg}-\mathrm{K} \\
& -\alpha_{\mathrm{w}}=1.53 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

- For a WC die:

$$
-k_{D}=42 \mathrm{~W} / \mathrm{m}-\mathrm{K}
$$

## Shear temperature $\left(\theta_{\mathrm{s}}\right)$

$$
\theta_{s}=\frac{u_{s}}{\rho_{w} c_{w}}=\frac{71.4 \times 10^{6}}{7850 \times 500}=18.2^{\circ} \mathrm{C}
$$

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## Frictional heat (Q)

$Q$ represents all heat generated by friction

$$
Q=u_{f} \frac{\pi D^{2} v}{4}
$$

- $\mathrm{v}=$ velocity
- (1-r)Q goes into the die

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## Die and wire temperatures ( $\theta$ )

ref: Carslaw and Jaeger

- For the die (steady):

$$
\theta=\theta_{o}+\frac{(1-r) Q}{2 \pi k_{D} l} \cdot \ln \frac{D_{o}}{D}
$$

- For the wire (moving):

$$
\theta=\theta_{o}+\theta_{s}+1.07\left(\frac{r Q}{\pi D l k_{w}}\right) \sqrt{\alpha_{w} l / 2 v}
$$

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## Q calculation

$$
\begin{aligned}
Q & =u_{f} \frac{\pi D^{2} v}{4} \\
& =84.6 \times 10^{6} \cdot \frac{\pi \cdot 0.012^{2} \cdot 1.5}{4} \\
Q & =14352 \mathrm{~W}
\end{aligned}
$$

## Dimensions

- $\mathrm{D}=12 \mathrm{~mm}$
- from $D_{0} / D \approx 6$
$-D_{0}=72 \mathrm{~mm}$ in this example
- $l=$ contact length
= reduction in radius $/ \sin \alpha$

$=0.5 \mathrm{~mm} / \sin 4^{\circ}=7.17 \mathrm{~mm}$


## Die temperature

$$
\begin{aligned}
& \theta=\theta_{o}+\frac{(1-r) Q}{2 \pi k_{D} l} \cdot \ln \frac{D_{o}}{D} \\
& \theta=20+\frac{(1-r) \cdot 14352}{2 \pi \cdot 42 \cdot 0.00717} \cdot \ln \frac{72}{12} \\
& =20+13591 \cdot(1-r)
\end{aligned}
$$

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## Wire temperature

$$
\theta=\theta_{o}+\theta_{s}+1.07 \cdot\left(\frac{r Q}{\pi D l k_{w}}\right) \cdot \sqrt{\alpha_{w} l / 2 v}
$$

$$
\theta=20+18.2+1.07 \cdot\left(\frac{r \cdot 14352}{\pi \cdot 0.012 \cdot 0.00717 \cdot 60}\right)
$$

$$
\times \sqrt{1.53 \times 10^{-5} \cdot 0.00717 / 2 \cdot 1.5}
$$

$$
=38.2+181 \cdot r
$$

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## Heat flow ratio and Temperature

- Equating the previous equations yields:

$$
r=0.99
$$

- hence

$$
\begin{gathered}
\theta=156^{\circ} \mathrm{C}=429 \mathrm{~K} \\
\mathrm{~T}_{\text {melt }}=1500^{\circ} \mathrm{C}=1723 \mathrm{~K} \\
\text { So } \theta / \mathrm{T}_{\text {melt }}=0.25, \text { cold (below } \\
\text { recrystallization point }
\end{gathered}
$$

## Temperature in practice

- In practice, $r \approx 1$
- all heat goes into wire

$$
\theta=\theta_{o}+\frac{u_{s}}{\rho_{w} c_{w}}+0.19 \cdot\left(\frac{u_{f}}{\rho_{w} c_{w}}\right) \cdot \sqrt{\frac{D^{2} \cdot v}{\alpha_{w} \cdot l}}
$$

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## Tube drawing



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## Tube Sinking



## Tube Drawing

Tethered Plug Drawing


## Fixed Plug Drawing



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## Plane strain / Slab analysis

$$
\begin{aligned}
\frac{\sigma_{x a}}{2 \tau_{\text {flow }}} & =\frac{1+B^{*}}{B^{*}}\left[1-\left(\frac{t_{f}}{t_{i}}\right)^{B^{*}}\right] \\
B^{*} & \equiv \frac{\mu_{\text {die }}+\mu_{\text {mandrel }}}{\tan \alpha-\tan \beta}
\end{aligned}
$$

$\alpha=$ semicone angle of die $\beta$ semicone angle of plug

## Tube Drawing - Special Cases

$$
B^{*} \equiv \frac{\mu_{\text {die }}+\mu_{\text {mandrel }}}{\tan \alpha-\tan \beta}
$$

Fixed mandrel- same
friction at both interface (plane - tube is modeled as a flat section)

$$
B^{*} \equiv \frac{2 \mu}{\tan \alpha} \quad \begin{aligned}
& \text { Fixed mandrel } \\
& \text { (slab - circular tube) }
\end{aligned} B^{*} \equiv \frac{2 \mu}{\tan \alpha}-\mu^{2}
$$

Moving mandrel - No friction at interface of mandrel and tube

$$
B^{*} \equiv \frac{\mu}{\tan \alpha}
$$ (plane and slab)

Moving mandrel with friction towards exit, takes into account motion between mandrel and tube (B may be negative) (plane)

$$
B^{*} \equiv \frac{\mu_{d i e}-\mu_{\text {mandrel }}}{\tan \alpha}
$$

## Summary

- Description
- Characteristics
- Mechanical analysis
- Thermal analysis
- Tube drawing

Prof. Ramesh Singh, Notes by Dr. Singh/ Dr. Colton

