Answer 1

In[1]:= **Final width at h / 2 will be w**

In[2]:= $$\text{pave1} = \frac{1}{w/2}\int_0^{w/2} 1.15\, Y_f\, e^{\frac{2\mu x}{h/2}}\, dx$$

Out[2]= $$\frac{0.575\left(-1. + e^{\frac{2 w \mu}{h}}\right) h\, Y_f}{w\,\mu}$$

The force from exact solution for case1

In[3]:= $F = \text{pave1} * 2\, w * d$

Out[3]= $$\frac{1.15\, d\left(-1. + e^{\frac{2 w \mu}{h}}\right) h\, Y_f}{\mu}$$

The final width will be 2 w

In[4]:= $$\text{pave2} = \frac{1}{w}\int_0^{w} 1.15\, Y_f\, e^{\frac{2\mu x}{h/2}}\, dx$$

Out[4]= $$\frac{0.2875\left(-1. + e^{\frac{4 w \mu}{h}}\right) h\, Y_f}{w\,\mu}$$

The force from exact solution for case2

In[6]:= $F = \text{pave2} * 2\, w * d$

Out[6]= $$\frac{0.575\, d\left(-1. + e^{\frac{4 w \mu}{h}}\right) h\, Y_f}{\mu}$$

The approximate average for case 1 is,

$$\text{pave1} = 1.15\ Y_f\left(1 + \frac{\mu\, w}{h}\right)$$

The approximate average for case 2 is,

$$\text{pave1} = 1.15\ Y_f\left(1 + \frac{2\,\mu\, w}{h}\right)$$

Answer 2

The cavity should be full at the lowest slug height 0.97 L (-3 %)

$$\text{Solve}\left[\frac{\pi}{4}\, 0.15^2\, 0.97\, L == \frac{4}{3}\,\pi\, 0.1^3,\ L\right]$$

$\{\{L \to 0.244368\}\}$

Maximum length of flash can be calculated by,
Max. volume of wire = volume of ball + volume of flash

```
L = 0.244
```

0.244

$$\text{Solve}\left[\frac{\pi}{4}\,0.15^2\,1.03\,L == \frac{4}{3}\,\pi\,0.1^3 + \frac{\pi}{4}\left(D_f^2 - 0.2^2\right)0.005,\; D_f\right]$$

$\{\{D_f \to -0.322914\}, \{D_f \to 0.322914\}\}$

$r_f = 0.323 / 2;$
$r = 0.1;$
$\tau = 60\,000;$
$\mu = 0.1;$
$h = 0.005;$

$$\text{pave} = \frac{2\,\tau}{\pi\left(r_f^2 - r^2\right)} \int_r^{r_f} e^{\frac{2\,\mu\,(r_f - r1)}{h}}\, 2\,\pi\, r1\, dr1$$

476 277.

$$F = \text{pave}\,\pi\, r_f^2$$

39 026.1

Note that 39, 026 psi is overestimation. We could approximate ball and flash separately as well.

#2 (a) First calculate the final part dimensions

$$h_f = \frac{1}{2} h_i = 12.5\,mm$$

$$r_f = \sqrt{\frac{\pi r_i^2 h_i}{\pi h_f}} = \sqrt{\frac{300^2 \times 25}{12.5}} = 424.3\,mm$$

now calculate the sticking/sliding transition

$$r_k = R - \frac{h}{2\mu} \ln \frac{1}{2\mu}$$

$$= 424.3 - \frac{12.5}{2 \times 0.4} \ln \frac{1}{2 \times 0.4}$$

$r_k = 420.8$ mm , so its almost all sticking with a little sliding

plugging into the correct equations

For sliding

$$\frac{P_{ave}}{2Y_{flow}} = \frac{2}{R^2 - r_k^2} \left(\frac{h}{2\mu}\right)^2 \left[\exp\left(\frac{2\mu(R - r_k)}{h}\right) \cdot \left(\frac{2\mu r_k}{h} + 1\right) - \left(\frac{2\mu R}{h}\right) - 1 \right]$$

$$\frac{P_{ave}}{2Y_{flow}} = \frac{2}{424.3^2 - 420.8^2} \left(\frac{12.5}{2 \times 0.4}\right)^2 \left[\exp\left(\frac{2 \times 0.4 (424.3 - 420.8)}{12.5}\right) \times \left(\frac{2 \times 0.4 \times 420.8}{12.5} + 1\right) - \left(\frac{2 \times 0.4 \times 424.3}{12.5}\right) - 1 \right]$$

$$= 1.11$$

for Sticking

$$\frac{P_{ave}}{2\tau_{flow}} = \left(\exp \frac{2\mu}{h}(R-r_k) + \frac{r_k}{3h}\right)$$

$$= \exp\left[\frac{2\times 0.4}{12.5}(424.3-420.8)\right] + \frac{420.8}{3\times 12.5}$$

$$= 12.47$$

We need $2\tau_{flow} = K\varepsilon^n$

$$\varepsilon = \ln\left(\frac{h_i}{h_f}\right) = \ln\left(\frac{25}{12.5}\right) = 0.69$$

$$2\tau_{flow} = 250\,(0.69)^{0.13} = 238.2 \text{ MPa}$$

so,

$$F = P_{slide}A_{slide} + P_{stick}A_{stick}$$

$$= 238.2\left[1.11\times\pi(424.3^2 - 420.8^2) + 12.47\times\pi(420.8)^2\right]$$

$$\underline{\underline{F = 1.65\text{ GN}}}$$

(b) If no friction Then

$$F = 2\tau_{flow}\times \text{Area}$$

$$= 238.2\times\pi(424.3)^2$$

$$= 134 \text{ MN}$$

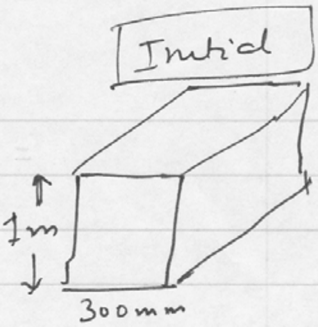
so 1.52 GN extra is required

(c) $\text{Power} = \frac{1}{\text{efficiency}} \times \text{Force} \times \text{velocity}$

$$= \frac{1}{0.4} \times 1.65 \times 10^9 \times \frac{12.5 \times 10^{-3}}{3}$$

$$= \underline{\underline{17.2 \text{ MW}}} = 23{,}066 \text{ Hp}$$

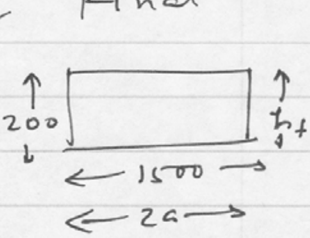
4



3. $Y_f = C(\dot{\epsilon})^m$

$$\dot{\epsilon} = \frac{V}{h}$$

Use final height to get conservative estimate of $\dot{\epsilon}$.



$h_i = 1m$ $\mu = 0.15$

$C = 45 MPa$

$\omega_i = 300 mm$

$l = 2m$

$V = 10 m/min$

$h_f = 200 mm$

Use volume constancy.

$$h_i \times \omega_i \times \cancel{l} = h_f \times \omega_f \times \cancel{l}$$

$$1000 mm \times \cancel{300} mm = \cancel{200} mm \times \omega_f$$

$$\omega_f = 1500 mm$$

$$\boxed{\omega_f = 2a}$$

$\cancel{\frac{Y}{h_f}} \cancel{=}$

$$x_K = a - \frac{h_f}{2\mu} \ln\left(\frac{1}{2\mu}\right)$$

$$= 750 - \frac{200}{2 \times 0.15} \ln\left(\frac{1}{0.3}\right)$$

$= -52.6 < 0$

$\therefore$ No sticking

$$\dot{\epsilon} = \frac{V}{h_f} = \frac{10\ m/min \times \frac{min}{60\,sec}}{0.2\,m} = 0.83/s$$

$$\therefore y_f = C\dot{\epsilon}^m = 45\,(0.833)^{0.1} = 44.2\ MPa$$

Assume plane strain (Average Pressure)

$$\overline{\sigma_y} = 1.15\, y_f \left(1 + \frac{\mu a}{h}\right)$$

$$= 1.15\,(44.2)\left(1 + \frac{0.15\,(0.75)}{0.2}\right)$$

$$= 79.4\ MPa$$

$$F_{Avg} = \overline{\sigma_y}\,(2)\,(1.5) = 79.4 \times 2 \times 1.5$$

$$= 238.2\ MN$$

(or)

By using Actual Pressure

$$F = \underbrace{1.15\, Y_f}_{\text{Plane strain}} \left(\frac{h}{2\mu a}\right)\left[\exp\left(\frac{2\mu a}{h}\right) - 1\right] 2 \times \frac{w}{2} \cdot \text{depth}$$

$$= 1.15 \times 44.2 \times 10^6 \times \left(\frac{200}{2 \times 0.15 \times 750}\right)\left[\exp\left(\frac{2 \times 0.15 \times 750}{200}\right) - 1\right] \times 1.5 \times 2$$

$$F_{forging} = \underline{\underline{281.75}} \text{ MN}$$

$$P_x = 1.15\, Y_f \, e^{\left(\frac{2\mu x}{h}\right)}$$

At the centre

$$P_x = 1.15 \times 44.2 \times 10^6 \times e^{\left(\frac{2 \times 0.15 \times 750}{200}\right)}$$

$$= 156.4 \text{ MPa}$$

∴ For a Safety factor of 3

$$\sigma_y = 3 \times 156.4 = \underline{\underline{469.2}} \text{ MPa}$$