(1) $\mu^2 \cdot R = (\Delta h)_{max}$

$\therefore \Delta h = \mu_{min}^2 \cdot R$

$\Delta h = 1\,mm$

$R = 100\,mm$

$\mu_{min} = \sqrt{\frac{\Delta h}{R}} = \sqrt{0.01} = 0.1$

(b) $\alpha = \tan^{-1}(0.1)$

$\alpha = 5.71^\circ$ or $\alpha = 0.1$ rad

Use radians every where.

(c)

$$H = 2\sqrt{\frac{R}{h_f}}\ \tan^{-1}\left(\frac{\sqrt{R}}{\sqrt{h_f}}\cdot\phi\right)$$

$$H_n = \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ln\left(\frac{h_0}{h_f}\right)\right)$$

$$H_0 = 2\sqrt{\frac{R}{h_f}}\cdot\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\cdot\alpha\right)$$

$$H_0 = 2\sqrt{\frac{100}{4}}\ \tan^{-1}\left(\ldots\ 0.1\right)$$

$$H_0 = 2\times 5\times 0.4636 = 4.63$$

(2)

$$H_n = \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ln\left(\frac{h_0}{h_f}\right)\right)$$

$$H_n = \frac{1}{2}\left(4.63 - \frac{1}{0.1}\ln\left(\frac{5}{4}\right)\right)$$

$$H_n = \frac{1}{2}\left(4.13 - 2.23\right)$$

$$= 1.199$$

$$\phi_n = \sqrt{\frac{h_f}{R}}\tan\left(\sqrt{\frac{h_f}{R}}\cdot\frac{H_n}{2}\right)$$

$$\phi_n = \frac{1}{5}\tan\left(\frac{1}{5}\times\frac{1.199}{2}\right)$$

$$\phi_n = .024\ \mathrm{rad} \Rightarrow 1.379^\circ$$

(2) a) $\Delta h_{max} = \mu^2 R = 375 \times (0.2)^2 \simeq 15\ mm$

Feasible

(b) $\epsilon = \ln\left(\frac{h_i}{h_f}\right)$

$= \ln\left(\frac{75}{60}\right) = 0.223$

$\bar{Y_f} = \frac{K\epsilon^n}{n+1} = \frac{800 \times (0.223)^{0.14}}{1.14} = 568.83\ MPa$

$\bar{Y_f}' = 1.15 \times 568.83 = 654.16\ MPa$

$L = \sqrt{R \cdot \Delta h}$

$= \sqrt{375(75-60)}\ mm$

$= 75\ mm$

$h_{av} = \frac{60+75}{2} = 67.5\ mm$

$F = \bar{Y_f}' \times L \times w \left(1 + \frac{\mu L}{2 \times h_{av}}\right)$

$F = 654.16 \frac{N}{mm^2} \times 75\ mm \times 250\ mm \times \left(1 + \frac{0.2 \times 75\ mm}{2 \times 67.5\ mm}\right)$

$F = 13.62 \times 10^6\ N$

$T = F \times 0.4L = 408.85 \times 10^3\ N \cdot m$

$Power = T \times \omega$

$\omega = 2N\pi/60$

(3)

$$\text{Power} = 408.85 \times 10^3 \times \frac{2 \times \pi \times 100}{60} = 4.281 \times 10^6 \text{ W}$$

$$= 4.281 \text{ MW}$$

$\therefore$ The process is feasible.

3

For ideal deformation,

$$\sigma_d = u = \int_0^{\epsilon} K t^n \cdot dt = \frac{K\epsilon^n}{n+1} \cdot \epsilon$$

$$\frac{K\epsilon^n}{n+1} \Rightarrow \quad \text{Not Valid}$$

This material is perfectly plastic

$\therefore$ $Y = 30{,}000$

$$\sigma_d = u = \int_0^{\epsilon} Y \cdot d\epsilon \qquad \epsilon = \ln\left(\frac{A_o}{A_f}\right) = 2\ln\left(\frac{D_o}{D_f}\right)$$

$$\sigma_d = 30{,}000\,(\epsilon)$$

$$= 30{,}000 \times 2\ln\left(\frac{0.1}{0.07}\right)$$

$$= 2 \times 10700.24 \text{ psi} = 21{,}400 \text{ psi}$$

$$F = \sigma_d \cdot A_f$$

$$F = 2 \times 10{,}700 \times \frac{\pi}{4} \cdot d_f^2$$

$$= 2 \times 10{,}700.24 \times \overline{\pi} \times (0.07)^2 = \cancel{41.17}\ 82.35 \text{ lb}$$

$$= 21{,}400 \times \frac{3.14}{4} \times$$

For friction,

$$\sigma_d = \overline{Y_f}\left(1 + \frac{\tan\alpha}{\mu}\right)\left[1 - \left(\frac{A_f}{A_o}\right)^{\mu\cot\alpha}\right]$$

(7)

$$\sigma_d = Y\left(1+\frac{\tan\alpha}{\mu}\right)\left(1-\left(\frac{A_f}{A_o}\right)^{\mu\cot\alpha}\right]$$

$$= 30{,}000\left(1+\frac{\tan 15^\circ}{0.1}\right)\left(1-\left(\frac{0.07^2}{0.12}\right)^{0.1\cdot\cot 15^\circ}\right]$$

$$\sigma_d = 25{,}791 \text{ psi}$$

$$F = \sigma_d \cdot A_f$$

$$= 25{,}791 \times \frac{\pi}{4} \times (0.07)^2 = 99.25$$

The friction increases the drawing force.

(4)

Extrusion force is given by

$$P_x = \bar{Y}_f \left(3.414 \ln\left(\frac{D_1}{D_2}\right) + \frac{2x}{D_1} \right)$$

Given

$L = 2m$ $\quad$ $K = 965$

$D_1 = D_0 = 75mm$ $\quad$ $n = 0.19$

$D_2 = D_f = 20mm$

$$\bar{Y}_f = \frac{K \epsilon_1^n}{n+1}$$

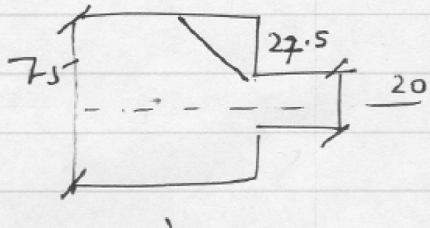
$$\epsilon = \ln\left(\frac{A_0}{A_f}\right) = 2 \ln\left(\frac{D_0}{D_f}\right) = 2 \ln\left(\frac{75}{20}\right)$$

$$\epsilon = 2.6435$$

$$\bar{Y}_f = \frac{965 \times (2.635)^{0.18}}{1.18} = 973.94 \text{ MPa}$$

$x = 2 - .0275$

$= 1.972m$



$$P_x = 973.94 \times 10^6 \times \left[3.414 \ln\left(\frac{75}{20}\right) + \frac{2 \times 1.972}{75 \times 10^{-3}} \right]$$

$= 55.62 \times 10^9 \simeq 55.62$ GPa

$F = P \times Area_{initial} = P \cdot A_0$

$= 55.6 \times 10^9 \times \frac{\pi}{4}\left(\frac{.075}{}\right)^2$

$= 246$ MN

Power $= F \times V = 246 \times 10^6 \times 1.5 = 368$ MW

(b) If the die wears 10%

$D_f = 22$ mm

$\epsilon = 2 \ln\left(\frac{22}{75}\right) = 2.45$

$Y_f' = \frac{965 \times (2.45)^{0.19}}{1.19} = 961.4$

$X = 2000 - \frac{75-22}{2} = 1973.50$

$P_x = 961.4 \left[3.414 \ln\left(\frac{75}{22}\right) + 2 \times \frac{1973.5}{75}\right]$

$= 54.6$ GPa

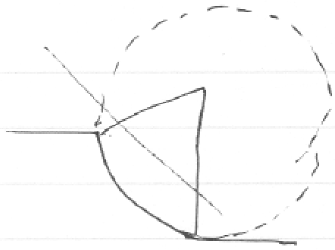
$F = 54.6 \times 10^9 \times \left(\frac{\pi}{4} 0.075^2\right) = 241$ MN

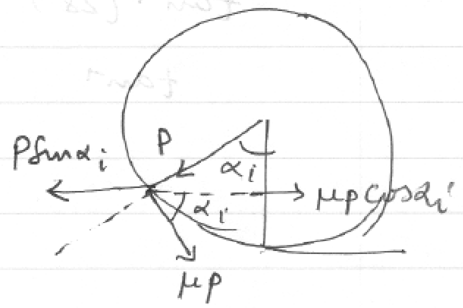
Power $= 241 \times 10^6 \times 1.5 = 362$ MW

So maximum Power is reduced.

Exam 2 solution:

2





Take a small angle α_i

$\mu p \cdot \Delta l \cos\alpha_i \cdot w \simeq$ Friction force pulling the sheet in

$p \cdot \Delta l \sin\alpha_i \cdot w$ = pressure component pulling the sheet out

For unaided bite, i.e., rolling without any external force,

$\mu p \Delta l \cos\alpha_i \cdot w \geq p \cdot \Delta l \sin\alpha_i \cdot w$

$\mu \geq \tan\alpha_i$

$\mu_{min} = \tan\alpha_i$

$\alpha_i = \dfrac{\text{arc}}{\text{radius}}$

For small α_i

$\alpha_i \simeq \tan\alpha_i$, Arc $\simeq L = \sqrt{R \cdot \Delta h}$

$\tan\alpha_i = \alpha_i = \dfrac{L}{R} = \dfrac{\sqrt{R \cdot \Delta h}}{R}$

$\therefore \mu = \frac{\sqrt{R \cdot \Delta h}}{R}$

$\mu^2 = \frac{\cancel{R} \cdot \Delta h}{\cancel{R^2} R}$

$\mu^2 R = \Delta h$

$\mu^2 R = \Delta h|_{max}$

$Power = T \cdot \omega$

$T = F \cdot L/_2$

$\omega = \frac{2 \cdot N \cdot \pi}{60}$

$Power = \frac{F \cdot L}{2 \times 60} \times 2 \cdot N \cdot \pi$

Given Power = 100 kW

F = 1000 KN

N = 60 RPM

$\cancel{100}^{10} \times \cancel{1000} = \frac{\cancel{1000} \times \cancel{1000} \times L \times \cancel{60 \times 2\pi}}{\cancel{120}}$

$L = \frac{1}{10\pi} = .0318 \text{ m} = 31.8 \text{ mm}$

$\sqrt{R \cdot \Delta h} = .0318$ — (i)

$\mu^2 R = \Delta h$ — (ii)

$\sqrt{\mu^2 R^2} = .0318$ & $\mu = 0.1$

$\Rightarrow \mu R = .0318 \therefore R = 0.318 \text{ m} = 318 \text{ mm}$

$R = 0.318$

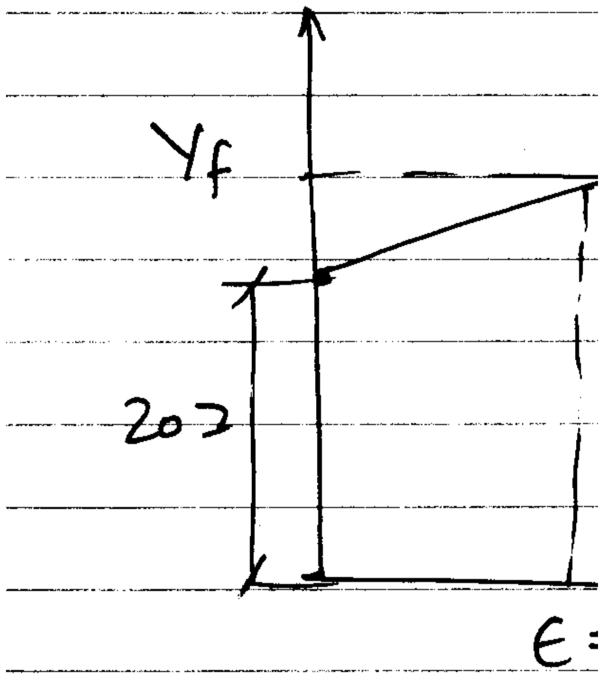
$\Delta h = \mu^2 R$

$\Delta h = (0.1)^2 \times 0.318 = 0.00318 = 3.18 \text{ mm}$

$h_i - h_f = 3.18$

$h_f = 4.0 - 3.18 = 0.82 \text{ mm}$

$$F = \bar{Y}_f' \left(1 + \frac{\mu L}{2 h_{ave}}\right) L \times w$$



σf = Yf = 207 + 414 ε; Use the value of strain, ε =1.58
Yf_aver = (207 + 861)/2 = 534.06 MPa
Yf_aver" = 1.15 Yf_aver and maximum value of Force = 10^6. The only unknown is width, which can be calculated