

Laser Optics-II



ME 677: Laser Material Processing
Instructor: Ramesh Singh

Outline

- Absorption
- Modes
- Irradiance



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Reflectivity/Absorption

- Absorption coefficient will vary with the same effects as the reflectivity
- For opaque materials:
 - $\text{reflectivity} = 1 - \text{absorptivity}$
- For transparent materials:
 - $\text{reflectivity} = 1 - (\text{transmissivity} + \text{absorptivity})$



Reflectivity

- In metals the radiation is predominantly absorbed by free electrons in an “electron gas”
- Free electrons are free to oscillate and reradiate without disturbing the solid atomic structure
- The reflectivity increases from visible to high wavelength
- As a wavefront arrives at a surface all the free electrons in the surface vibrate in phase generating an electric field 180° out of phase with the incoming beam
- The sum of this field will be a beam whose angle of reflection equals the angle of incidence



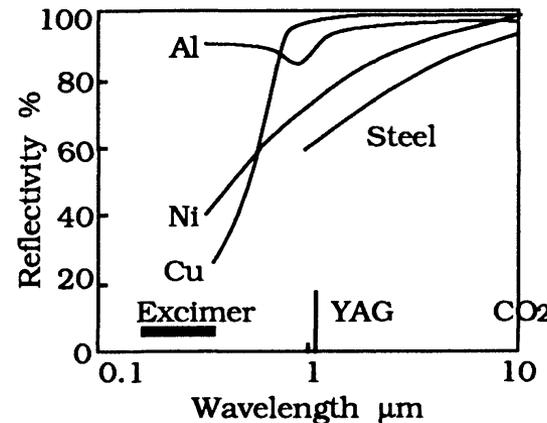
Effect of Wavelength

- Reflectivity is a function of the refractive index, n , and the extinction coefficient, k

$$R = \frac{\{(1-n)^2 + k^2\}}{\{(1+n)^2 + k^2\}}$$

$$A = 1 - R$$

$$A = \frac{4n}{\{(n+1)^2 + k^2\}}$$



Complex refractive index and reflection coefficient for some materials to 1.06μm radiation (8).

Material	k	n	R
Al	8.50	1.75	0.91
Cu	6.93	0.15	0.99
Fe	4.44	3.81	0.64
Mo	3.55	3.83	0.57
Ni	5.26	2.62	0.74
Pb	5.40	1.41	0.84
Sn	1.60	4.70	0.46
Ti	4.0	3.8	0.63
W	3.52	3.04	0.58
Zn	3.48	2.88	0.58
Glass	0	1.5	0.04

- At shorter wavelengths, the more energetic photons can be absorbed by a greater number of bound electrons
 - Reflectivity decreases and absorptivity increases



Reflectivity of Metals

Table 1. Reflectivity of Metals

Metal						
Wavelength (μm)	Copper	Nickel	Steel (99% Fe, 1% C)	Tungsten	Chromium	Silver
0.251	0.259	0.378	0.38	0.15	0.32	0.341
0.305	0.253	0.442	0.44	0.25	0.37	0.091
0.357	0.273	0.488	0.50	0.28	0.41	0.745
0.500	0.437	0.608	0.560	0.493	0.55	0.927
0.700	0.834	0.688	0.580	–	0.56	0.946
1.0	0.901	0.720	0.63	0.623	0.57	0.964
2.0	0.955	0.835	0.77	0.846	0.63	–
3.0	0.971	0.887	0.83	0.905	0.70	0.981
5.0	0.979	0.944	–	0.940	0.81	0.981
9.0	0.984	0.956	0.93	0.905	0.92	–

Source: Adapted from G. G. Gubareff, J. E. Janssen, and R. H. Torborg. *Thermal Radiation Properties Survey*, 2nd ed., Minneapolis Honeywell Regulator Co., Minneapolis, MN, 1960.



Reflectivity of Non-metals

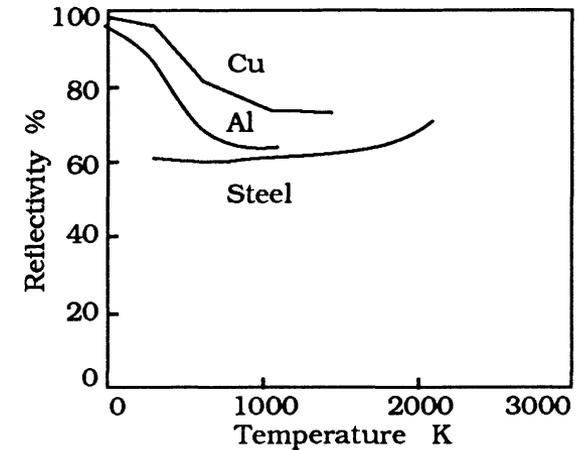
Table 2. Reflectivity of Some Nonmetals

Wavelength (μm)	0.60	4.4	8.8
Asphalt	0.148	-	-
Indiana limestone	0.429	0.203	0.050
Quartz powder	0.810	0.079	0.090
Slate	0.067	0.134	0.200
White marble, unpolished	0.535	0.064	0.051
Black velvet	0.0187	0.037	0.027
MgO white pigment	0.86	0.16	0.03



Effect of Temperature

- Temperature increase results in increase in phonon population and phonon-electron energy exchanges
 - Reflectivity decreases
 - Absorption increases

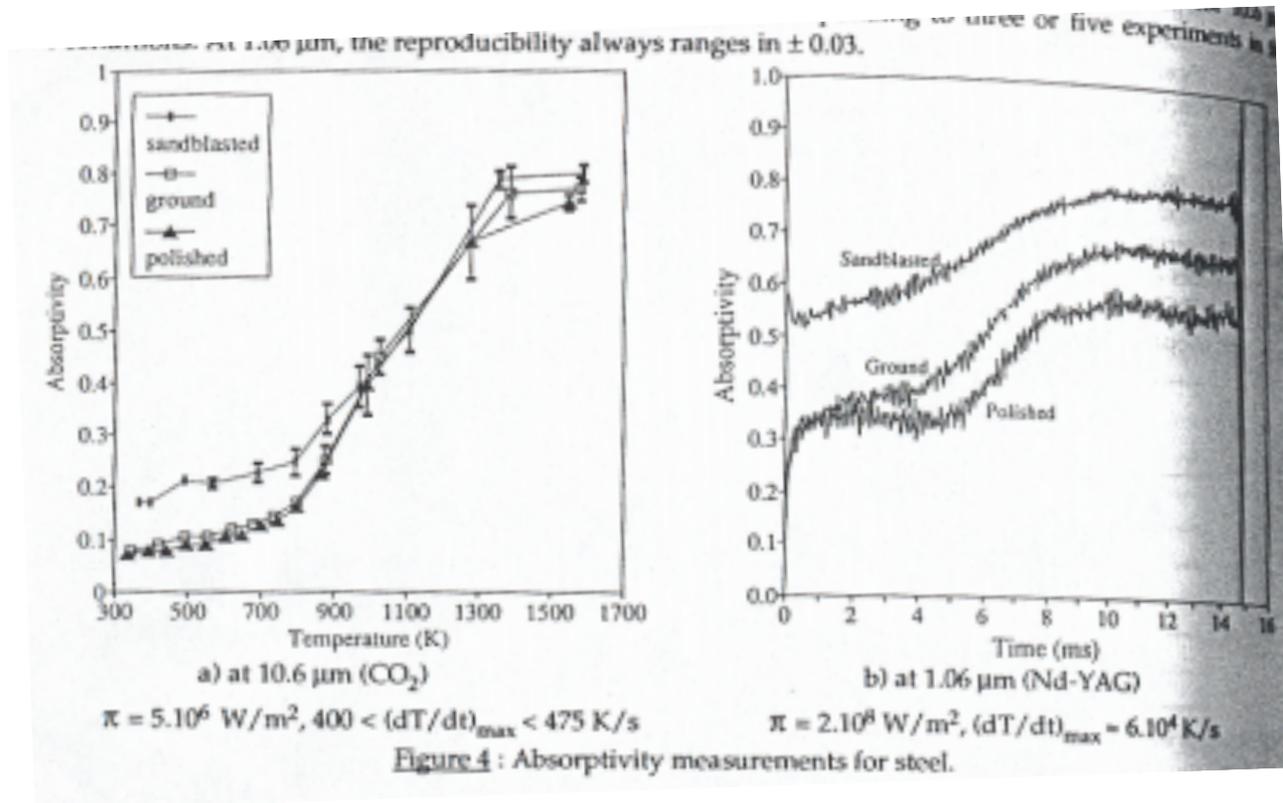


Surface Roughness

- Surface Roughness has a large effect on absorption due to:
 - The multiple reflections in the undulations
 - Also some "stimulated absorption" due to beam interference with sideways reflected
- If roughness is less than the beam wavelength, the light will perceive the surface as flat

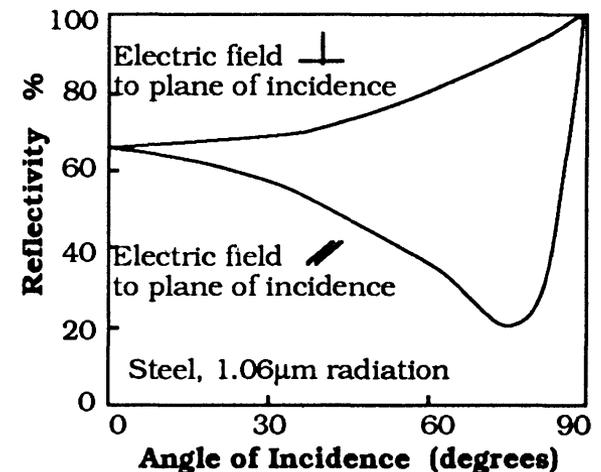


Case Study-1045 steel



Angle of Incidence

- At certain angles the surface electrons may be constrained from vibrating since to do so would involve leaving the surface. This they would be unable to do without absorbing the photon
- The electric vector is in the plane of incidence, the vibration of the electron is inclined to interfere with the surface and absorption is thus high



Refraction

- On transmission the ray undergoes refraction described by Snell's law:

$$\sin\phi/\sin\varphi = n = v_1/v_2$$

n = Refractive index.

ϕ = Angle of incidence.

φ = Angle of refraction.

v_1 = Apparent speed of propagation in medium 1.

v_2 = Apparent speed of propagation in medium 2.



Refraction

- Scattered intensity is a function of $1/\lambda^4$ Rayleigh Scattering Law.
- The normal form of a dispersion curve (refractive index vs wavelength) is known as a Cauchy Equation:

$$n = A + B/\lambda^2 + C/\lambda^4.$$



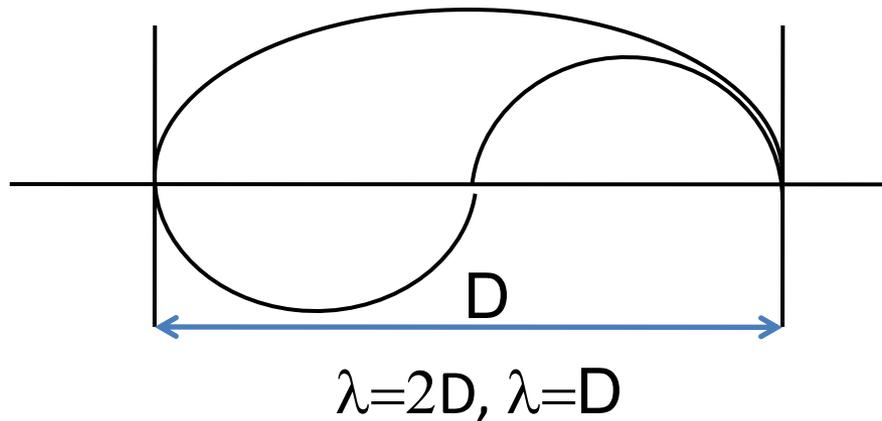
Beam Mode

- Two spatial modes describe the beam
 - Longitudinal
 - Transverse
- Essentially independent of each other
 - Transverse dimension in a resonator is normally considerably smaller than the longitudinal
- The standing wave condition will be amplified, i.e., there can be only integer number of half wavelengths in the cavity,
 - $D = q \cdot \lambda/2$ or $q\lambda = 2D$
 - q is a large integer referring to the number of nodes in the longitudinal standing, D is the cavity length (mirror separation), and λ is the wavelength.
- The longitudinal mode number is large in industrial lasers and is normally ignored on beam characteristics and performance.
- The transverse electromagnetic mode (TEM) is more important.



Longitudinal Mode

- Longitudinal Mode (integral multiples of $\lambda/2$)



Longitudinal Mode

- The frequency of the axial mode of the cavity,

$$\nu = \frac{qc}{2D}$$

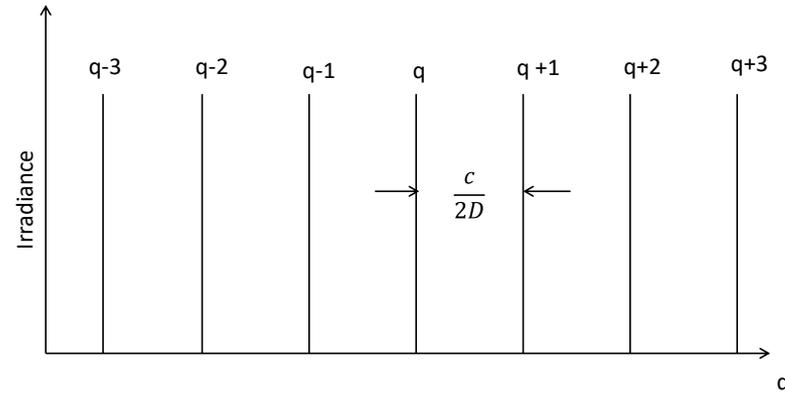
- If λ is replaced by $\frac{c}{\nu}$, where c is the speed of light and ν is the frequency, the frequency separation between two adjacent nodes $\Delta\nu$ between adjacent modes ($\Delta q = 1$) is given by,

$$\Delta\nu = \frac{c}{2D}$$

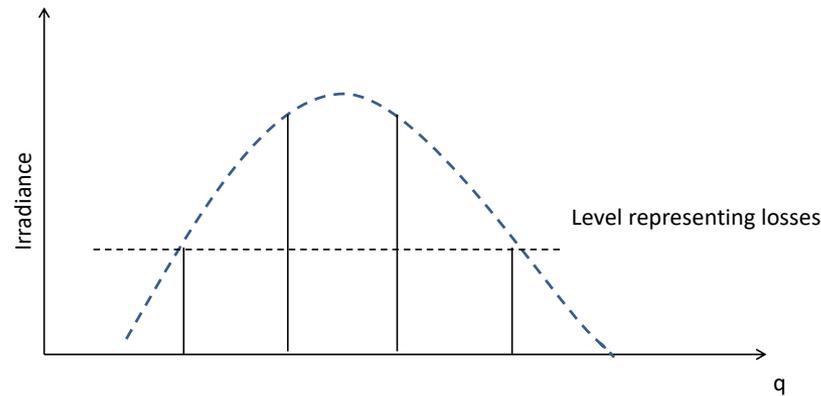
- The axial modes of the laser cavity consists of a large number of frequencies spaced apart by $\frac{c}{2D}$ as illustrated. However, the given mode can oscillate if the gain exceeds the losses at that frequency



Spectral lines due to axial mode



(a)



(b)



Transverse Mode

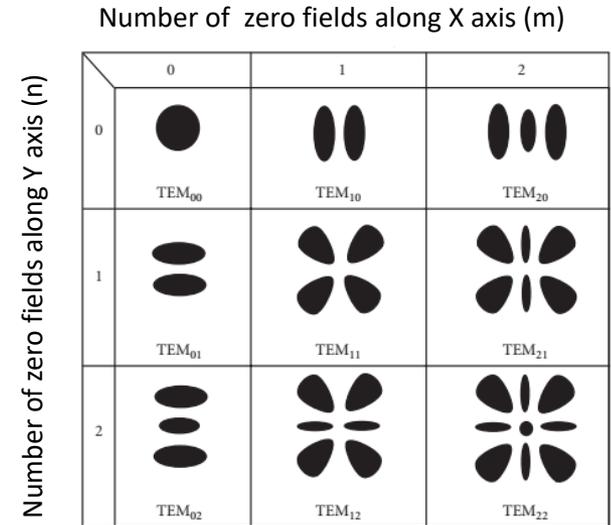
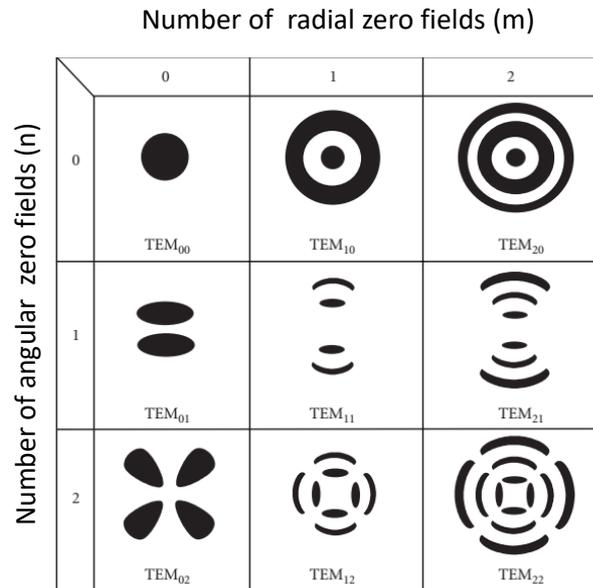
- TEM describes the variation in beam intensity with position in a plane perpendicular to the direction of beam propagation
 - It characterizes the intensity maxima in the beam
- The TEM is determined by:
 - The geometry of the cavity
 - Alignment and spacing of internal cavity optics
 - Gain distribution and propagation properties of the active medium
 - Presence of apertures in the resonator

$$P(r,\phi) = E^2(r,\phi)$$

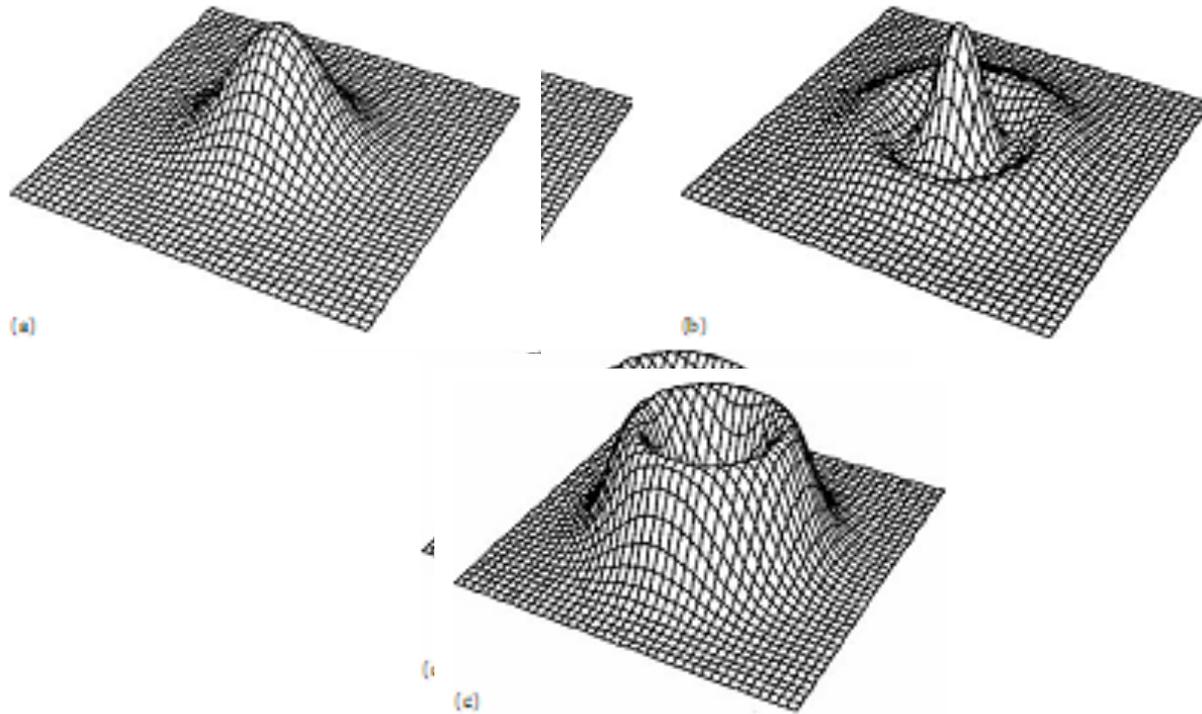


TEM

- Rectangular Modes
 - X axis
 - Y axis
- Circular Modes TEM_{pl}
 - Radial, p
 - Angular, l



Intensity Plots



a. TEM₀₀; b. TEM₁₀; c. TEM₀₁*



Propagation...

- Generic equation for propagation:

$$E(x, y, z)$$

$$= E_{mn} H_m \left(\frac{\sqrt{2} x}{w(z)} \right) H_n \left(\frac{\sqrt{2} y}{w(z)} \right) \frac{w}{w(z)} e^{-\left(\frac{x^2 + y^2}{w^2} \right)} e^{-j \left(kz - (1+m+n) \tan^{-1} \left(\frac{z}{z_R} \right) \right)} e^{-j \left(\frac{kr^2}{2R(z)} \right)}$$

Equations for Hermite Gaussian beam in (x-y coordinates) at beam waist (z=0)

$$E(x, y) = E_0 H_m \left(\frac{\sqrt{2} x}{w} \right) H_n \left(\frac{\sqrt{2} y}{w} \right) e^{-\left(\frac{x^2 + y^2}{w^2} \right)}$$

where

E = Electric _ field _ amplitude

E_0 = No min al _ amplitude

x = x _ dis tan ce _ from _ axis

y = y _ dis tan ce _ from _ axis

w = No min al _ beam _ radius

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Propagation...

- $H_n(x)$ is a Hermite polynomial

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

$$H_0(x) = 1$$

$$E(x, y) = E_0 \cdot e^{-\left(\frac{x^2 + y^2}{w^2}\right)}$$

Circular _ coordinates,

$$E(r) = E_0 e^{\left(\frac{-r^2}{w^2}\right)}$$

$$I(r) \propto E(r)^2$$

$$I(r) = I_0 e^{\left(\frac{-2r^2}{w^2}\right)}$$

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Gaussian Beam

- Gaussian function goes out to infinity
- Low powered lasers mimic the TEM_{00}
- TEM_{00} beams can be focused to smallest spot as compared to any other distribution



Beam Properties

- The point where irradiance drops to $1/e^2$ of the peak
- The radius containing $1 - 1/e^2$ power
- A two dimensional plot, the x value of which 95% of the plot area is contained between x and $-x$.

$$\text{In[1]:= } \int_0^w I_0 e^{-2r^2/w^2} 2\pi r dr$$
$$\text{Out[1]= } \frac{(-1 + e^2) I_0 \pi w^2}{2 e^2}$$



Gaussian Distribution

For different gaussian beams:

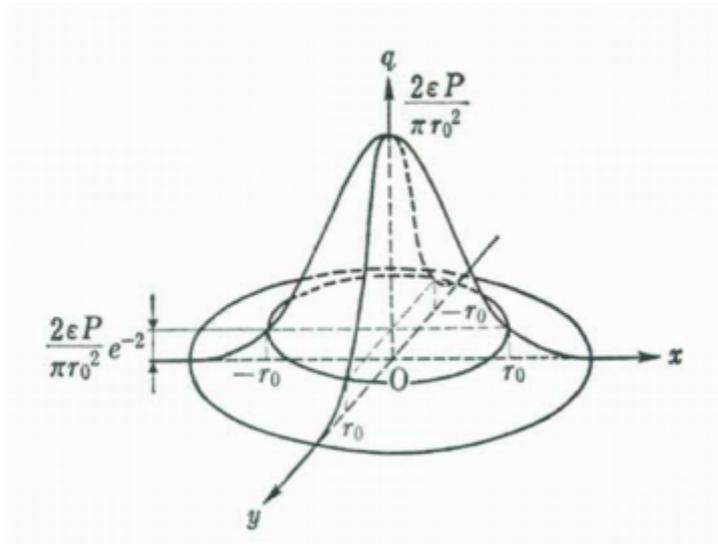
$$I(r) = I_0 e^{\left(-\frac{2r^2}{w^2}\right)}$$

$$I_0 = \frac{2P}{\pi w^2}$$

Other beam,

$$I(r) = I_0 e^{\left(-\frac{9r^2}{w^2}\right)}$$

$$I_0 = \frac{9P}{\pi w^2}$$



Equations for a generic intensity

Intensity $I(x, y, z)$ propagating along z axis. The second moments in x and y axes at a given location, z , can be defined as

$$\sigma_x^2(z) = \frac{\iint (x-\bar{x})^2 I(x,y,z) dx dy}{\iint I(x,y,z) dx dy}$$

$$\sigma_y^2(z) = \frac{\iint (y-\bar{y})^2 I(x,y,z) dx dy}{\iint I(x,y,z) dx dy}$$

The centroids, \bar{x} and \bar{y} are defined by,

$$\bar{x} = \frac{\iint x I(x,y,z) dx dy}{\iint I(x,y,z) dx dy}$$

$$\bar{y} = \frac{\iint y I(x,y,z) dx dy}{\iint I(x,y,z) dx dy}$$

The beam dimensions (widths) in x and y are given by,

$$D_{\sigma x} = 4 \sigma_x(z)$$

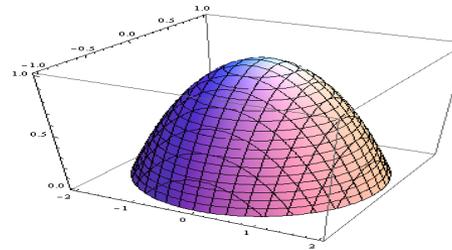
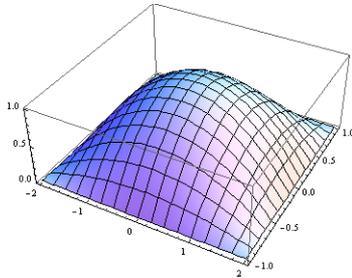
$$D_{\sigma y} = 4 \sigma_y(z)$$



Examples

$$I(x', y') = \frac{2*P}{A} \left[1 - \frac{y'^2}{b^2}\right] \left[1 - \frac{x'^2}{a^2}\right] \quad (1)$$

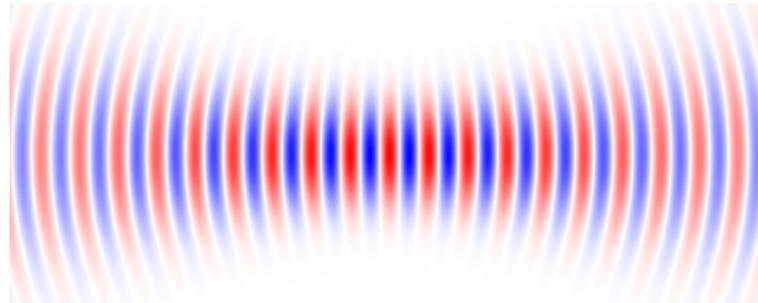
$$I(x', y') = \frac{2*P}{A} \left[1 - \frac{y'^2}{b^2}\right] \left[1 - \frac{x'^2}{a^2 \left(1 - \frac{y'^2}{b^2}\right)}\right] \quad (2)$$



Propagation of laser beam

For a monochromatic beam propagating in z the complex electric field amplitude

$$E(r, z) = E_0 \frac{w_0}{w_z} \text{Exp} \left(-\frac{r^2}{w_z^2} \right) \text{Exp} \left(-i \left[kz - \tan^{-1} \left(\frac{z}{z_R} \right) + \frac{kr^2}{2R_z} \right] \right)$$

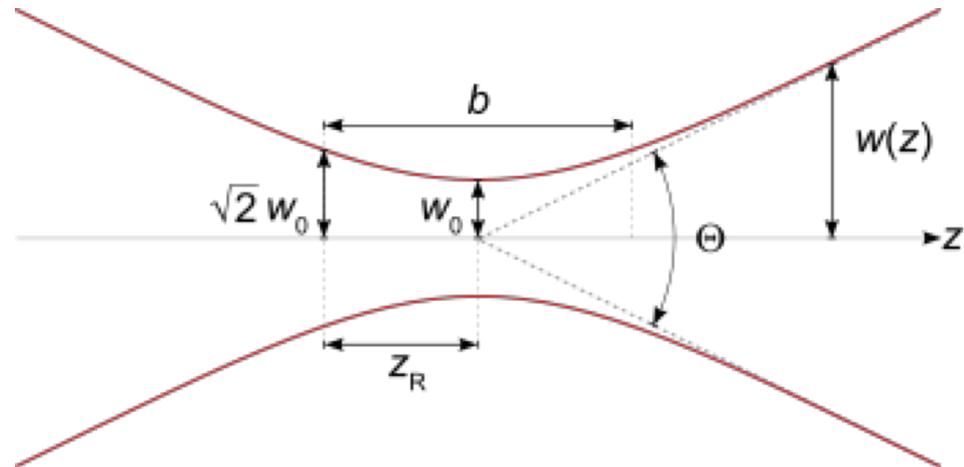


where E_0 is the peak amplitude; w is beam waist radius;
 $k = 2\pi/\lambda$; Z_R is the Rayleigh length; R_z is the radius of curvature
of the wave front



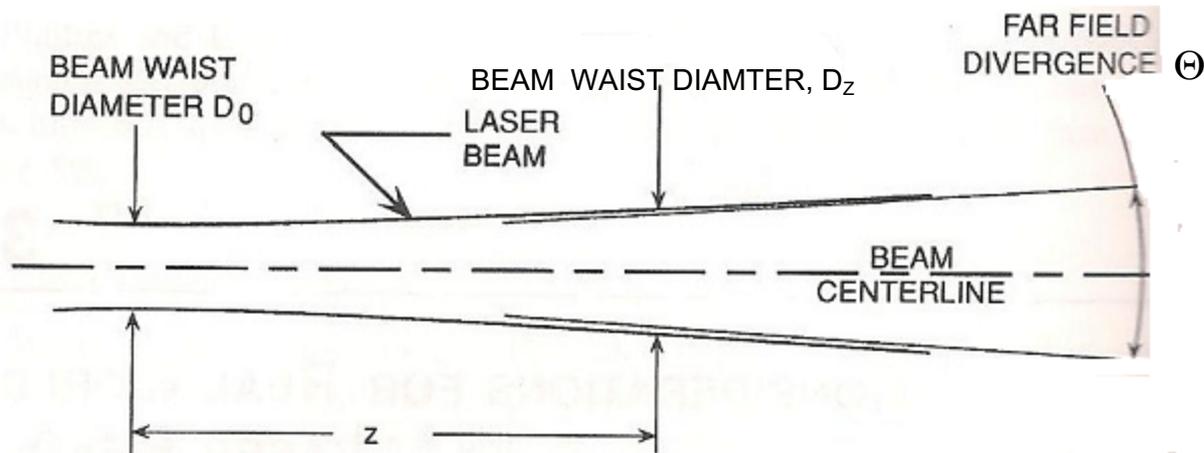
The variation of beam radius in propagation

$$w_z = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}$$
$$z_R = \frac{\pi w_0^2}{\lambda}$$



Propagation of Laser Beams

- A laser beam propagating in space (lower case for TEM_{00} and upper case for real beams)
 - Beam waist or minimum diameter, d_0/D_0
 - Beam waist diameter, d_z/D_z at a location z from the waist
 - Beam waist or minimum radius, w_0/W_0
 - Beam waist radius, w_z/W_z at a location z from the waist
 - θ/Θ = Full-angle beam divergence
 - λ = Wavelength of light



Propagation of Ideal Beam

- For a TEM₀₀ beam, the diameter d_z for any distance z from the waist is a hyperboloid

$$d_z = d_0 \sqrt{1 + \left(\frac{4\lambda z}{\pi d_0^2} \right)^2}$$

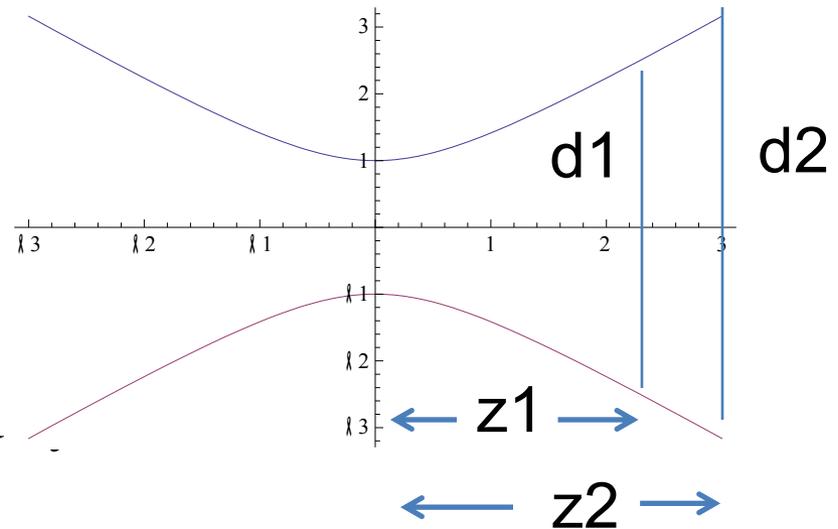
$$d_z^2 = d_0^2 + z^2 \theta^2$$

$$\theta = \frac{4\lambda}{\pi d_0}$$

$$\theta = \text{Divergence}_{TEM_{00}}$$

Divergence can be approximated

$$\theta = 2 \tan^{-1} \left(\frac{d_2 - d_1}{2(z_2 - z_1)} \right)$$



Real Beam

- Real beams can be defined in terms of TEM₀₀
- It can be postulated a fictitious “embedded Gaussian beam” having a smaller dia d exists in the real beam; ∴ D=M.d , where M>1

$$d_z = d_0 \sqrt{1 + \left(\frac{4\lambda z}{\pi d_0^2} \right)^2}$$

$$\frac{D_z}{M} = \frac{D_0}{M} \sqrt{1 + \left(\frac{4.\lambda.z.M^2}{\pi.D_0^2} \right)^2}$$

$$D_z^2 = D_0^2 \left(1 + \left(\frac{4.\lambda.z.M^2}{\pi.D_0^2} \right)^2 \right)$$

$$D_z^2 = D_0^2 + z^2 \left(\frac{4.\lambda.M^2}{\pi.D_0} \right)^2$$

$$D_z^2 = D_0^2 + z^2 \Theta^2$$

$$\Theta = \frac{4.\lambda.M^2}{\pi.D_0} = M^2 \theta$$

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Focused Beam Calculations

For ideal beam,

$$z_2 = f + \frac{(z_1 - f)f^2}{(z_1 - f)^2 + \left(\frac{\pi d_{01}^2}{4\lambda}\right)^2}$$

$$\frac{1}{d_{02}^2} = \frac{1}{d_{01}^2} \left(1 - \frac{z_1}{f}\right)^2 + \frac{1}{f^2} \left(\frac{\pi d_{01}}{4\lambda}\right)^2$$

For real beam with $\Theta = \frac{4.\lambda.M^2}{\pi.D_0}$,

$$z_2 = f + \frac{(z_1 - f)f^2}{(z_1 - f)^2 + \left(\frac{D_{01}^2}{\Theta_1^2}\right)}$$

$$\frac{1}{D_{02}^2} = \frac{1}{D_{01}^2} \left(1 - \frac{z_1}{f}\right)^2 + \frac{1}{f^2 \Theta_1^2}$$

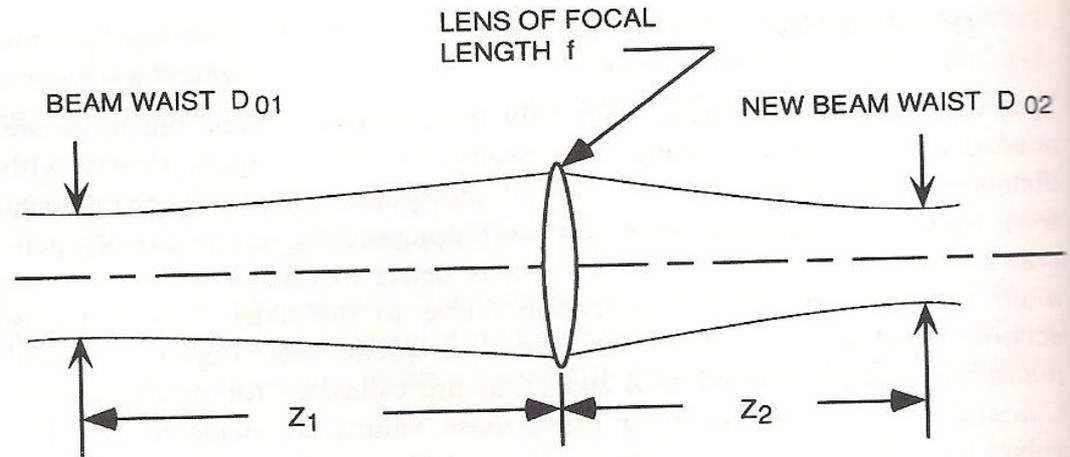


Figure 3.4 Focusing a laser beam.



Final Calculation

- Once D_{02} is calculated, Θ_2 could be found

$$\Theta = \frac{4.\lambda.M^2}{\pi.D_{02}}$$

$$D_{2z}^2 = D_{02}^2 + z^2 \Theta^2$$

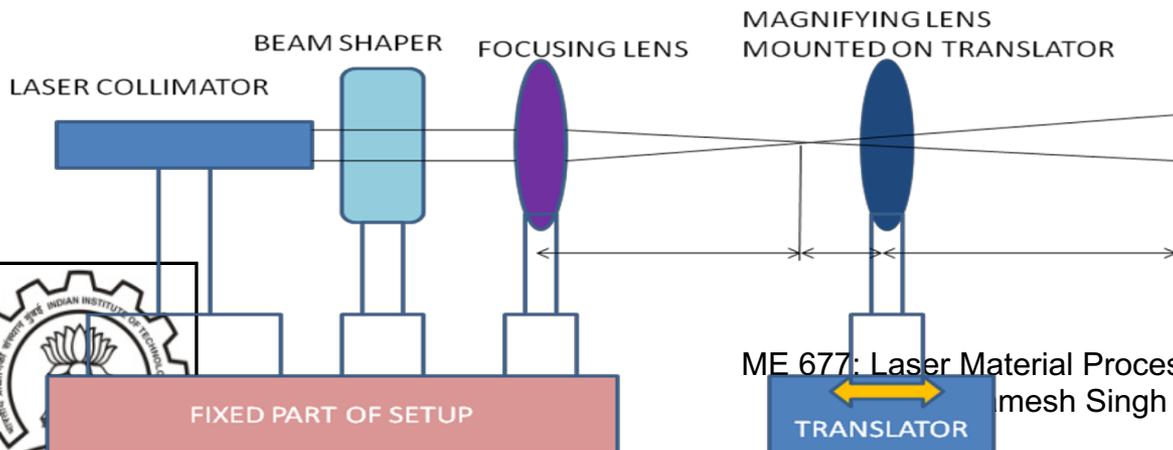
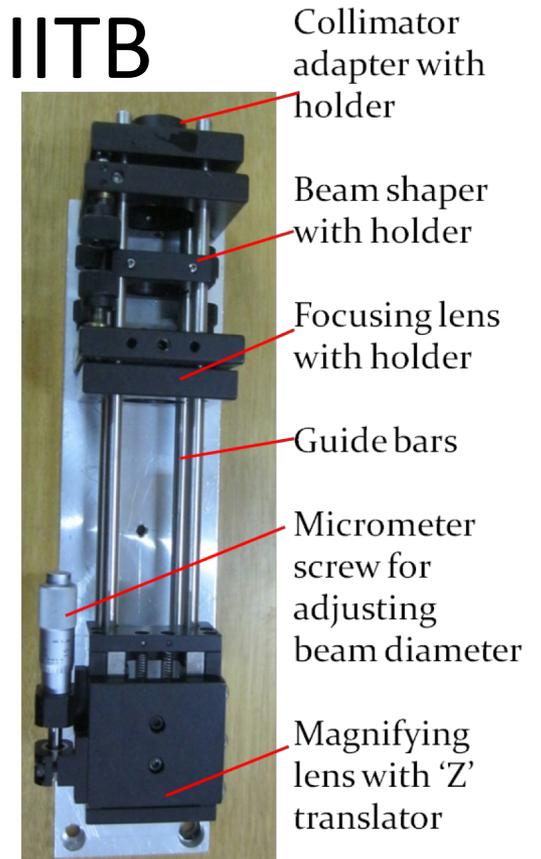
- Depth of focus where focal spot size changes by $\pm 5\%$.
- Approximate solution for focused beam diameter if lens is placed at z from the beam waist

$$D_{02} = \frac{4.f.\lambda.M^2}{\pi.D_z}$$

If unfocused beam diameter at z , is D_z .



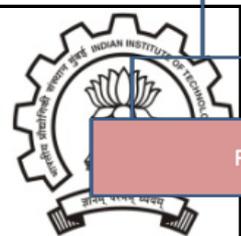
Laser Optics Setup at IITB



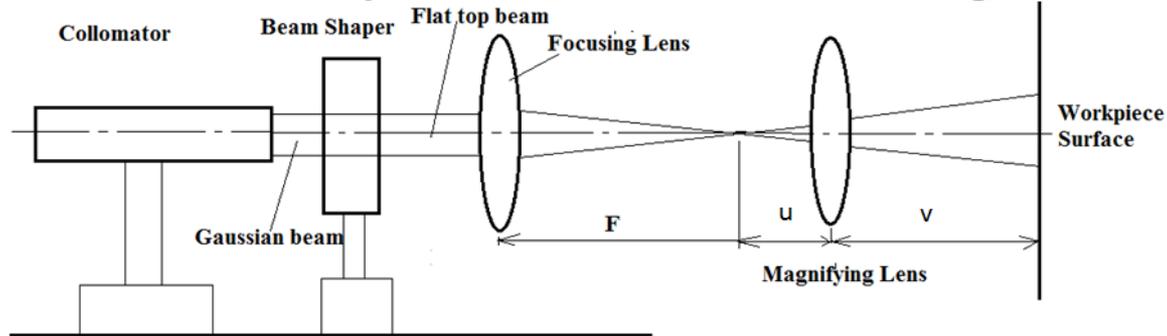
Indian Patent Application No 442/MUM/2011 Filed on 17 February 2011

Method and device for generating laser beam of variable intensity distribution and variable spot size

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mesh Singh



Compact Focusing



The magnifying lens can be selected for a given magnification(m) and length ($l=u+v$).

$$m = \frac{v}{u}$$

$$v + u = l$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Solving the above set of equations, the focal length of magnifying lens (a biconvex lens) can be determined in terms of length(l) and magnification(m),

$$f = \frac{ml}{(m+1)^2}$$



Aberrations

- Spherical Aberration
- Thermal Distortion
- Astigmatism
- Damage



Spherical Aberration

- There are two reasons why a lens will not focus to a theoretical point
 - Diffraction limited problem
 - Spherical lens is not a perfect shape.
 - Most lenses are made with a spherical shape since this can be accurately manufactured economically
 - The alignment of the beam is not so critical as with a perfect aspheric shape



Thermal Distortion

- High power laser beams are absorbed by lenses/optics
 - Selection of right optics ZnSe with CO2
 - The power distribution in TEM00 causes more severe gradients than Donut
 - Shape change of lens
 - Varies the refractive index, specially in ZnSe



Astigmatism and Damage

- Due to optical misalignment
- Damage
 - Due to dirt accumulation and burning on lens surface



Summary

- Absorption
- Beam Modes
- Propagation
- Focusing
- Aberrations

