

1. Mathematica solution for necking,  
Necking should occur where  $dP = 0$

$$\epsilon = \text{Log}[(A_0/A)]$$

$$\sigma = K \epsilon^n$$

$$P = K \epsilon^n A_0 e^{-\epsilon}$$

Taking derivative of P with respect to  $\epsilon$  and equating it to 0,

$$\text{Solve}[D[P, \epsilon] == 0, \epsilon]$$

$$\{\{\epsilon \rightarrow n\}\}$$

Given

$$\sigma = K \epsilon^n$$

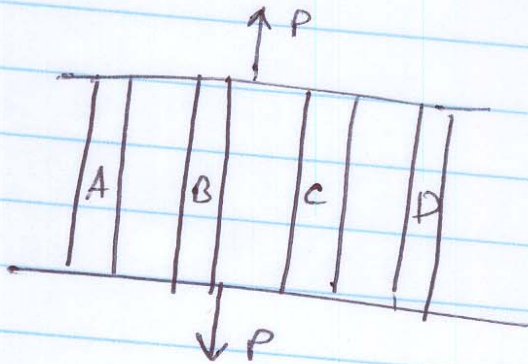
$$n = 0.3$$

Material A:  $K_A = 450 \text{ MPa}$ ,  $A_{0A} = 7 \text{ mm}^2$

B:  $K_B = 600 \text{ MPa}$ ,  $A_{0B} = 2.5 \text{ mm}^2$

C:  $K_C = 300 \text{ MPa}$ ,  $A_{0C} = 3 \text{ mm}^2$

D:  $K_D = 760 \text{ MPa}$ ,  $A_{0D} = 2 \text{ mm}^2$



At maximum tension prior to necking,  
 $\epsilon = n$

$$P = \sigma_A \cdot A_{A \text{ final}} + \sigma_B \cdot A_{B \text{ final}} + \sigma_C \cdot A_{C \text{ final}} + \sigma_D \cdot A_{D \text{ final}}$$

Where  $A_{\text{final}}$  is the area before necking

$$\sigma_A = K_A \epsilon^n$$

$$\sigma_B = K_B \epsilon^n$$

$$\sigma_C = K_C \epsilon^n$$

$$\sigma_D = K_D \epsilon^n$$

$$\ln\left(\frac{l_f}{l_i}\right) = \epsilon = \ln\left(\frac{A_0}{A_{\text{final}}}\right)$$

$$\frac{A_0}{A_{\text{final}}} = e^\epsilon$$

$$A_{\text{final}} = A_0 \cdot e^{-\epsilon}$$

$$P = K_A \epsilon^n \cdot A_{0A} e^{-\epsilon} + K_B \epsilon^n \cdot A_{0B} e^{-\epsilon} \\ + K_C \epsilon^n \cdot A_{0C} e^{-\epsilon} + K_D \epsilon^n \cdot A_{0D} e^{-\epsilon}$$

$$P = \epsilon^n \cdot e^{-\epsilon} [K_A \cdot A_{0A} + K_B \cdot A_{0B} + K_C \cdot A_{0C} \\ + K_D \cdot A_{0D}]$$

$$P = \epsilon^n \cdot e^{-\epsilon} [450 \times 7 + 600 \times 2.5 + 300 \times 3 \\ + 760 \times 2] \text{ N}$$

$$P = 0.3^{0.3} \cdot e^{-0.3} [450 \times 7 + 600 \times 2.5 + 300 \times 3 + 760 \times 2] \\ = 3649.78 \text{ N}$$

If  $n$  values were different then the necking will start at the strand with lowest  $n$  value.

2. The total moment,

$$M_t = \int (\tau dA)r$$

From Mathematica,

$$\int_r^{r+t} \int_0^{2\pi} \tau r \, d\theta \, dr$$

$$2\pi r^2 t \tau + 2\pi r t^2 \tau + 2/3 \pi t^3 \tau$$

Since t is small, higher orders are ignored

$$M_t = \int (\tau dA)r = \tau 2\pi r^2 t$$

$$\tau = \frac{M_t}{2\pi r^2 t}$$

Axial stress,

$$\sigma = \frac{F_z}{2\pi r t}$$

$$\tau = M_t / (2\pi r^2 t)$$

$$\sigma = F_z / (2\pi r t)$$

$$\sigma_1 = \sigma/2 + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_3 = \sigma/2 - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = F_z / (4\pi r t) + \sqrt{\frac{F_z^2}{16\pi^2 r^2 t^2} + \frac{M_t^2}{4\pi^2 r^4 t^2}}$$

$$\sigma_3 = F_z / (4\pi r t) - \sqrt{\frac{F_z^2}{16\pi^2 r^2 t^2} + \frac{M_t^2}{4\pi^2 r^4 t^2}}$$

$\sigma_2 = 0$  ; Plane stress state;

You can find maximum shear stress =  $(\sigma_1 - \sigma_3) / 2$

All other problems should be relatively straight forward.