ME 677
Laser material Processing
HW#2

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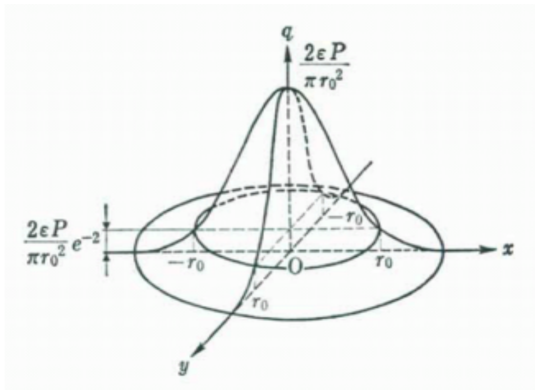
Assigned Date: 09/02/2019
Due Date: 21/02/2019

1. The lasers can be modeled as moving heat sources and they have different spatial modes such as Gaussian distribution (shown in Fig. 1) and uniformly distributed on a rectangle or a circle. The solutions could be found by integrating the quasi-steady state moving point solution over the limits.



Gaussian distribution

The solution for moving point heat source in moving and fixed coordinates in a semi-infinite body are given by:
Eq. (1) and Eq. (2), respectively:

$$dT'(X,Y,Z,t) = \frac{2\delta q}{\rho C(4\pi a(t-t'))^{\frac{3}{2}}} \exp[-\frac{(X-x')^2+(Y-y')^2+(Z)^2}{4a(t-t')}] \qquad (1)$$

$$dT'(x,y,z,t) = \frac{2\delta q}{\rho C(4\pi a(t-t'))^{\frac{3}{2}}} \exp[-\frac{(x-vt'-x')^2+(y-y')^2+(z)^2}{4a(t-t')}] \qquad (2)$$

where, δq = heat (x', y', z'), r = beam radius, k = thermal conductivity, a = k/ρc = diffusivity, ρ = density, c = specific heat capacity. x-y-z are fixed co- ordinates tied to the center of the moving heat source.

Find the solutions for:

a. Gaussian moving disk heat source for a given depth or z:

Also for the given parameters find out the temperature distribution on surface.

Parameters: Q = 1300W, r = 1.5mm, v = 1m/min, t = 0.18sec, α = 5.1 mm/sec, ρ = 0.000008 Kg/mm3, C = 674 J/Kg K. and -50 < (x, y) < 50

(**NOTE: Numerical integration may be required for solving the final equation. Extra credits will be awarded for reaching the correct solution. Feel free to use Mathematica, Maple, Matlab or programming environment of your choice**)

b. How will the answer change if you use Beer-lambert's law and assume that the beam is not acting on the surface but penetrates the sub-surface up to a depth of 0.1 mm. (Hint use a volumetric heat source and solve numerically)

2. With the help of above two cases, Derive the solution for uniform moving circular heat source for a given depth or z:

   Note that in case of Gaussian moving circular heat source, the limits vary from -∞ to ∞ but values die down at certain distance from center. Consequently, in case of circular heat source put appropriate limits.

3. Derive the following solution for Uniform moving rectangular heat source for given depth or z:

$$T - T_0 = \frac{2P}{8(4bl)\rho C\sqrt{\pi a}} \int_0^t \frac{dt'}{\sqrt{(t-t')}} \exp[-\frac{z^2}{4a(t-t')}] \times$$
$$[erf(\frac{x+l+vt'}{\sqrt{4a(t-t')}}) - erf(\frac{x-l+vt'}{\sqrt{4a(t-t')}})][erf(\frac{y+b}{\sqrt{4a(t-t')}}) - erf(\frac{y-b}{\sqrt{4a(t-t')}})]$$

   where rectangular heat source dimensions varies from $-l < x < l$ and $-b < y < b$
   For numerical integration use the parameters given above.

4. Derive the solution for moving elliptical heat source for a given depth (z).
   Hint: Intensity distribution for elliptical distribution is given by,

$$I(x', y') = \frac{2*P}{A}[1 - \frac{y'^2}{b^2}][1 - \frac{x'^2}{a^2\left(1-\frac{y'^2}{b^2}\right)}]$$

where P=laser power, A=area of ellipse, the bounds of both the rectangle and ellipse are $-a \le x' \le a$ and $-b \le y' \le b$.

5. A hemispherical volumetric moving heat source in a semi-infinite body (semi-infinite in z-axis) encountered during laser surface hardening is given by: $Q_v(x, y, z) = Q_m \text{ Exp}[-k(x^2 + y^2 + z^2)$; where Q_v is volumetric power density (Power/volume) at any given location (x, y, z). Prove that it is same as a spherical heat source an infinite body if there is no heat transfer at the plane where the sphere is cut into two hemispheres. Use this condition to derive the solution.

   At the radius of the hemisphere, r, the volumetric power density drops to 5% of the maximum heat intensity, Q_m. Formulate the solution for moving volume (Do not solve.)

6. Derive the equation of temperature rise under steady state conditions for infinite line heat source. Determine the expression for maximum temperature rise and its location, can it be evaluated? A steel plate is welded using a high power laser at a speed of 40 mm/s. The material has a thickness of 5 mm. Determine the power required for a weld bead of 2 mm and determine the extent of HAZ. The weld is formed al through the plate thickness. Assume density = 7870 kg/m3; specific heat = 452 J/kg K; thermal conductivity = 0.073 W/mm K; melting temperature is 1538deg C. Assume transformation temperature to be 850 deg C.

7. Derive the flux formulation for one dimensional heat conduction.