Study of Handling Characteristics of Four-wheeled Vehicles with Independently Steered Front wheels

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Dissertation Approval Sheet

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Abstract

In this thesis, we undertake the study of steering action of four-wheel automobiles with front-wheels steering. We explore the possibilities of improvement in handling behavior of an automobile given the freedom to steer the front two wheels independent of each other. As compared to the conventional steering system where there is only a single steering input, independent steering system provides an opportunity to have two control inputs in the form of steering angles. This concept has also become practically realizable due to the advent of technologies like steer-by-wire.

In order to study the effects of independent steering, we develop a 3 degree-of-freedom model of a vehicle in which we consider the freedom of yaw, side-slip and roll. For this, we derive equations of motion of a vehicle based on the first principles and develop a linearized model based on them. We identify situations that impose particular requirements on vehicle response and find that it would be highly beneficial if we can control the yaw rate and lateral velocity of a vehicle independently. Hence, we devise method to achieve the above objective.

We devise a decoupling control strategy such that the overall plant get decomposed into two seperate single-input single-output loops. Subsequently, we design a controller based on the model obtained for a sample prototype car which can independently control the yaw rate and lateral velocity of a vehicle. We perform simulation of this system by taking the example of external force and moment disturbance acting on the vehicle due to cross-wind in which we aim to attenuate its effects on the yaw rate without changing its effect on the lateral velocity. We observe that the change in yaw rate due to disturbance is completely abated with zero steady-state error and at the same time there is no change in the lateral velocity response of the vehicle.

Acknowledgements

I take this opportunity to thank my guide, 'Shashi' as he is known as, for giving me complete freedom for working in the fashion i wanted to and also at the same time, always providing with the guidance and clues whenever approached for the same. He made himself available at any point of time in the day whenever he was approached with some task.

I also thank my friends, Raj, Vikalp and especially Amit for providing me enough help whenever i used to get stuck up with Matlab or with some 'Controls' fundas!

Finally, gratitude should go to my family, especially my brother, since its due to their support that i could pursue my academic endeavours.

Contents

List of Figures

List of Tables

List of Symbols

a	Distance of CG of a vehicle from front axle
b	Distance of CG of a vehicle from rear axle
p	Distance of CG of a vehicle from outside wheels
q	Distance of CG of a vehicle from inside wheels
U	Forward velocity of a vehicle
V	Lateral velocity of a vehicle
C_f	Coefficient of slip of the front tires
C_r	Coefficient of slip of the rear tires
R, r	Yaw velocity of a vehicle about vertical axis
R_d	Radius of turn of a vehicle
M	Total Mass of a vehicle
M_s	Sprung mass of a vehicle
I_z	Moment of inertia of a vehicle about z axis
I_x	Moment of inertia of a vehicle about x axis
P	Product of moment of inertia of a vehicle about z and x axis
CG	Centre of gravity of a vehicle
Y_D	External force disturbance
N_D	External moment disturbance
L_{if}	Roll stiffness at front roll centre
L_{ir}	Roll stiffness at rear roll centre
L_{pf}	Roll damping at front roll centre
L_{pr}	Roll damping at rear roll centre
G	Transfer Function

E_f	Roll steer at front wheels
E_r	Roll steer at rear wheels
$A_{\alpha f}$	Self-aligning moment at front wheels
$A_{\alpha r}$	Self-aligning moment at rear wheels
Y_{ff}	Lateral force due to camber at front wheels
Y_{fr}	Lateral force due to camber at rear wheels
δ_h	Steering Hand-wheel angle
δ_s	Reference steering angle at the centre of the front axle
δ_1	Steering wheel angle of the inside wheel during a turn
δ_2	Steering wheel angle of the outside wheel during a turn
α_f	Tire slip angle at the front wheels
α_r	Tire slip angle at the rear wheels
ϕ	Roll angle of a vehicle about longitudinal axis
β	Side-slip angle of a vehicle

Chapter 1

Introduction

1.1 Problem Statement

This project focuses on the study of steering operation of four-wheeled, front steered automobiles. The steering system of a vehicle provides a means to change its direction during its operation. In conventional vehicles, the driver turns the hand-wheel whose motion is transmitted to the front wheels by mechanical means and steering is affected by turning of front wheels. In independently steered vehicles, the mechanical linkages between the hand-wheel and the front two wheels are removed and the wheels are turned independent of each other by, say, electromechanical means. This method has also become practically realizable with the advent of technologies like steer-by-wire [1]. The purpose of this project is to study effects of such independent action of front wheels on the dynamics of a vehicle and to investigate the possible improvements, if any, in its handling behavior. The term 'Handling' refers to the ease of controlling the direction of a vehicle from driver's point of view.

The steering sub-system takes input from the driver and gives direction to a moving vehicle. It has to do this in a fashion that the resulting outcome (vehicle response) is acceptable to the driver and the external environment (other people on the road). The goal of any steering system should be to satisfy driver demand as well as take care of any uncertainties arising from with-in the vehicle and external environment (road and wind interaction). The response of a vehicle depends on its own dynamics and the dynamics of the steering system. It will depend on the way the front wheels are turned, i.e. the amount and rate at which wheels are turned. Therefore, it is required of the steering system to actuate the front wheels in such a manner that the desired outcome (vehicle response) is achieved. The problem at hand is first to identify and quantify the desirable response of the vehicle under various conditions and then devise ways to control the steering inputs such that the desired response is obtained. Our problem can be stated as follows: "Given the driver input, vehicle parameters like its mass, inertial and geometric properties, tire characteristics, and external disturbances on the vehicle from road and wind, develop control strategies to control the front two wheels' steer angles in order to obtain a specified dynamic response characteristics of the vehicle."

1.2 Motivation

In a general driving scenario, the driver has to regularly provide inputs to the steering-system. Apart from the spatial response of a vehicle, the driver has to do this in order to counter the effects of disturbances from external world like road irregularities and wind flow which try to deviate the vehicle from its normal behavior. These disturbances come as a surprise to the driver and moreover, the driver can make things worse by over-reacting or re-correcting them. It is desirable therefore that the steering-system by itself take care of these disturbances so that the driver remains concerned only with the nominal response of the vehicle. Secondly, certain characteristics in vehicle response are considered to be particularly desirable. For example, it is desirable to have a zero or small side-slip angle for better control of a vehicle. It is desired to have small roll angle for the sake of passenger comfort as well as safety against vehicle roll-over (over-turning) as it causes lateral shift in the CG. The achievement of above objectives can have significant impact on the handling quality of an automobile from both driver comfort and vehicle-safety point of view. Moreover, the technology of independent actuation of front wheels by electro-mechanical means do not impose any limitations on the magnitude and rate at which thay can be actuated. This also makes the problem suitable for control-oriented study. The goal to achieve the above stated objectives provide us with the motivation to carry out an attempt to investigate the means by which it can be done.

1.3 Methodology and Main Results

In order to solve the above problem, the following tasks are carried out:

1. The study of the dynamic and motion characteristics of a vehicle and development of steady-

state and transient models of a vehicle suitable for our application.

2. Identification of the handling criteria and performance metrics for the evaluation of the response characteristics of a vehicle.

3. Development of a control strategy and synthesis of controller for suitably controlling the variables of interest in vehicle's dynamic response.

4. Performing simulation studies in order to improve and evaluate the performance of the controller.

The aim is here is to show in principle that it is possible to have a desired control over dynamic response variables of a vehicle. Our simulation results show that it is possible to control the yaw rate and lateral velocity of a vehicle separately. Therefore, in case of vehicle subjected to external disturbances like external moment or force, it is possible to attenuate its effects on the yaw response of a vehicle without affecting its side-slip behavior. It is also observed that it is possible to reduce the side-slip angle of the vehicle without affecting its yaw response. The controller seems to work well with parametric uncertainties such as vehicle mass and forward velocity up to considerable extent.

1.4 Contribution of the Thesis

1. We have introduced a relatively new concept of independent steering system in a vehicle and shown in principle that its application can deliver significant benefits in terms of improving vehicle handling qualities. We have also introduced a method of controlling the chosen two metrics of vehicle response through two steering inputs by means of a multivariable controller that can decouple the two variables. This also opens up a research area for further investigation into the relationship between pair of variables of motion and the effect of their manipulation on the overall improvement in vehicle response quality.

2. In preliminaries section, we have summarized fundamental aspects related to the lateral dynamics of a vehicle. It can provide a good insight into the subject of vehicle dynamics for a beginner. In this thesis, we have derived the equations of motion which govern vehicle's dynamic behavior, developed a mathematical model based on that, and finally designed a

controller for the model. This material can provide a good background or a base for a person who wants to work further in the same area.

1.5 Organization of the report

The report is organized as follows: All the preliminary information required to understand the work in this report is presented in the Chapter 2. This includes the working of the steering systems and the elements of the vehicle dynamics. It also brings out the main differences in the conventional and independent steering systems. Chapter 3 is devoted to development of a mathematical model of a vehicle. A set of vehicle response graphs are also included in this chapter. Our main problem of developing a control strategy and subsequent controller design for controlling vehicle motion variables is included in Chapter 4. The simulation results for our choosen problem and corresponding observations are included in the Chapter 5. Finally, the conclusion and future work in the area is listed in the last chapter.

Chapter 2

Preliminaries

2.1 Steering Kinematics

2.1.1 Vehicle Axis System

As seen in the Figure 2.1 [3], we assume the origin of a co-ordinate axis system to be located at the centre of gravity (CG)of the vehicle. The longitudinal axis points in the direction of the forward velocity. The rotation around the vertical axis is known as "yaw" while that around the longitudinal axis is known as "roll". If we define the ratio of lateral velocity to forward velocity at the CG as "side-slip angle", the motion of a vehicle in a plane can be completely defined by the forward velocity, yaw rate and side-slip angle of the vehicle. It is also useful to note here that in most of the modern front-engined cars, CG may lie very close to driver's location along the longitudinal direction.

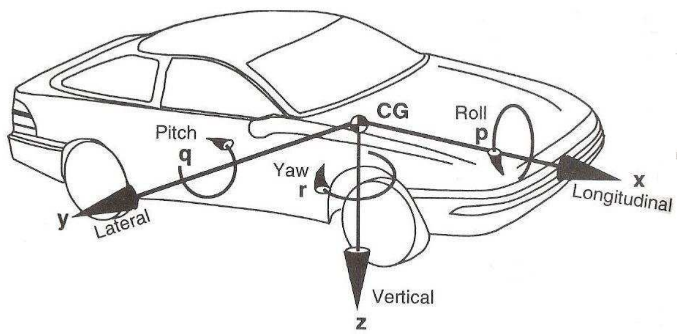

Figure 2.1: Vehicle Axis System [3]

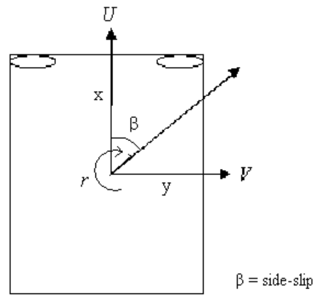

Figure 2.2: Vehicle Side-Slip

2.1.2 Fundamentals of Steering

If a vehicle has to negotiate a turn without slip at any of its wheels, it is required that all wheels have a pure rolling motion. This is possible if all wheels turn about a common centre. For this condition to occur, the ideal turning angles ($\delta_o(\delta_2)$, $\delta_i(\delta_1)$) can be obtained from the geometry of turn. The term 'Ackerman Geometry' is often used to denote the geometry of the front wheels that result in the kinematic condition showed in Figure 2.3 (Ackerman Model).

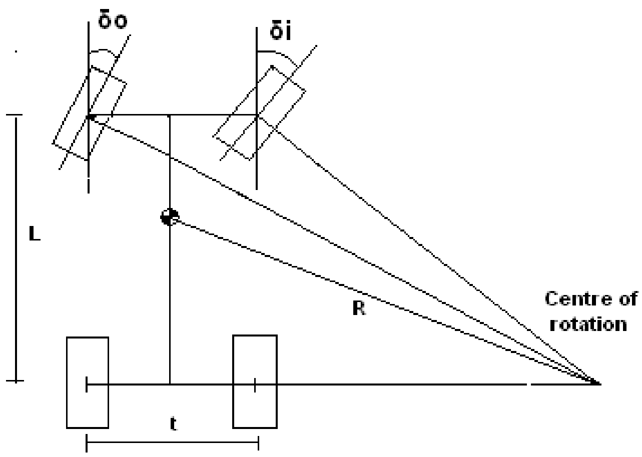

Figure 2.3: Ackerman Geometry

Here, δ_s is the reference steering angle at the centre of front axle that comes from the driver input. If δ_h is the steering hand-wheel angle and i is the steering ratio, then, We have,

$$\delta_s = \delta_h / i \tag{2.1}$$

$$\delta_s \approx (L/R_d) \tag{2.2}$$

$$\delta_1 \approx \left(\frac{L}{R_d - t/2} \right) \tag{2.3}$$

$$\delta_2 \approx \left(\frac{L}{R_d + t/2} \right) \tag{2.4}$$

2.1.3 Conventional and Independent Steering Mechanisms

In a conventional steering mechanism, oftentimes, the motion of the hand-wheel is transmitted to a gear box which contains a pinnion and a rack to convert the rotary motion into translatory motion. This translatory motion is then transmitted to the wheels through mechanical linkages. The kinematics of the steering gear are such that the Ackerman geometry is closely retained by the vehicle during turning (Refer figure 2.4 [3]).

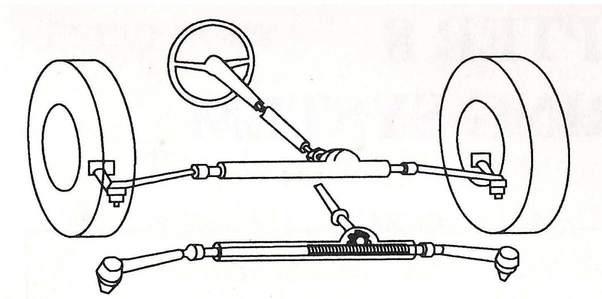

Figure 2.4: Conventonal Rack and Pinnion Arrangement [3]

In independent steering systems, an actuator is mounted near the front wheel, typically an electro-mechanical one such as an electric motor driving a ball screw-nut arrangement, whose translatory motion is transmitted to the wheel by linkages. This arrangement is made available for each of the wheels. Refer Figure 2.5.

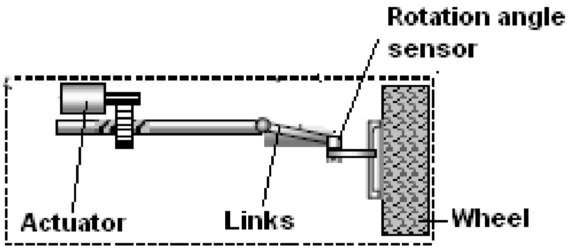

Figure 2.5: Independent Actuation System

2.2 Tire Dynamics

It is through the medium of tires that a vehicle experience forces and is steered, driven and supported. The pneumatic tire is an elastic body and hence generates forces and moments when in motion. Consider that a tire in motion is constrained to move in a direction at an angle (α) to its heading. This angle is known as the 'slip angle'. This causes a lateral distortion of the treads in contact with the ground and produces a force in the lateral direction which is known as the 'Cornering force' (Figure 2.6). It has been shown that this force may be assumed to be proportional to the slip angle [3].

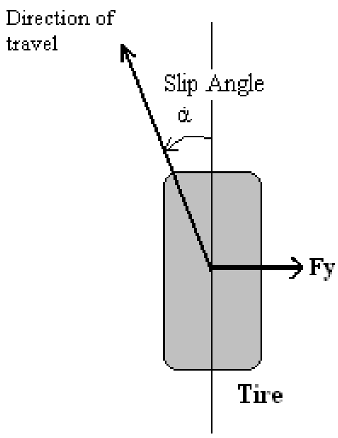

Figure 2.6: Tire Slip

$$F_y = C_\alpha \times \alpha \tag{2.5}$$

As the cornering force acts unequally along the length of the tire-road contact area and is more

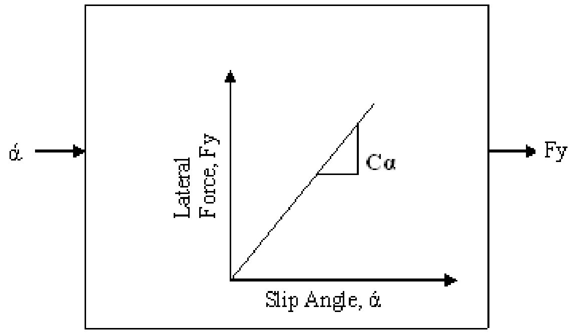

Figure 2.7: Linear Tire Model

near the rear end, the resultant of the force passes through the point at a certain distance behind the centre of contact area. This situation gives rise to a moment around the wheel turning axis which is called as 'Self-Aligning' moment since it tries to reduce the slip angle.

2.3 Vehicle Handling

2.3.1 Handling Criteria

The need to improve vehicle handling arises from the following requirements:

1. Driver comfort: It pertains to the performance of vehicle steering system to provide overall comfort and ease of operation to the driver.

2. Safety: This translates to increased probability of the vehicle to remain stable and under the control of the driver under different manuvering conditions.

In order to quantify the requirements of good handling, it is necessary to define certain measurable quantities or metrics on the basis on which vehicle's handling performance can be evaluated. The question of which metrics are important has in itself attracted a lot of attention in the past [4].

Evaluation of vehicle dynamic behavior can be carried out by predicting the response through modeling and simulation during vehicle development stage or assessment of actual vehicle through vehicle testing and experimentation.

2.3.2 Handling Metrics

The handling metrics that are considered in this work are the following steady-state values and transient response characteristics of variables of motion:

1. Steady-state values of lateral velocity and lateral acceleration of a vehicle for the given forward velocity and steering input along with the time to achieve them.

2. Steady-state yaw velocity and time to achieve it.

3. Vehicle side-slip angle for given yaw rate. Vehicle side-slip should be as small as possible.

4. Roll angle for the given forward velocity and steering input. Roll angles should be as small as possible.

5. Amount of oscillations in the yaw velocity and roll angle response and maximum overshoot for step steering input.

2.4 Vehicle Control Loop

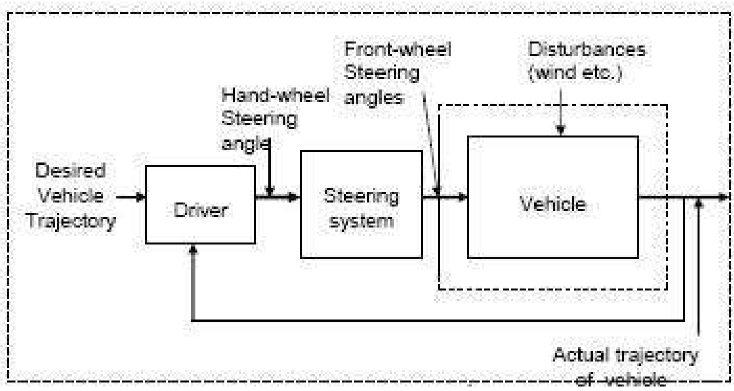

Figure 2.8: Vehicle control loop without controller

In any driving situation, the driver guides the vehicle through a course pre-decided by him. This decision of driver is based on the his needs and the road conditions ahead of him. Apart from the driver inputs, the vehicle is subjected to external disturbances and responds in a fashion depending upon its own dynamics. His decision may be considered as being based on utilizing feedback information about the actual trajectory of the vehicle (Figure 2.8).

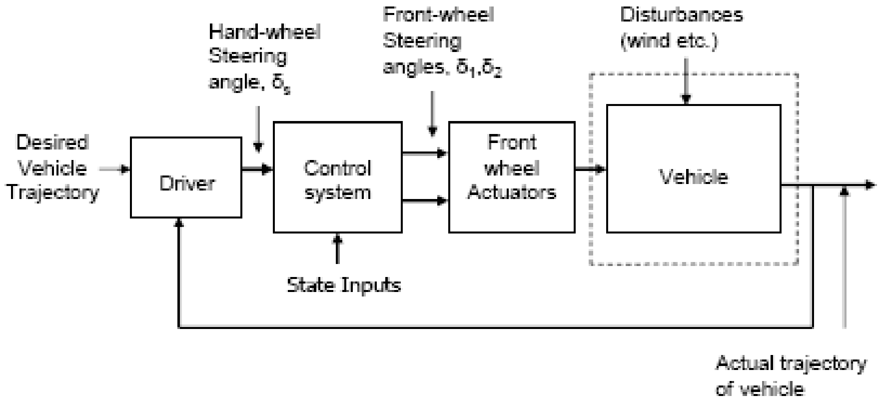

Figure 2.9: Vehicle control loop with controller

In the case of a vehicle with say, an independent steering controller (Figure 2.9), the inputs of the driver are interpreted by the controller and a desired steering action is communicated to the front wheel actuators. Thus the controller is responsible for the task of matching the vehicle response to the driver's requirements. It is useful to note that there is only one commanding input that comes from the driver in the form of steering hand-wheel angle which has to be interpreted by the controller.

2.5 Stability of a Vehicle

One of the important considerations from the point of view of vehicle stability is the amount of slip-angles generated at the front wheels and the rear wheels. The amount of slip angles go on increasing with the amount of lateral acceleration required. With the increase in slip-angles, the corresponding lateral forces increase untill a point is reached where the lateral force exceeds the limiting frictional force at the tire-ground contact and the vehicle skids. This may occur first at the front or the rear wheels depending on which has more slip-angles. As the slip-angle at the front wheel is in direct control by means of the steering angle whereas the slip-angle at the rear is not, it is desirable to have more slip-angles at the front. This condition is known as under-steer. Under-steer is dependent on the slip-angle coefficients of the front and rear tires and longitudinal position of the centre of gravity (CG) of a vehicle.

Chapter 3

Mathematical Modeling of Vehicle Dynamics

In this chapter, we describe mathematical models describing the vehicle lateral dynamics. For the purposes of controller design, we focus on a 3 degree-of-freedom model with freedoms of yaw, side-slip and roll.

3.1 Modeling

Conventionally, vehicle lateral dynamic behavior has been modeled with the help of a two-wheel model known as the 'Bicycle Model'. In this model, the front and rear axles are represented by an equivalent single wheel at the centres of the axles [5]. In developing such models, the principle of body fixed inertial co-ordinate system is used whose origin is assumed to be located at the centre of gravity. In our case, it is necessary to develop model that can accomadate the conditon of seperate steering input to the front two wheels. Therefore, we focus on a four-wheel model for our study in this thesis.

3.1.1 A Four-Wheel Model

Consider a Four-Wheel model as shown in the Figure 3.1. The total lateral force generated is equal to the sum of the lateral forces acting on all the four wheels. Moreover, each wheel may have a different slip-angle.

We make the following assumptions in developing the model,

1. Forward velocity of vehicle remains constant
2. No forces act on the tires due to traction
3. Steering angles are small
4. Slip angles remain small
5. Vehicle track-width is small as compared to the radius of turn
6. Road surface is smooth, i.e. no external forces or moments act on the tires due to road irregularities

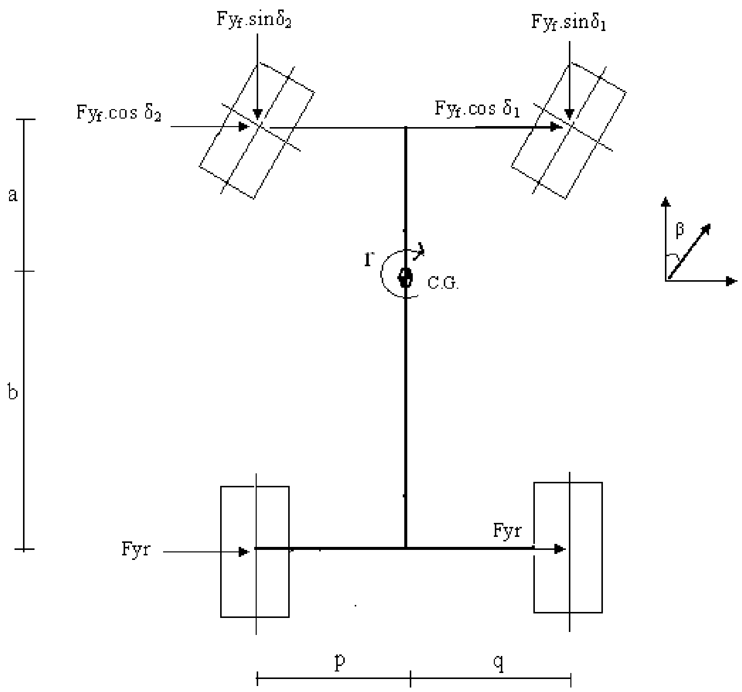

Figure 3.1: Four-Wheel Model

The following degrees-of-freedom are considered,

1. Motion of a vehicle in the lateral direction which causes vehicle side-slip (Side-slip freedom)

2. Motion of a vehicle around the vertical axis (Yaw freedom)

3. Motion of a vehicle around longitudinal axis (Roll freedom)

Applying Newton's Second law one by one along lateral axis, about vertical axis, and about longitudinal axis, we get a set of corresponding three dynamically coupled equations as below,

$$m\left(\dot{V} + Ur\right) + M_s h\dot{\phi} = Y_b\beta + Y_r r + Y_d\left(\delta_1 + \delta_2\right) \tag{3.1}$$

$$I_z\dot{r} - P\ddot{\phi} = N_\beta\beta + N_r r + N_\phi\phi + N_\delta\delta_1 + N_\delta\delta_2 + N_g\beta\delta_1 - N_h\beta\delta_2 + N_q r\delta_1 - N_p r\delta_2 - N_g\delta_1^2 + N_h\delta_2^2 \tag{3.2}$$

$$I_x\ddot{\phi} - P\dot{r} + M_s h\dot{V} + M_s hUr = L_\phi\phi + L_p\dot{\phi} \tag{3.3}$$

The above equations are known as the "equations of motion". In the above equations, we have used certain coefficients for the sake of simplification [5]. These coefficients are defined and listed in Appendix A. The detailed derivations of the equations of motion can also be found in Appendix A.

From the equations of motion, we observe that our model is a fourth order system. We define the following state variables:

$$State\,Vector,\ x = \begin{bmatrix} r \\ V \\ \phi \\ \dot{\phi} \end{bmatrix} \tag{3.4}$$

As seen above, the dynamic equations that describe the model are non-linear in nature. For the purposes of controller design, it is often useful to work with linear descriptions of dynamics. We therefore build a linearized model as follows, As seen from the Figure 3.2, the front wheels steering angles are the input to a set of non-linear steady-state equations which are used to determine the steady-state values of the motion variables. This steady-state of a vehicle is considered as an operating point for linearizing the equations and constructing the transient model of a vehicle. This procedure has been described in the next sections.

Figure 3.3 shows the block diagram of the transient model from input-output perspective. The inputs to the model are the change in front wheels steering angles and disturbance inputs of external force and external moment. The outputs are the change in yaw rate, lateral velocity and roll angle.

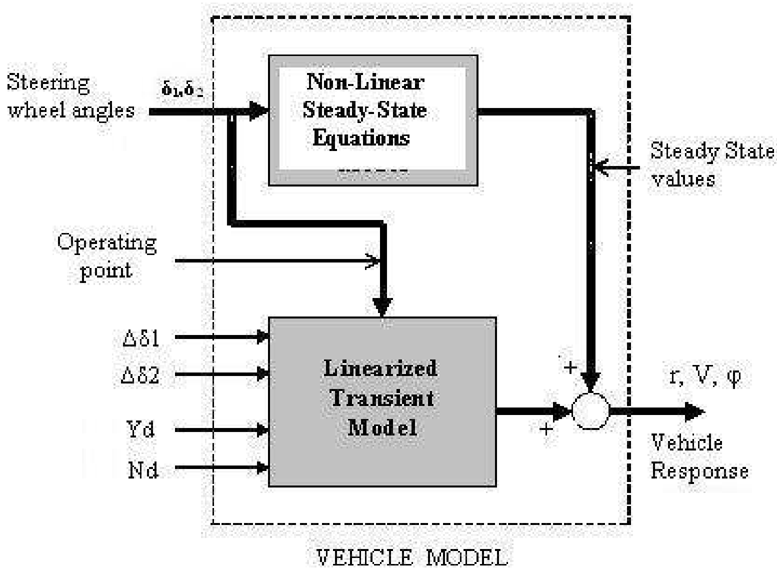

Figure 3.2: Linearization procedure

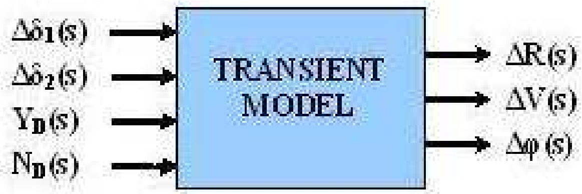

Figure 3.3: Transient Model

3.1.2 Steady-State Behavior

The steady-state equations of a vehicle are obtained by substituting all the time-derivative terms in the equations of motion as zero.

$$\dot{r} = \dot{V} = \dot{\phi} = \ddot{\phi} = 0 \tag{3.5}$$

We get,

$$mUr = Y_b\beta + Y_r r + Y_d\left(\delta_1 + \delta_2\right) \tag{3.6}$$

$$0 = N_\beta\beta + N_r r + N_\phi\phi + N_\delta\delta_1 + N_\delta\delta_2 + N_g\beta\delta_1 - N_h\beta\delta_2 + N_q r\delta_1 - N_p r\delta_2 - N_g\delta_1^2 + N_h\delta_2^2 \tag{3.7}$$

$$M_s hUr = L_\phi \tag{3.8}$$

By seperating the variables and rearranging, the above equations can be written in the vector matrix form. Consider the matrix equation shown below,

$$[X][K] = [Z]$$

Here, [Z] is the input vector and [K] is the output vector. The input to the steady-state model are the steer-angles of the inside and outside front wheels obtained from the Ackerman model.

$$\begin{bmatrix} Y_r - MU & Y_\beta & Y_\phi \\ N_r + N_q\delta_1 - N_p\delta_2 & N_\beta + N_g\delta_1 - N_h\delta_2 & N_\phi \\ M_s hU & 0 & -L_\phi \end{bmatrix} * \begin{bmatrix} r \\ \beta \\ \phi \end{bmatrix} = \begin{bmatrix} -Y_\delta\left(\delta_1 + \delta_2\right) \\ N_g\delta_1^2 - N_h\delta_2^2 - N_\delta\left(\delta_1 + \delta_2\right) \\ 0 \end{bmatrix} \tag{3.9}$$

By solving the above equation, we can obtain the steady-state values of the variables. Thus, the output of the non-linear model is the steady-state values of the Side-slip angle, β, Yaw rate, r, and Roll angle, ϕ.

3.1.3 Transient Behavior

The transient model of a vehicle is constructed by linearizing the dynamic equtions of the vehicle (Equations 3.1 to 3.3) about the steady-state operating condition. Thus, the linearizing parameters in our model are the steady-state values of the steering angles, δ_1 and δ_2, and the corresponding output variables, β, r and ϕ. These values are obtained from the non-linear steady-state equations of the vehicle described above. Four inputs have been considered in the case of transient model, two of which are the change in steering wheel angles about the operating point and remaining two are those of external force and external moment acting on the vehicle due to enviormental disturbances.

We obtain transfer functions for each input-output pair. Describing the model in this form also makes it suitable from the point of view of controller synthesis. Thus we have a set of twelve transer functions corresponding to four inputs and three outputs. The Transfer Matrix for the model can be written in the following way,

$$\begin{bmatrix} \Delta R \\ \Delta V \\ \Delta \phi \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \end{bmatrix} * \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \\ Y_D \\ N_D \end{bmatrix} \tag{3.10}$$

3.2 Steady-State and Dynamic Response of the vehicle model

We simulate the response of the vehicle model for a small car [5]. The values of parameters of the car are listed in the Appendix B. The transient model obtained for this car is also included in the Appendix B.

3.2.1 Steady-State Response and Observations

3.2.1.1 Variation of front wheel angles with steering angle

Figure 3.4 shows the variation of inside and outside wheel angles, δ_1 and δ_2 with the change in the reference steering angle, δ_s according to the Ackerman geometry (model).

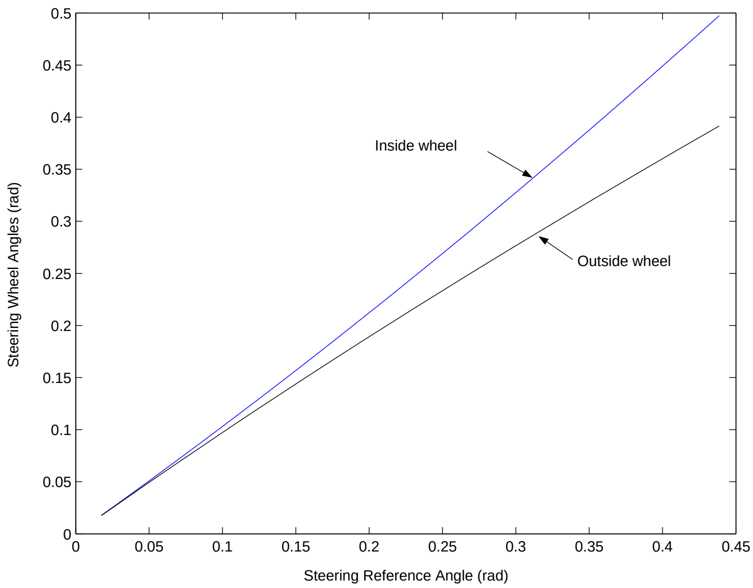

Figure 3.4: Variation of front wheels angles with reference steering angle (Ackerman Model)

3.2.1.2 Comparison of steady-state response of a Four-wheel model and a Bicycle model

Figures 3.5, 3.6 and 3.7 compares the variation of side-slip, yaw rate and roll angles with steering angle for the Bicycle model and a Four-wheel model. It is observed that at lower steering angle, there is a very little difference in the steady-state response of the two models. Here, U = 15 m/s.

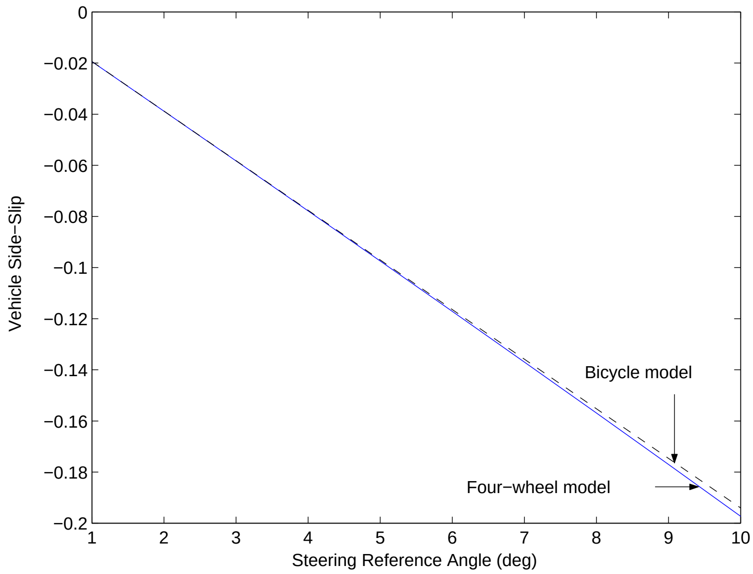

Figure 3.5: Four-wheel model vs Bicycle model: Side-slip

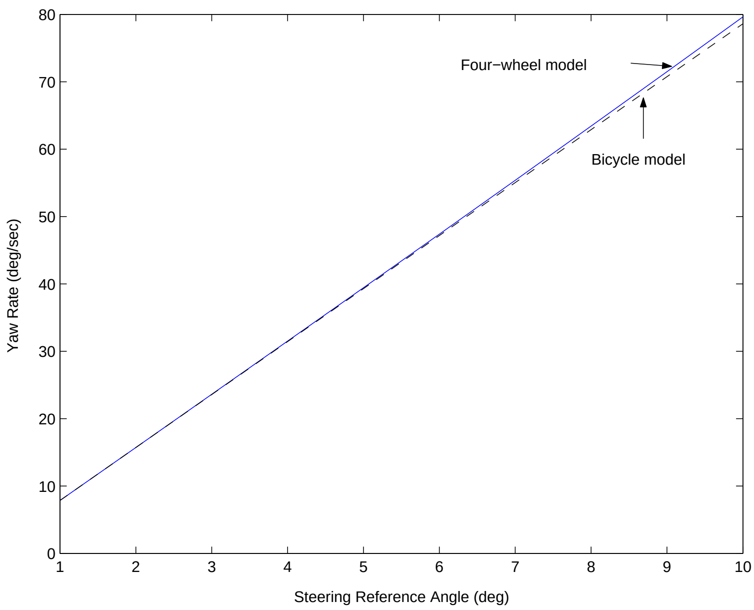

Figure 3.6: Four-wheel model vs Bicycle model: Yaw Rate

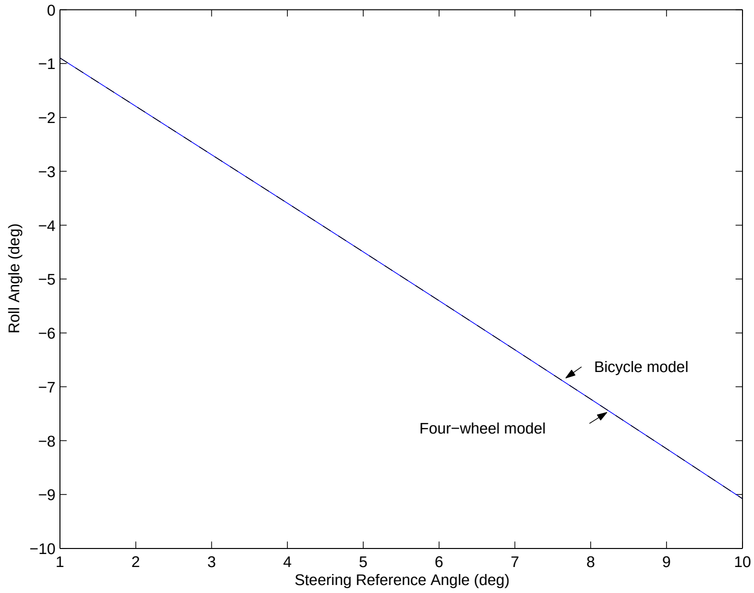

Figure 3.7: Four-wheel model vs Bicycle model: Roll angle

3.2.1.3 Variation of steady-state response with steering angle and forward velocity

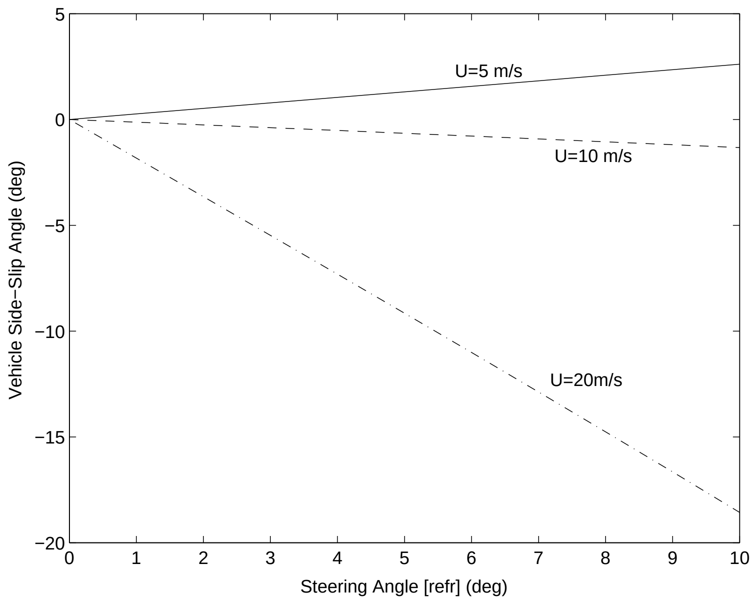

Figure 3.8: Steady-state Sideslip values for different steering angles

We perform steady-state analysis for our Four-wheel model. Refering to the Figure 3.8, it is observed that for lower forward velocity the Side-slip angle is positive while it is negative for higher velocities. From the above observation, it can be inferred that at small lateral acclerations, the rear wheel tracks inboard of the front wheel but when the lateral acceleration increases with forward velocity, the rear wheels must drift outboard to develop necessary slip. Also, there exists a particular velocity at which the slip-angle becomes zero and is independent of radius of turn (or steering angle).

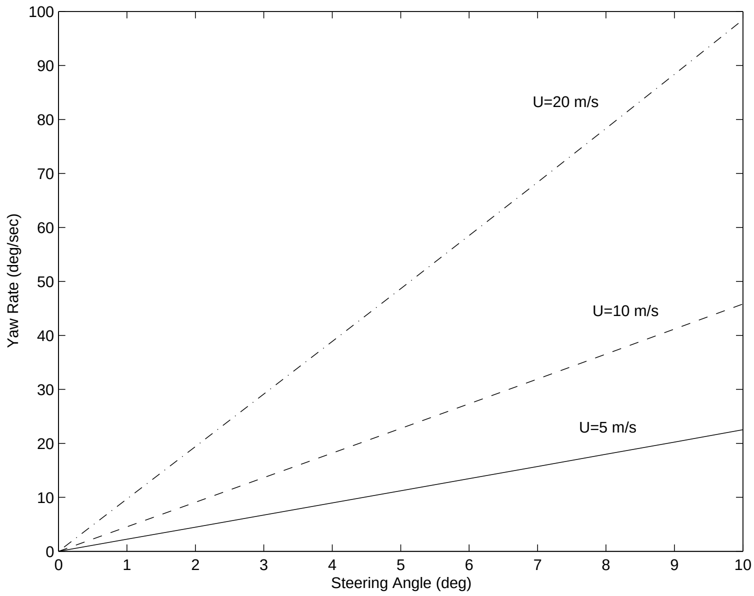

Figure 3.9: Steady-state Yaw Rate values for different steering angles

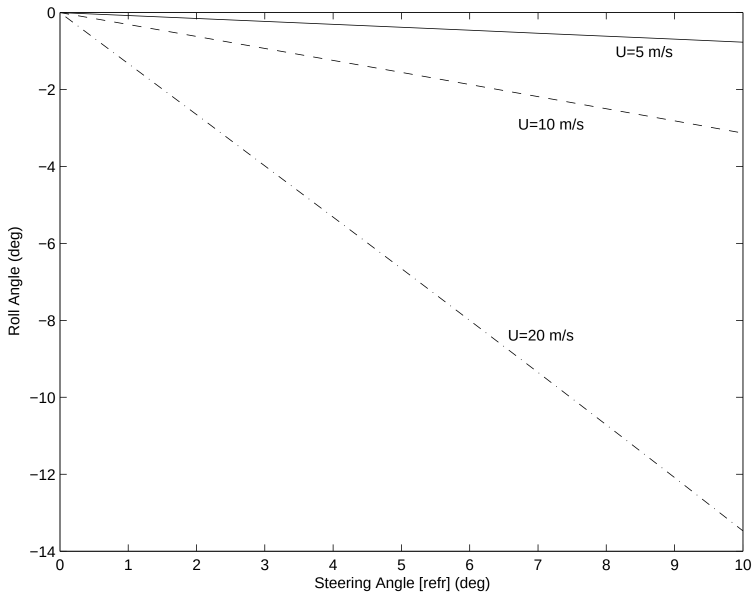

Figure 3.10: Steady-state Roll Angle values for different steering angles

From the Figure 3.9, it is inferred that the yaw velocity increase almost linearly with the steering angle and non-linearly with the forward velocity. In Figure 3.10, it is observed that variation of roll angle is proportional to the steering angle. Also, there is a substantial change in roll angle of a vehicle with increase in forward velocity.

3.2.2 Transient Response and Observations

3.2.2.1 Transient Response characteristics of Linear Model

In order to the study the transient response characteristics of the vehicle, an operating point of $\delta_s = \delta_1 = \delta_2 = 0$ is taken and a step-input of change in steering angle, $\Delta\delta_s = 5^o$ is applied. The response curves are obtained for different forward velocities. Figure 3.11 shows the lateral velocity response of a vehicle. It can be observed that the final steady state values of the lateral velocity can be positive or negative depending upon the forward velocity as discussed in previous section. Also, when the input is applied, the immediate lateral acceleration response is positive which later changes to negative. However, the total final lateral acceleration of a vehicle in a turn is always positive. It is also observed that the settling time of the vehicle response also increases with the forward velocity.

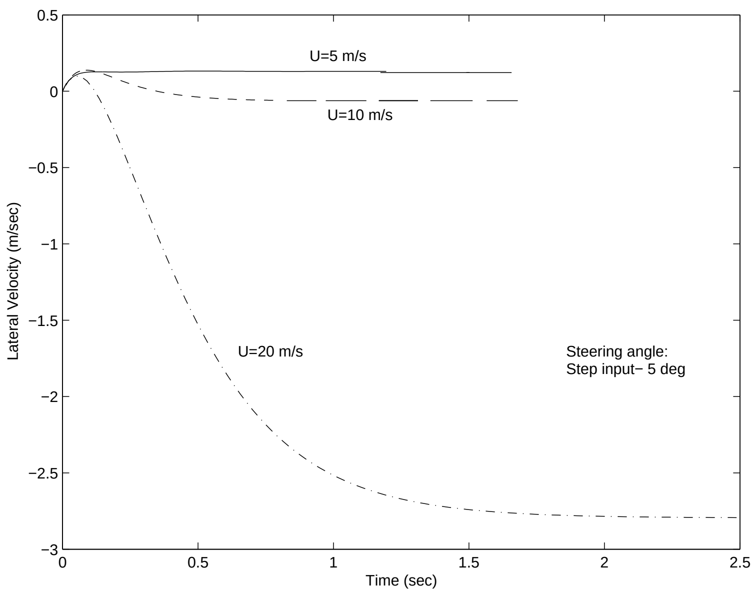

Figure 3.11: Lateral velocity response for step steer-angle input

The yaw velocity response of a vehicle is shown in the Figure 3.12. It is observed that the yaw rate and also the settling time increases with forward velocity. From Figure 3.13, it is observed that at lower forward velocities, the roll angle response is oscillatory. At higher forward velocities, the

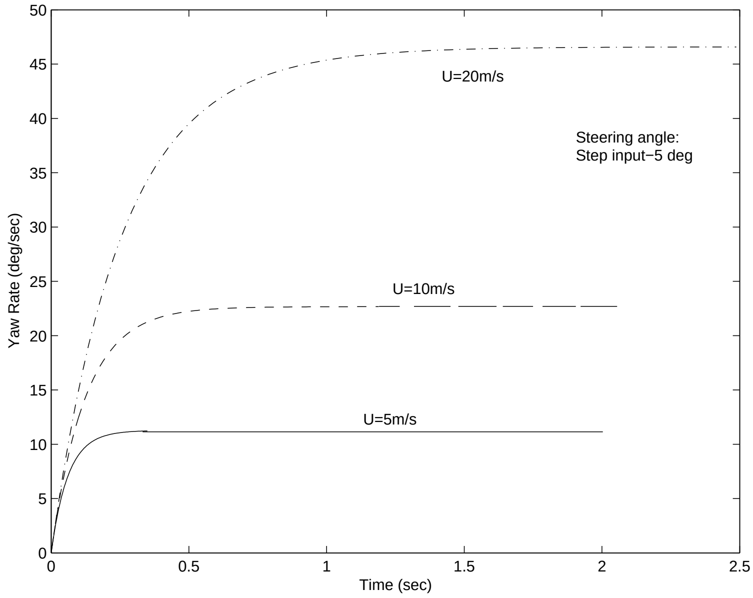

Figure 3.12: Yaw velocity response for step steer-angle input

roll angle of a vehicle increases and it takes more time for the vehicle to reach steady-state. It can be observed that the steady-state values reached in these response match with the steady-state graphs shown in the previous section

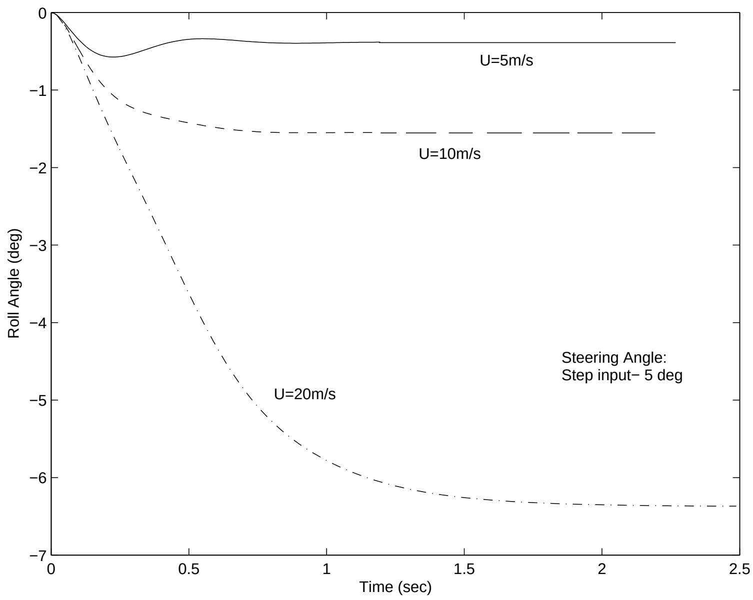

Figure 3.13: Roll angle response for step steer-angle input

3.2.2.2 Variation of Transient Response characteristics with vehicle properties

As shown in Figure 3.14, with the increase in the roll stiffness, the steady-state roll angle obtained for the same conditions reduces as expected. Also, for the lower stiffnesses, the roll response becomes oscillatory.

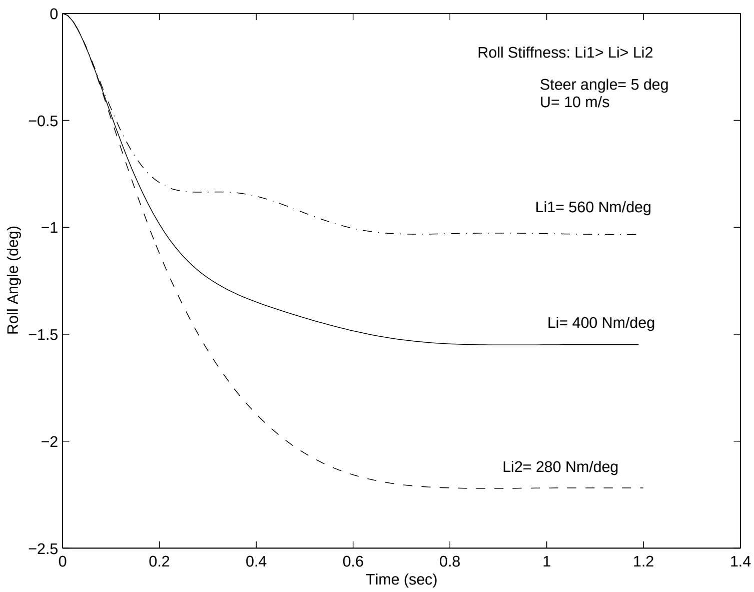

Figure 3.14: Effect of Roll stiffness on Roll Angle response of a vehicle

Chapter 4

Controller Development

4.1 Control Problem

In case of turning of a vehicle, the driver commands the car to enter a curve by providing lateral acceleration to the car. By commanding the steering angle of the front wheels, the driver controls slip angles developed at the wheels. These slip angles generate a lateral force which is the cause of the lateral acceleration. Moreover, as the vehicle is a rigid body, this lateral motion implies a corresponding yaw velocity. Thus the driver mainly controls the lateral motion of a vehicle and is related to yaw motion only indirectly [6].

Apart from the driver inputs, yaw motion of a vehicle is also affected by the external disturbances like due to wind, road inclinations and 'μ-split' braking where the braking force occuring at the four wheels may be different. These external disturbances tend to deviate a vehicle from its normal behavior. This makes the driver's task more difficult as apart from the task of path following, driver has to keep on providing inputs to compensate for the effects of these disturbances. Also, these disturbances come to him as a surprise and it takes sometime for him to recognize the situation and react. Then, he may overreact and can make things worse. Hence, it required to have a control system that can make vehicle's behavior immune to the external disturbances and also do not interfere with the driver's task. Also, it has been suggested in the literature that for improved control of the vehicle, it is beneficial to have small or zero side-slip angle (zero side-slip conditon) [2].

Clearly, it is only possible to achieve the above goals if we can have a seperate control over the lateral motion and yaw motion of a vehicle.

In relation to the above discussion, there are two important considerations,

1. For the discrete control of output variables, the number of input variables should be more or atleast equal to the number of output variables. This is known as an "equally-actuated" system. In conventional steering, we have only one independent input in the form of a steering angle hence it is impossible to control two variables separately, i.e. it is an "under-actuated" system. In independent steering system, we have two steering inputs and therefore, we have the opportunity to control two choosen output variables independently.

   Conversely, as we have only two independent inputs, we can control only two output variables at a time. For example, in our case, we try to control yaw rate and lateral velocity, and therefore, we have no control over the third degree of freedom that is of roll angle.

2. The output variables to be controlled may be coupled to each other due to the dynamics of the system. Therefore, while choosing the output variables to be controlled, it is necessary that the decoupling of the two variables should be possible.

Our objective is to devise a control strategy so that we can have a seperate control over the chosen variables of interest, i.e., lateral velocity and yaw rate. These variables of motion are dynamically coupled to each other as can be seen from the equations of motion (see chapter on modeling) or in other words, there exist an interaction among these variables. Therefore, we try to achieve a decoupling between these two motions of a vehicle by exploiting independent steering action. The control design problem can be stated as: "Design a controller that can decouple the yaw rate and lateral velocity of a vehicle and seperately control them in order to meet the desired transient response characteristics." The 'Decoupling control' scheme used for the above purpose is described in detail in the next section.

4.2 Decoupling Control

A popular approach to dealing with control loop interactions is to design non-interacting or decoupling control schemes [7]. This is achieved by means of compensation networks known as decouplers. The role of a decoupler is to decompose a multivariable process into series of independent single-loop sub-systems that can be controlled using independent loop controllers. Refer Figure 4.1, [8]. These sub-systems will, of course, be feedback control systems.

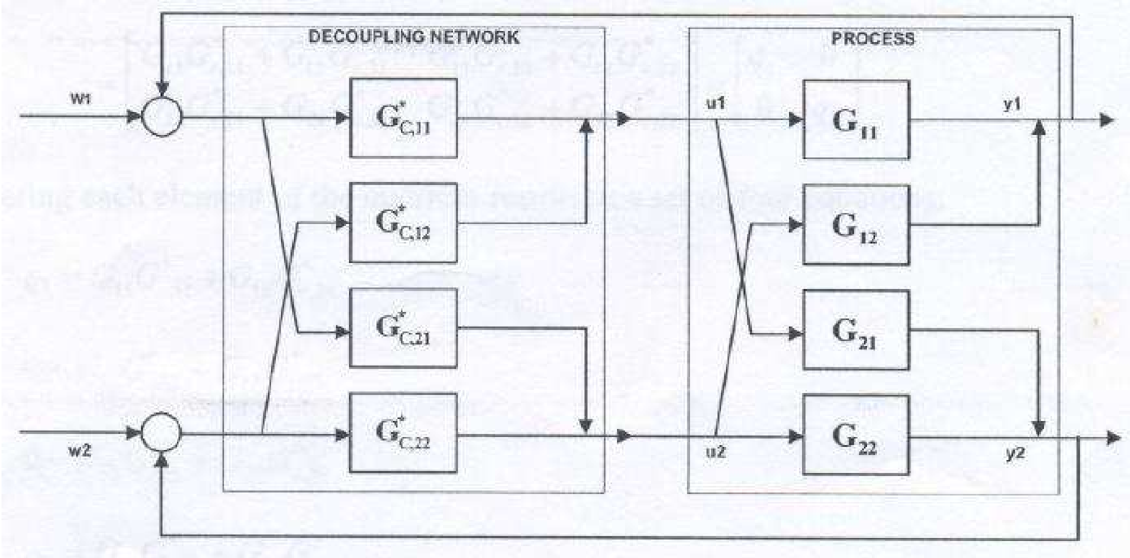

Figure 4.1: Decoupling Control Network [8]

Consider that the process to be controlled is a two-input two-output system, $[G]$.

$$\begin{bmatrix} Y \end{bmatrix} = \begin{bmatrix} G \end{bmatrix} * \begin{bmatrix} U \end{bmatrix} \tag{4.1}$$

$$[G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{4.2}$$

We have to design the controller $[G_c]$ such that the overall plant becomes a set of two single-input single-output systems.

$$[G][G_c] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} * \begin{bmatrix} G_{c11} & G_{c12} \\ G_{c21} & G_{c22} \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \tag{4.3}$$

By comparing the elements, we get the following four equations,

$$q_1 = G_{11}G_{c11} + G_{12}G_{c21} \tag{4.4}$$

$$0 = G_{11}G_{c12} + G_{12}G_{c22} \tag{4.5}$$

$$0 = G_{21}G_{c11} + G_{22}G_{c21} \tag{4.6}$$

$$q_2 = G_{21}G_{c12} + G_{22}G_{c22} \tag{4.7}$$

The overall plant becomes,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} * \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{4.8}$$

where w_1 and w_2 are the control inputs for controlling y_1 and y_2 respectively. In this case, we choose G_{c11} and G_{c22} as the controllers for controlling y_1 and y_2 respectively, while the controller G_{c21} minimizes the effect of input to the plant, u_2 on y_1 and, G_{c12} minimizes the effect of u_1 on y_2. Thus, by choosing G_{c11} and G_{c22}, the other two transfer functions can be obtained by using equations (4.5) and (4.6).

4.3 Design of a decoupling controller for the 3 d-o-f model

In our case, since we have two control inputs to the plant in the form of two steering angles of front wheels, we can have an independent control over a maximum of two output variables. Such a system is known as equally-actuated system. We consider the vehicle model with two steering angles as inputs and yaw rate and lateral velocity as output as shown in Figure 4.2. Thus we design the controller for the vehicle model described previously and obtained for a prototype small car.

$$\begin{bmatrix} \Delta R \\ \Delta V \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} * \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix} \tag{4.9}$$

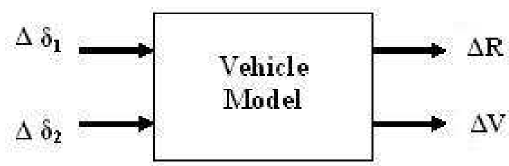

Figure 4.2: Actual Plant

The two output variables are the functions of both the inputs, i.e. there is an interaction between the input and output variables. Therefore, we design a decoupling controller so that we can have an independent control over the two outputs using the two input variables.

$$\begin{bmatrix} \Delta R \\ \Delta V \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} * \begin{bmatrix} \Delta R_D \\ \Delta V_D \end{bmatrix} \tag{4.10}$$

$$\Delta R = q_1 * \Delta R_D \tag{4.11}$$

$$\Delta V = q_2 * \Delta V_D \tag{4.12}$$

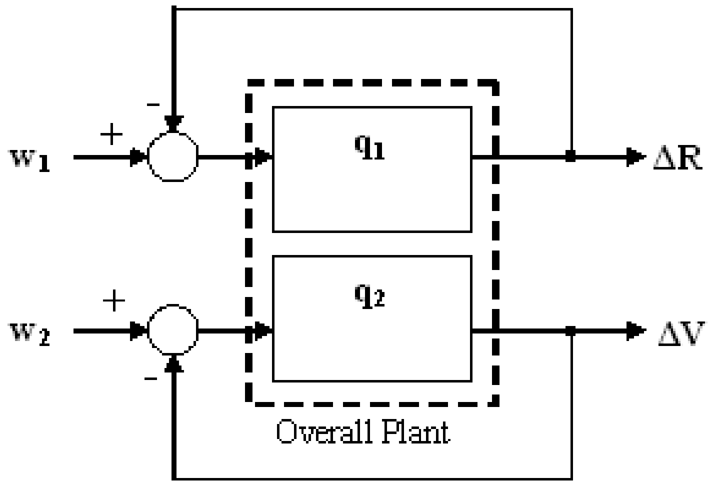

Figure 4.3: Overall plant with two SISO loops

As seen from the equations above, the overall plant gets transformed into two single-input single-output (SISO) loops where $\Delta R_D(w_1)$ and $\Delta V_D(w_2)$ are the change in desired values of the yaw rate and lateral velocity and ΔR and ΔV are the change in actual values respectively. The change in desired values come from the "reference model" that generate these values depending upon the driver input and forward velocity. The q_1 and q_2 are obtained from the equations described in the previous section.

For the purpose of controlling the output variables, we have to design the controllers, G_{c11} and G_{c22} as stated in the previous section. Any suitable configuration can be chosen for both the controllers. We choose a PID (Proportional-Integral-Derivative) configuration for both of them. The Transfer Function of a PID controller is given by,

$$TF = \frac{(K_p + K_d A)S^2 + (K_p A + K_i)S + K_i A}{S^2 + AS} \tag{4.13}$$

Here, K_p, K_d and K_i are the proportional, derivative and integral gains which need to be synthesized. A is a parameter for filter with derivative, D. The value of A is taken as 10. The values of gains are obtained by trial and error method by suitably varying the values and observing the transient response characteristics until the optimum response characteristics are obtained. The main considerations in response characteristics are that the settling time should be as small as possible, and there should be minimum osillations and overshoot.

We obtain the following values of gains,

K_p= 14

K_i= 28

K_d= 1

Therefore, we obtain the transfer functions for controllers as follows,

$$G_{C11} = \frac{24s^2 + 168s + 280}{s^2 + 10s} \tag{4.14}$$

$$G_{C22} = \frac{24s^2 + 168s + 280}{s^2 + 10s} \tag{4.15}$$

Using equation 4.3, we get the Transfer Function for the controller G_{C12},

$$G_{C12} = \frac{-3.957e077s^9 - 2.159e079s^8 - 5.865e080s^7 - 1.023e082s^6 - 1.233e083s^5}{1.832e076s^9 + 1.055e078s^8 + 2.957e079s^7 + 5.264e080s^6 + 6.421e081s^5} \tag{4.16}$$

$$\frac{-1.048e084s^4 - 6.187e084s^3 - 2.366e085s^2 - 5.105e085s - 4.577e085}{+5.458e082s^4 + 3.153e083s^3 + 1.119e084s^2 + 1.818e084s}$$

Similarly, using equation 4.4, we get the Transfer Function for the controller G_{C21},

$$G_{C21} = \frac{-1.608e078s^9 - 6.532e079s^8 - 1.388e081s^7 - 1.858e082s^6 - 1.594e083s^5}{6.69e076s^9 + 2.978e078s^8 + 6.654e079s^7 + 9.387e080s^6 + 8.588e081s^5} \tag{4.17}$$

$$\frac{-8.265e083s^4 - 1.501e084s^3 + 8.549e084s^2 + 4.859e085s + 6.649e085}{+4.905e082s^4 + 1.327e083s^3 - 1.986e083s^2 - 1.67e084s}$$

Chapter 5

Simulation Results

Simulations were performed in order to evaluate the disturbance attenuation capability of the controller. The simulation was carried out for the vehicle subjected to the external disturbance due to cross-wind. Simulations were performed in the *Simulink* enviornment.

Consider a vehicle taking a turn and subjected to sudden gust of side wind. The force experienced by the vehicle due to wind will be,

$$F_w = 1/2 * \rho * V_w^2 * C_s * A \tag{5.1}$$

Here, V_w is the side wind velocity and C_s is a side-force coefficient that varies between 0.03 to 0.06 [3]. We take V_w of 20 m/s and side area, $A = 3m^2$. Also, $\rho = 1kg/m^3$(Air density) and $C_s = 0.05$. We assume that the centre of wind-force lies at 0.5 metres behind the CG. We get the following values for the external force and moment,

Table 5.1: Magnitude of external disturbances acting on the car

Disturbance	.	.
External Force	30	Kgf
External Moment	15	Kgf-m

The control loop for the yaw rate takes the reference yaw rate value from the reference model and actual yaw rate is fedback through the closed loop. The controller tries to maintain the actual yaw rate of the vehicle as close as possible to the reference yaw rate. If the actual yaw rate deviates from its reference value, the controller tries to minimize the error between the two. The control

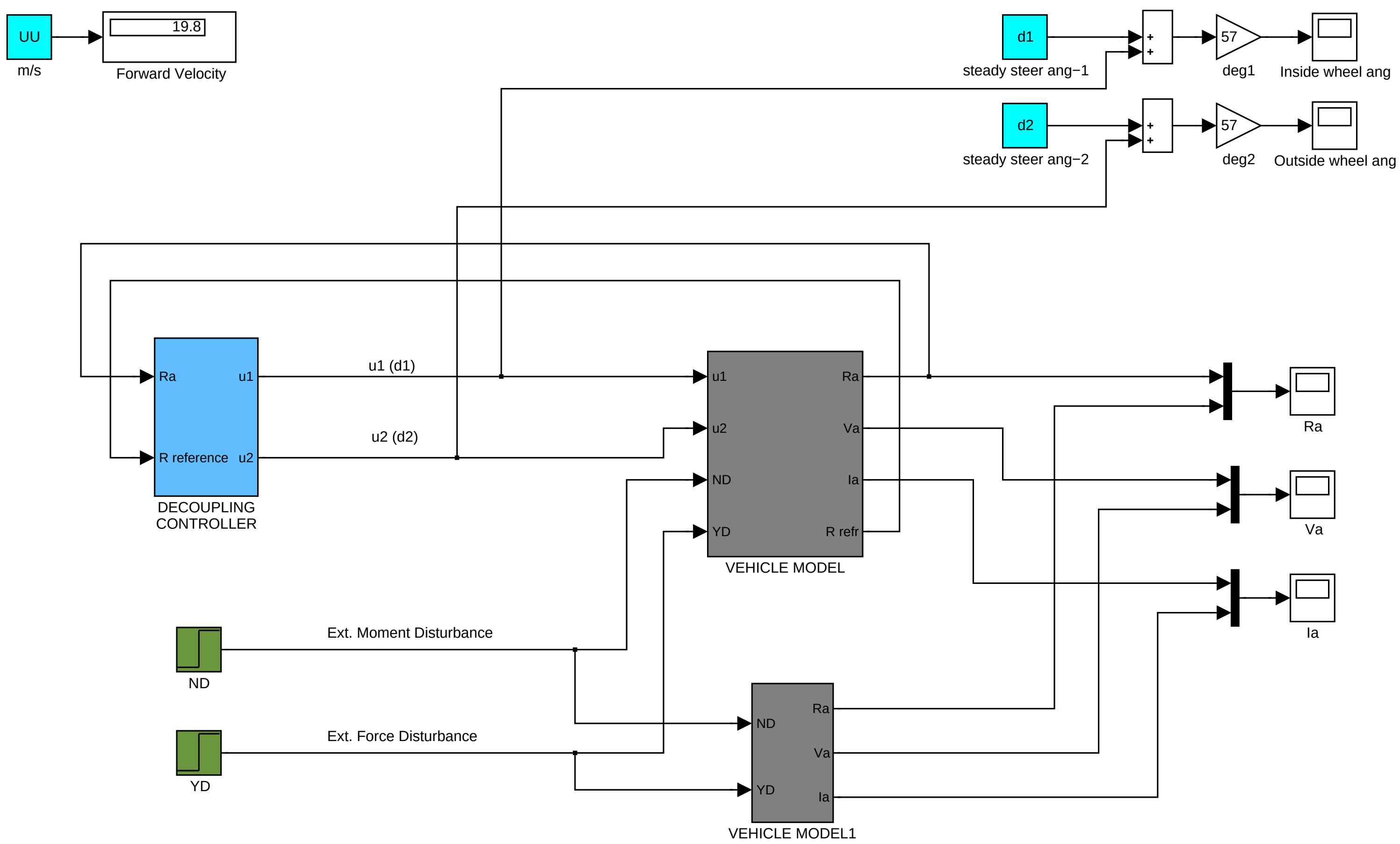

Figure 5.1: Simulink Control Block

loop for the lateral velocity is fed with the error signal, $V_{error} = 0$. Thus, any change in yaw rate of a vehicle due to disturbance will be attenuated and there will be no change in the vehicle's lateral velocity response that occurs due to disturbance. The initial state here is at δ_s=4^o.

The external force and moment is applied as step-input at time, t= 3 seconds. The simulation time is 10 seconds. Forward velocity of the vehicle is taken to be 20 m/sec.

As seen from the Figure 5.2 and 5.3, the effect of disturbance on yaw rate is attenuated and the steady-state error is zero while the lateral velocity response in case of vehicle both with and without the controller is the same. Also, as it can be observed from the equations of motion that the yaw response and roll response of the vehicle are highly coupled, the effect of disturbance on roll angle is also attenuated along with the yaw rate. Thus, the change in the roll angle due to disturbance also reduces to zero.

Table 5.2: Response curves

Response without controller	———
Response with decoupling controller	- - - - - -

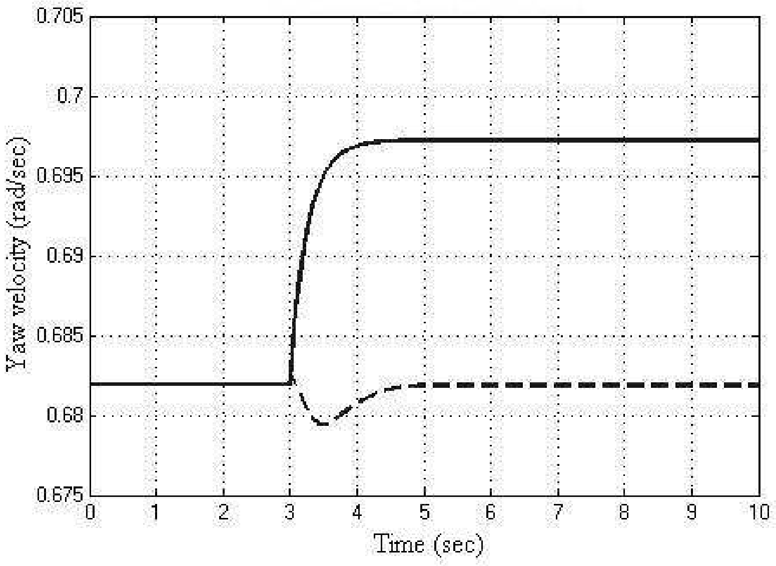

Figure 5.2: Variation of Yaw Velocity with time

Figures 5.5 and 5.6 show how the inside and outside steering angles vary with time. The steady-

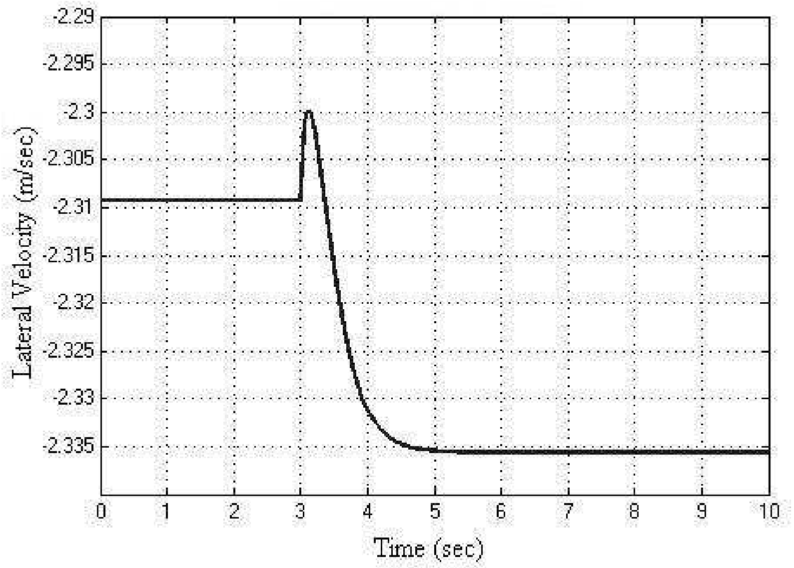

Figure 5.3: Variation of Lateral Velocity with time

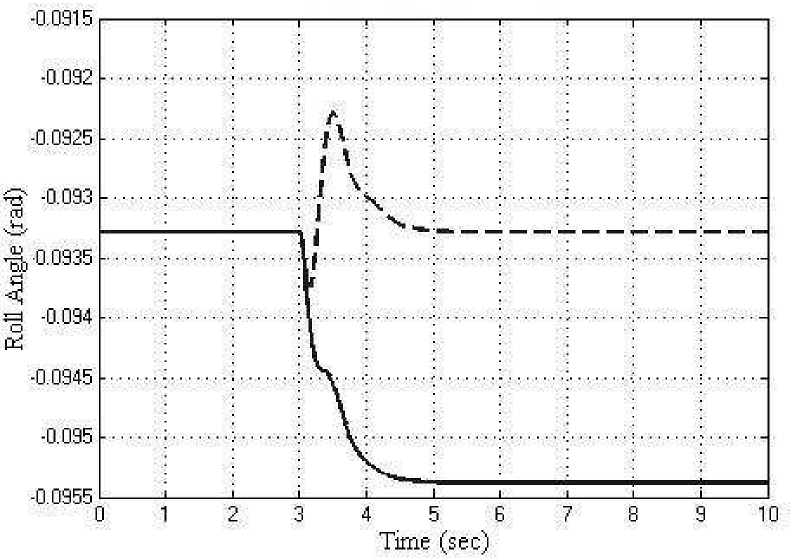

Figure 5.4: Variation of Roll Angle with time

state steering angles before the disturbance are, $\delta_1 = 4.1^o$ on the inside wheel and $\delta_2 = 3.9^o$ on the outside wheel. After the disturbance is applied, the angle on the inside wheel increases to $\delta_1 = 7.2^o$ and that on the inside wheel reduces to $\delta_2 = -0.1^o$ in order to produce the desired response.

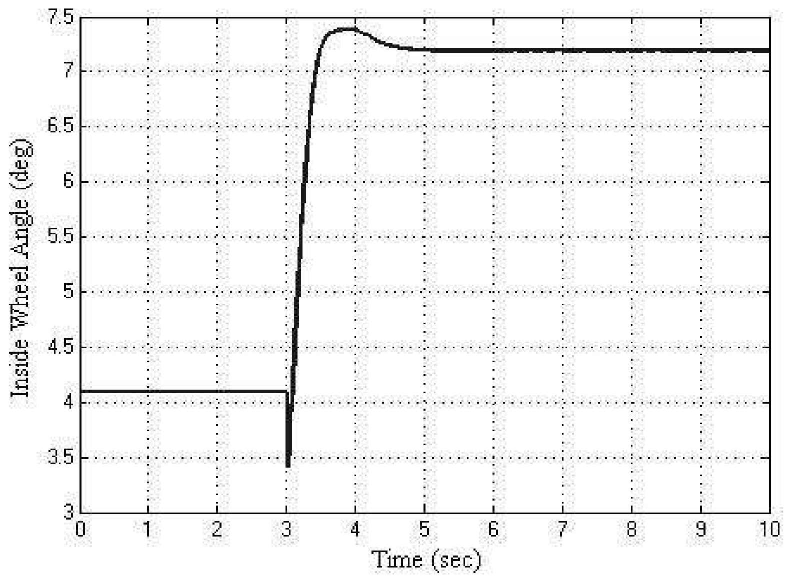

Figure 5.5: Variation of inside front wheel angle with time

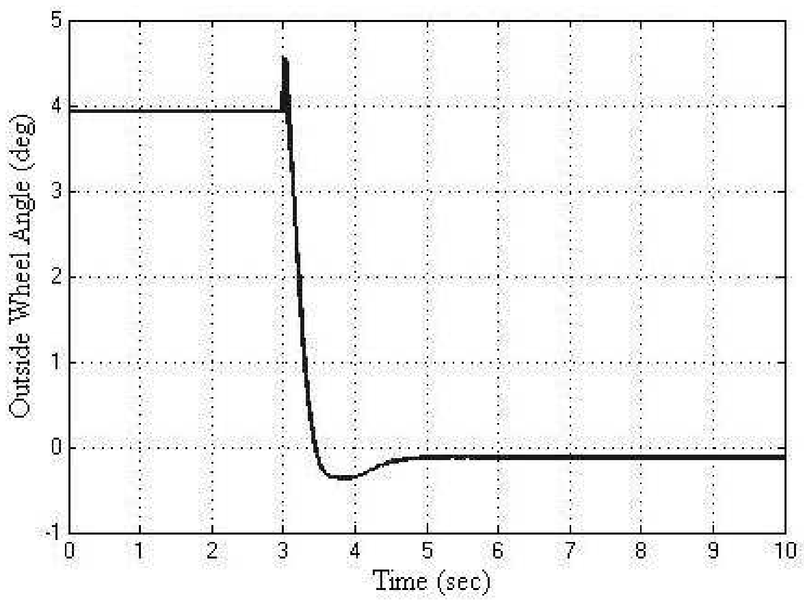

Figure 5.6: Variation of outside front wheel angle with time

It will be useful to note that in this example, we have not considered the following things which may be important in the case of real-life vehicles,

1. The amount and rate at which the front wheels can be actuated or steering-angles changed may be limited due to practical limitations of the actuation system used.

2. The parameters of the vehicle model on the basis of which the controller is designed may vary with time or situations such as mass of the vehicle. This problem can be resolved if the controller is made robust to these uncertainities.

3. There may be other unaccounted disturbances acting on the vehicle or on the controller itself and presence of noise etc.

Chapter 6

Conclusion and Future Work

In this thesis, we have studied the steering action of a vehicle which is responsible for its lateral dynamics. We studied that in order to have a control over more than one variables of the vehicle's handling characteristics, we need to have atleast as much number of inputs. We introduced the concept of independent steering system in which we have two independent steering inputs and therefore there is a possibility of controlling two output variables independently. We developed a 3 degree-of-freedom model and choose the two degrees of yaw rate and lateral velocity to be controlled independently. We showed that by means of decoupling process, we can decompose the plant into two seperate SISO loops and control them seperately. We designed the controller for our model and choose the area of disturbance attenuation to successfully show that on one side we can attenuate the effects of disturbance on yaw rate and on the other side do not disturb its affect on the lateral velocity of the vehicle.

We can say that this opens up a new area of study of relationship between the characteristics of response variables and developing strategies for relatively controlling them in order to achieve the overall improvement in vehicle handling. The future work in this field may include the following,

1. Conducting experiments on a actual vehicle to find out how the controller perform in the real-life situations. This may include some modifications in the controller depending on the vehicle instrumentation and feedback available.

2. Improving the considered model by including more degrees of freedom and taking other effects that occur during vehicle's lateral response into consideration. The more degrees of freedom may include that of pitching, different suspension characteristics on individual wheels etc. The

effects such as non-linear behavior of tire at higher slip angles and load transfer between inside and outside wheel due to shifting of centre-of-gravity can be taken into consideration.

3. Identifying other safety-critical or comfort-based real-life situations where the improvement in handling behavior of a vehicle needs attention. Analyzing the response characteristics involved and studying the interaction among them and how there manipulation can have an overall improvement in the vehicle's handling behavior.

4. Devising new methods by which we can have control over more than two response characteristics. This implies developing control strategies through which we can achieve an overall optimum balance between the choosen variables of interest in vehicle's response for the given situation.

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Appendix A

Derivation of Equations of Motion and coefficients used

A.1 Coefficients

In our derivation of the equations of motion, we have defined certain coefficients, which are functions of vehicle physical parameters, in order to simplify our equations. They are as defined as below:

$$Y_d = -C_f \tag{A.1}$$

$$Y_\beta = 2 * (C_f + C_r) \tag{A.2}$$

$$Y_r = 2 * (a * C_f - b * C_r)/U \tag{A.3}$$

$$Y_i = 2 * (-E_f * C_f - E_r * C_r + Y_{fi} + Y_{ri}) \tag{A.4}$$

$$N_d = -a * C_f \tag{A.5}$$

$$N_\beta = 2 * (a * C_f - b * C_r + A_f + A_r) \tag{A.6}$$

$$N_r = 2 * (a^2 * C_f + b^2 * C_r)/U \tag{A.7}$$

$$N_i = 2 * (-a * E_f * C_f + b * E_r * C_r - E_f * A_f - E_r * A_r + a * Y_{fi} - b * Y_{ri} \tag{A.8}$$

$$N_p = C_f * a * p/U \tag{A.9}$$

$$N_q = C_f * a * q/U \tag{A.10}$$

$$N_g = C_f * q \tag{A.11}$$

$$N_h = C_f * p \tag{A.12}$$

$$L_\phi = L_{if} + L_{ir} + M_s * g * h \tag{A.13}$$

$$L_p = L_{pf} + L_{pr} \tag{A.14}$$

A.2 Derivation of the equations of motion

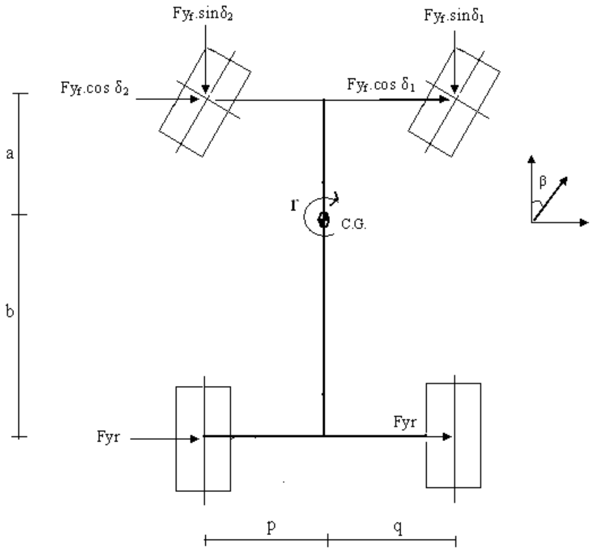

Figure A.1: Four-Wheel Model

Consider a vehicle taking a turn. Here, U is the forward velocity, V is the lateral velocity, and r is the yaw velocity of a vehicle. As the steering angles are small, we can take, $sin\delta = \delta$ and $cos\delta = 1$

Consider the slip angle at the front wheels.

Slip angle at the inside wheel is,

$$\alpha_{f1} = \frac{V + ar}{U} - \delta_1 \tag{A.15}$$

Slip angle at the outside wheel is,

$$\alpha_{f2} = \frac{V + ar}{U} - \delta_2 \tag{A.16}$$

Slip angle at both the rear wheels will be same given by,

$$\alpha_{r1,2} = \frac{V - br}{U} \tag{A.17}$$

We know that the Newton's Second law is given by,

$$Mass \times Acceleration = Force$$

Applying Newton's Second law in the lateral direction,

$$M\left(\dot{V} + Ur\right) + M_s h\ddot{\phi} = C_f\left(\alpha_{f1} + \alpha_{f2}\right) + 2.C_r\alpha_r \tag{A.18}$$

By substituting the relation for slip angles in the above equation and simplifying,

$$M\left(\dot{V} + Ur\right) + M_s h\ddot{\phi} = 2\left(C_f + C_r\right)\left(\frac{V}{U}\right) + 2\left(aC_f - bC_r\right)\left(\frac{r}{U}\right) - C_f\left(\delta_1 + \delta_2\right) \tag{A.19}$$

Substituting for the coefficients defined earlier, we finally get,

$$M\left(\dot{V} + Ur\right) + M_s h\ddot{\phi} = Y_b\beta + Y_r r + Y_\delta\left(\delta_1 + \delta_2\right) \tag{A.20}$$

The above equation gives the equation of motion in the lateral direction.
Applying Newton's Second law around vertical axis,

$$I_z\dot{r} - P\ddot{\phi} = a.C_f\left(\alpha_{f1} + \alpha_{f2}\right) - 2b.C_r\alpha_r + q.C_f\alpha_{f1}\delta_1 - p.C_f\alpha_{f2}\delta_2 \tag{A.21}$$

By simplification and substitution, we get,

$$I_z\dot{r} - P\ddot{\phi} = N_\beta\beta + N_r r + N_\phi\phi + N_\delta\delta_1 + N_\delta\delta_2 + N_g\beta\delta_1 - N_h\beta\delta_2 + N_q r\delta_1 - N_p r\delta_2 - N_g\delta_1^2 + N_h\delta_2^2 \tag{A.22}$$

The above equation is the equation of motion around the vertical axis.
Similarly, by the application of Newton's Second law around longitudinal axis, we get the third

equation of motion as follows,

$$I_x\ddot{\phi} - P\dot{r} + M_s h\dot{V} + M_s hUr = L_\phi + L_p\ddot{\phi} \tag{A.23}$$

Thus, the equations $A.20, A.22$ and $A.23$ give the equations of motion of a vehicle.

Appendix B

Vehicle parameters and transient model obtained for the prototype small car

B.1 Vehicle Parameters

The vehicle parameter values used for construction of our model are as listed below (in metric units). For more details, one can refer [5]. .

M = 875 kg

$M_s = 773kg$

a=1.3 m

b=0.8 m

p=0.61 m

q=0.61 m

h=0.3 m

$I_z = 1029kg - m^2$

$I_x = 277kg - m^2$

P=-37 kg-m

$E_f = 0$

$E_r = 0$

$C_f = -29103N/rad$

$C_r = -42720 N/rad$

$A_{\alpha f} = 768 N - m$

$A_{\alpha r} = 1241 N - m$

$Y_{ff} = 165 N/rad$

$Y_{fr} = 2430 N/rad$

$L_{\phi} = -31000 N - m/rad$

$L_p = -2100 N - m/rad/s$

B.2 Transient Model

The Transient Model of a vehicle is obtained by taking Laplace Transform of all the three equations of motion and writing the equations in the vector-matrix form in order to get the solution for $\Delta R(s), \Delta V(s)$ and $\Delta\phi(s)$ for four different inputs. This was done using *Symbolic Toolbox* of *Matlab*. Thus, we obtain a set of 12 different Transfer Functions which can be write in a Transfer Matrix form as below,

$$\begin{bmatrix} \Delta R \\ \Delta V \\ \Delta\phi \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \end{bmatrix} * \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ Y_D \\ N_D \end{bmatrix} \tag{B.1}$$

The Transfer functions obtained for each of the input-output pair are listed below. The forward velocity in this case was taken to be 10 m/sec.:

$$G_{11} = \frac{5.956e038s^3 + 1.192e040s^2 + 1.304e041s + 6.917e041}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.2}$$

$$G_{21} = \frac{2.177e039s^3 + 1.322e040s^2 + 1.771e041s - 9.036e041}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.3}$$

$$G_{31} = \frac{-5.786e038s^2 - 4.477e039s - 5.205e040}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.4}$$

$$G_{12} = \frac{5.359e038s^3 + 1.072e040s^2 + 1.173e041s + 6.22e041}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.5}$$

$$G_{22} = \frac{2.175e039s^3 + 1.512e040s^2 + 1.957e041s - 6.355e041}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.6}$$

$$G_{32} = \frac{-5.755e038s^2 - 4.433e039s - 4.68e040}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.7}$$

$$G_{13} = \frac{1.835e033s^3 + 8.972e033s^2 + 3.582e035s + 1.002e036}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.8}$$

$$G_{23} = \frac{6.577e035s^3 + 9.783e036s^2 + 1.102e038s + 5.335e038}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.9}$$

$$G_{33} = \frac{-1.675e035s^2 - 1.239e036s - 7.539e034}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.10}$$

$$G_{14} = \frac{4.061e034s^3 + 8.188e035s^2 + 8.901e036s + 4.741e037}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.11}$$

$$G_{24} = \frac{1.835e033s^3 - 1.293e036s^2 - 1.263e037s - 1.824e038}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.12}$$

$$G_{34} = \frac{-2.119e033s^2 - 2.937e034s - 3.567e036}{3.076e037s^4 + 8.479e038s^3 + 1.131e040s^2 + 8.589e040s + 2.628e041} \tag{B.13}$$