Manufacturing Process Optimization

Application to Machining

Selection of Cutting Conditions

- Selection of cutting speed, or cutting speed and feed can be made using unconstrained or constrained mathematical optimization methods.

- Optimization is based on time, cost, or profit rate criteria.

Optimization Criteria

- Commonly employed optimization criteria (objectives) are:
  - ***Max. production rate*** or ***min. production time***: aims to maximize number of parts produced in a unit time interval or minimizes the time per unit part. Neglects cost and/or profit.
  - ***Min. production cost***: aims to determine the least production cost. Coincides with max. profit rate criterion for constant unit revenue. Ignores time constraints.
- Usually one of the two criteria is used. Sometimes both criteria are used simultaneously.

Decision Variables

- Depth of cut (*d*)
- Feed (*f*)
- Cutting speed (*V*)

Depth of cut usually decided based on part geometry and material to be removed and can be assumed constant.

Definitions (1)

- **Unit production time, *t*** (min/pt): time to manufacture a unit of the product.

$$t = t_p + t_m + t_r$$

Where,

t_p = setup time (min/pt); includes time needed to load/unload parts into machine, tool setup time, etc.

t_m = machining time (min/pt)

t_r = total tool replacement time (min/pt) = $t_c(t_m/T)$

t_c = time needed to replace a worn cutting edge with a new one (min/edge)

T = tool life (min)

- **Production rate** = $(1/t)$

Definitions (2)

- **Unit production cost, *u*** (\$/pt): cost to manufacture a unit of the product.

$$u = u_o + u_m + u_t = k_o t_p + (k_o + k_m)\, t_m + [k_t + k_o t_c]\,(t_m/T)$$

Where,

u_o = capacity utilization cost (\$/pt); includes machine cost, labor cost, overhead etc. = $k_o t = k_o[t_p + t_m + t_c(t_m/T)]$

k_o = machine utilization rate (\$/min)

u_m = machining cost (associated with actual machining time); includes cost of electricity, cutting fluids etc. = $k_m\, t_m$

k_m = machining overhead (\$/min)

u_t = tool utilization cost; includes cost of cutting tool, tool resharpening, etc. = $k_t\,(t_m/T)$

k_t = cost per cutting edge (\$/edge)

Unconstrained Optimization (1)

- Basic problem formulation: Find cutting speed V that optimizes (Minimizes or Maximizes) Z, where Z = appropriate optimization criterion.

- In order to solve this problem, the objective function Z should be expressed in terms of V. Feed and depth of cut are assumed to be fixed.

- Note that only tool life T and the machining time t_m are functions of V.

Unconstrained Optimization (2)

- In general,

$$t_m = \lambda / V$$

Where λ is a constant based on the machining process and V is in m/min units.

- For turning, boring, drilling, and reaming operations:

$$t_m = L/(Nf) \text{ and } V = \pi \mathrm{D}N/1000$$

$$\lambda = \pi DL/(1000f)$$

where D = diameter of part/tool (mm), L = length of cut (mm), f = feed (mm/rev), N = spindle speed (rpm)

Unconstrained Optimization (3)

- Taylor's tool life equation:

$$VT^n = C$$

Where T = tool life (min), C = 1 min. tool life cutting speed, n = function of tool-workpiece material.

Unconstrained Optimization (4)

- Expressing *t*, *u* in terms of *V*, we get:

$$t = t_p + (\lambda/V) + t_c \left(\lambda V^{\frac{1-n}{n}} \Big/ C^{1/n} \right)$$

$$u = k_o t_p + (k_o + k_m)(\lambda/V) + (k_t + k_o t_c)\left(\lambda V^{\frac{1-n}{n}} \Big/ C^{1/n} \right)$$

Unconstrained Optimization (5)

- Optimal cutting speed (and corresponding tool life) can be obtained by standard calculus as:
- Min. time or max. production rate criterion:

$$V_{opt} = C \Big/ \left[\left(\frac{1-n}{n} \right) t_c \right]^n$$

$$T_{opt} = \left(\frac{1-n}{n} \right) t_c$$

Unconstrained Optimization (6)

- Min. cost criterion:

$$V_{opt} = C\left[\frac{(k_o + k_m)}{\left(\frac{1-n}{n}\right)(k_t + k_o t_c)}\right]^n$$

$$T_{opt} = \left(\frac{1-n}{n}\right)\left[\frac{(k_t + k_o t_c)}{(k_o + k_m)}\right]$$

Unconstrained Optimization Example (1)

- Given:
  - Taylor tool life equation $VT^{0.23} = 430$, V (m/min)
  - Machining (turning) parameters
    - Depth of cut, $d = 1.00$ mm
    - Feed, $f = 0.2$ mm/rev
  - Workpiece geometry
    - Workpiece diameter, $D = 50$ mm
    - Workpiece length, $L = 200$ mm
  - Time parameters
    - Setup time, $t_p = 0.75$ min/pt
    - Tool replacement time, $t_c = 1.5$ min/edge

Unconstrained Optimization Example (2)

- Given:
  - Cost parameters
    - Machine utilization rate, k_o = 0.50 \$/min
    - Machining overhead, k_m = 0.05 \$/min
    - Tool cost, k_t = 2.50 \$/edge

Unconstrained Optimization Example (3)

- Min. time or max. production rate cutting speed and tool life:

$$V_{opt} = C \Big/ \left[\left(\frac{1-n}{n} \right) t_c \right]^n = 430 \Big/ \left[\left(\frac{1-0.23}{0.23} \right) 1.5 \right]^{0.23} = 296 \text{ m/min}$$

$$T_{opt} = \left(\frac{1-n}{n} \right) t_c = \left(\frac{1-0.23}{0.23} \right) 1.5 = 5 \text{ min}$$

Unconstrained Optimization Example (4)

- Min. cost cutting speed and tool life:

$$V_{opt} = C\left[\frac{(k_o + k_m)}{\left(\frac{1-n}{n}\right)(k_t + k_o t_c)}\right]^n = 430\left[\frac{(0.5+0.05)}{\left(\frac{1-0.23}{0.23}\right)(2.5+0.5\times1.5)}\right]^{0.23} = 216 \text{ m/min}$$

$$T_{opt} = \left(\frac{1-n}{n}\right)\left[\frac{(k_t + k_o t_c)}{(k_o + k_m)}\right] = \left(\frac{1-0.23}{0.23}\right)\left[\frac{(2.5+0.5\times1.5)}{(0.5+0.05)}\right] = 19.8 \text{ min}$$

Summary

- Manufacturing process optimization
  - Application to machining
- Unconstrained optimization
  - Optimum cutting speeds
  - Production time and cost criteria