

# Diffusion model



# Developing diffusion model:

- kinetic strength of the heat cycle
- The transformation of pearlite to austenite
- The homogenization of austenite
- Volume fraction of martensite
- Hardness of transformed surface layer



## • kinetic strength of the heat cycle:

- Structural changes are diffusion controlled. Transformation of pearlite to austenite , homogenization of carbon in austenite and the decomposition of austenite to ferrite and pearlite.
- Extent of changes depends on kinetic strength of heat cycle.
- Kinetic strength of the heat cycle is given by,

$$I = \int_0^{\infty} \text{Exp}^{-\frac{Q}{R \cdot T(p)}} dt$$

Where Q = activation energy for transformation, R = gas constant.

Simplifying we get

$$I = \alpha * \tau * e^{-\frac{Q}{R \cdot T_p}}$$

where

$$\alpha = 3 \sqrt{\frac{R \cdot T_p}{Q}}$$

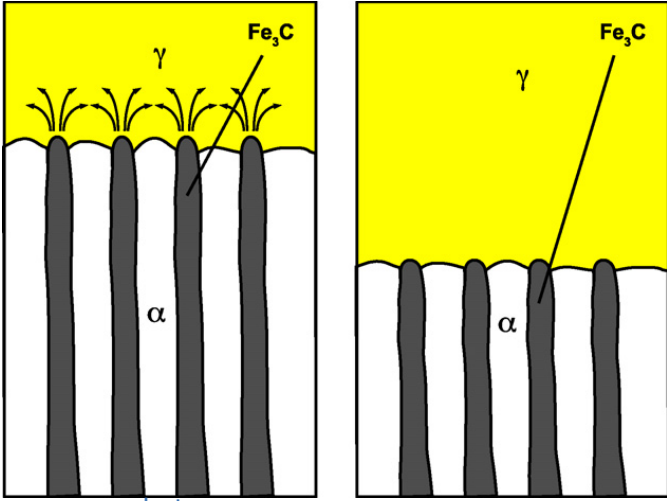
• Where  $T_p$  = peak temperature,  $\tau$  = thermal time constant



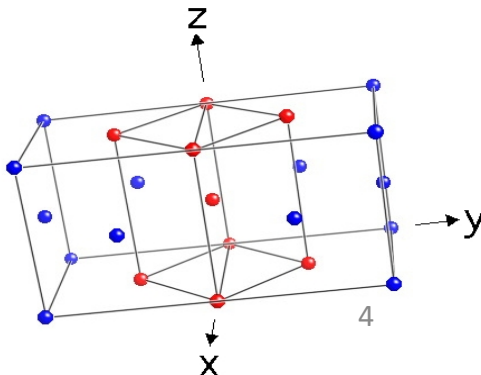
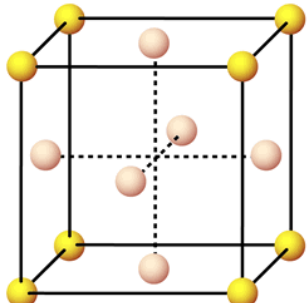
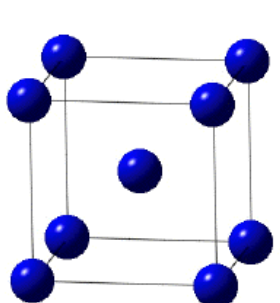
# The transformation of pearlite to austenite

- austenization process is conducted rising the temperature of bulk material 50-90 C above Ac3 temperature
- Pearlite colonies first transform to austenite. Carbon diffuses outward from these transformed zones into surrounding ferrite.

Ferrite (BCC)  
↓  
Austenite (FCC)  
↓  
Martensite(BCT)



$\lambda$



# • Formulation

- If the pearlite spacing within a colony is  $\lambda$ , carbon required sufficient time for lateral diffusion. This time is given by,

$$\lambda^2 = 2Dt$$

- In heat cycle the quantity  $Dt$  is replaced by,

$$D \cdot t = \int_0^{\infty} D_0 \text{Exp}^{-\frac{Q}{R \cdot T(t)}} dt = D_0 \alpha * \tau * e^{-\frac{Q}{R \cdot T}}$$

$$D \cdot t = D_0 \alpha \tau \exp\left[-\frac{Q}{R \cdot T_p}\right]$$

So that,  $\lambda^2 = 2 D_0 \alpha * \tau * e^{-\frac{Q}{R \cdot T_p}}$

where  $D_0$  is pre-exponential C-diffusion in austenite.



# Pearlite Dissolution contd.

$$\lambda^2 = 2D_0\alpha\tau \exp\left[-\frac{Q}{R^*T_p}\right] \quad \alpha = 3 \times \sqrt{\frac{R.T_p}{Q}}$$

- To determine the processing condition that will give the peak temperature,  $T_p$  and thermal time constant  $\tau$ .

The peak temperature is a function of interaction time,  $t$ , and the time to diffuse till the beam radius  $t_0$  where

$$t = \frac{r_b}{V}; t_0 = \frac{r_b^2}{4a}$$

If  $t \ll t_0$ , the peak temperature and thermal time constant is given by

$$T_{p1} = T_0 + \left(\frac{2}{e}\right)^{1/2} \frac{\eta P}{\pi \rho C_p r_b V (z + z_{01})}$$

$$\tau_1 = \frac{1}{\pi^2 k e \rho C_p} \left[ \frac{\eta P}{V r_b (T_p - T_0)} \right]^2$$

# Pearlite Dissolution contd.

- If  $t \gg t_0$ , the peak temperature and thermal time constant is given by

$$T_{p2} = T_0 + \frac{2\eta P}{\pi e \rho C_p V (z + z_{02})^2}$$

$$\tau_2 = \frac{1}{2\pi k e V} \left[ \frac{\eta P}{(T_{p2} - T_0)} \right]$$

$$z_{01}^2 = \frac{\pi a r_b}{2eV}; z_{02}^2 = \frac{r_b}{e} \left( \frac{\pi a r_b}{V} \right)^{1/2}$$

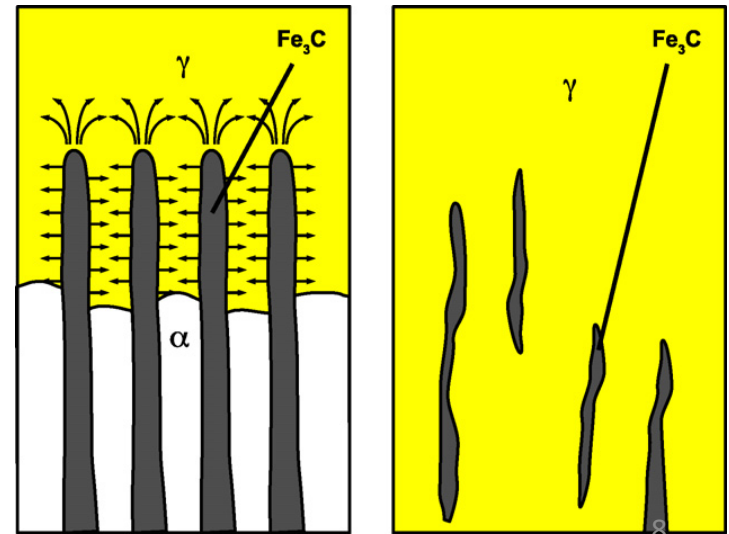
where P is power,  $r_b$  is the beam radius,  $\eta$  is absorptivity, V is scan velocity, a is diffusivity, k is conductivity,  $\rho$  is density Cp is specific heat

- The homogenization of austenite
- Modeling carbon redistribution in austenite.
- Carbon diffuses from the high to the low concentration regions, which depends on temperature and time.
- The boundary region where carbon % increased is given by  
(This region is converted to martensite)

$$x = \frac{2}{\sqrt{\pi}} \ln \left( \frac{C_e}{2C_c} \right) \sqrt{Dt}$$

$$D \cdot t = D_0 \alpha \tau \exp \left[ - \frac{Q}{R^* T_p} \right]$$

Where  $C_e$  = austenite C % (0.8%),  
 $C_c$  = critical carbon content C % (0.05%)





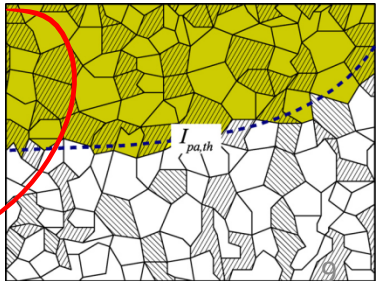
- Volume fraction of martensite:
- Extent of the martensite which forms when the surface layer is quenched.
- Volume fraction of martensite depends on grain size and volume fraction of pearlite colonies.
- Maximum volume fraction permitted by the phase diagram is,

$$\begin{aligned}
 f_m &= 0 && \text{if } T_p < A1 \\
 f_m &= f_i + (1 - f_i) \frac{T_p - A1}{A3 - A1} && \text{if } A1 < T_p < A3 \\
 f_m &= 1 && \text{if } T_p > A3
 \end{aligned}$$

- Volume fraction of martensite

$$f = f_m - (f_m - f_i) \exp \left[ - \left[ \frac{12 f_i^{2/3}}{\sqrt{\pi} g} \ln \left( \frac{C e}{2 C c} \right) \sqrt{D t} \right] \right]$$

Where  $f_i$  = volume fraction of pearlite =  $C/0.8$



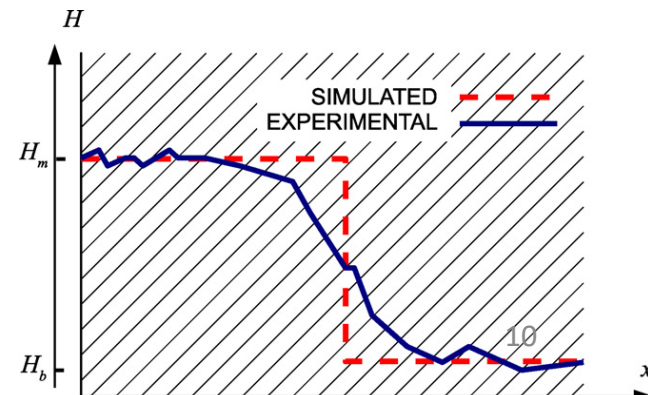
- Hardness of transformed surface layer
- The Vickers hardness of treated surface varies with depth.
- It is also depends on volume fraction of martensite and its carbon content and hardness given by rule of mixtures

$$H = fH_m + (1 - f)H_f$$

Also from carbon content and martensite volume hardness is given by,

$$H = 1667c - 926 \frac{c^2}{f} + 150$$

Where  $H_m$  = hardness of martensite,  
 $H_f$  = hardness of ferrite. = 150MPa.



# Example

- **Material and process variables:**

Material : AISI 4140 steel.

Laser power : 1000 W

Beam and distribution: rectangular(12 x 8 mm) with uniform distribution

Velocity: 2 mm/s

Thermal conductivity : 42.7 W/mk

Diffusivity : 11.24 mm<sup>2</sup>/s

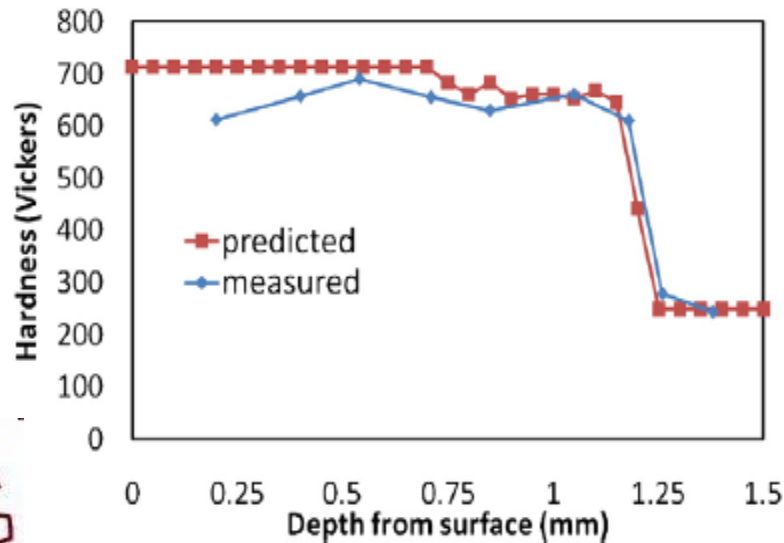
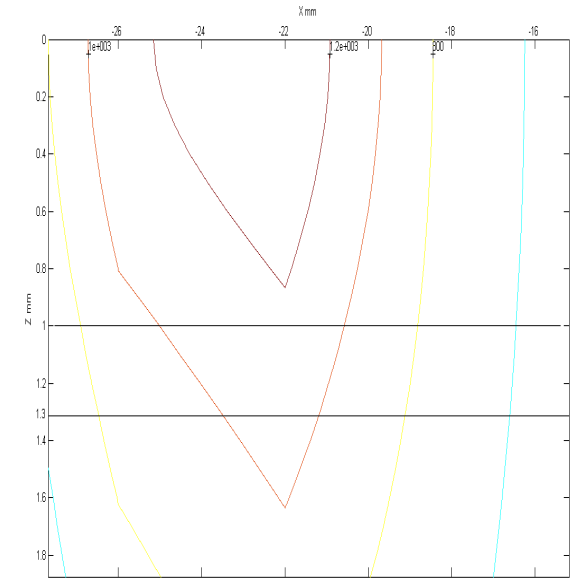
Specific capacity : 473 J/kgk

## Microstructural properties for AISI 4140 steel [33]

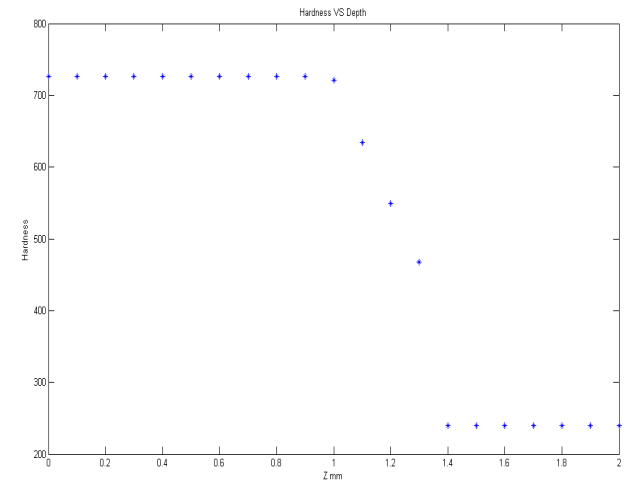
Property	Value
Pre-exponential carbon in ferrite, $D_{\alpha\alpha}$	$6 \times 10^{-5} \text{ m}^2/\text{s}$
Pre-exponential carbon in austenite, $D_{\alpha\gamma}$	$1 \times 10^{-5} \text{ m}^2/\text{s}$
Activation energy ferrite, $Q_{\alpha}$	80 kJ/mol
Activation energy austenite, $Q_{\gamma}$	135 kJ/mol
Universal gas constant, $R$	8.314 J/mol K
Initial volume fraction of pearlite, $f$	0.45
Critical carbon content value, $C_c$	0.05%

# Result

- Temperature profile along depth:
  1. At 1mm temperature above AC3
  2. At 1.3mm temperature above AC1
- Hardness profile along depth:



Maximum hardness = 712 HV



Maximum hardness = 725 HV<sub>12</sub>



# Example for AISI 1050 steel

