

# Deformation Processing - Rolling

ver. 1

Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Overview

- Process
- Equipment
- Products
- Mechanical Analysis
- Defects



# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



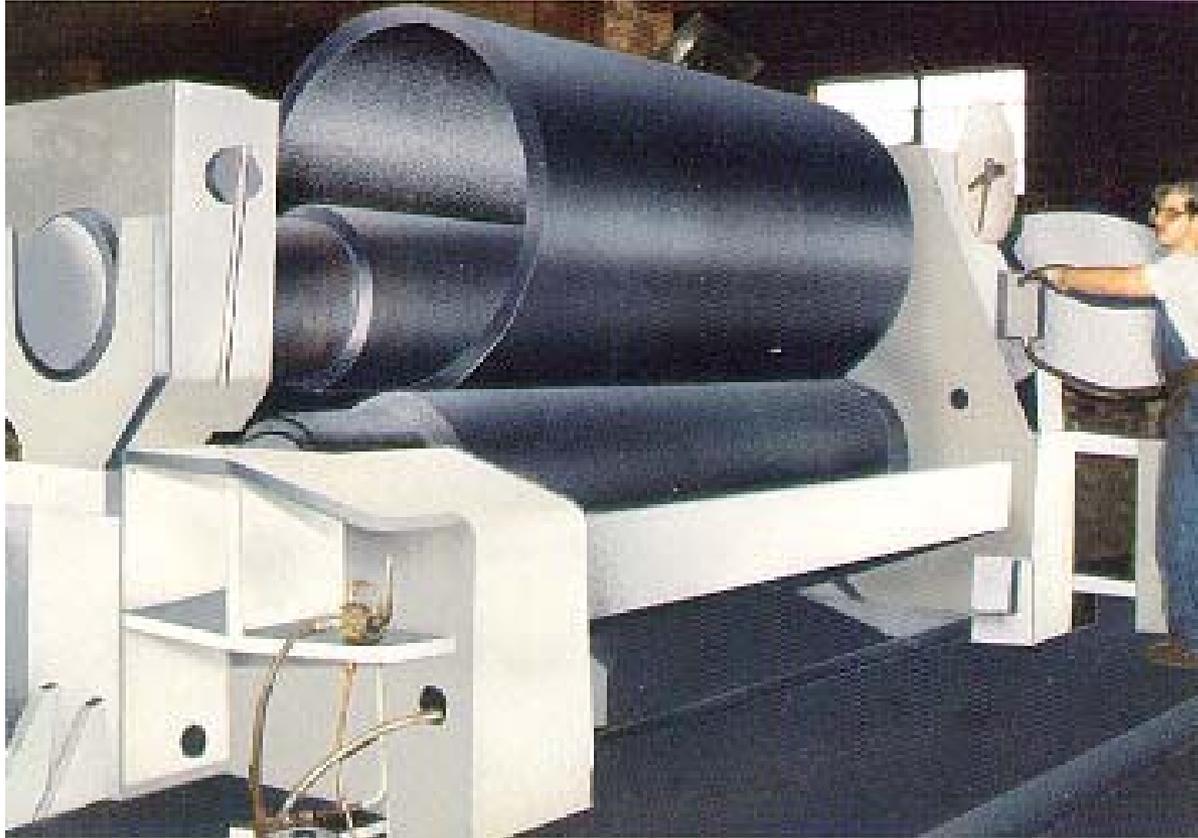
# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Ring Rolling



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



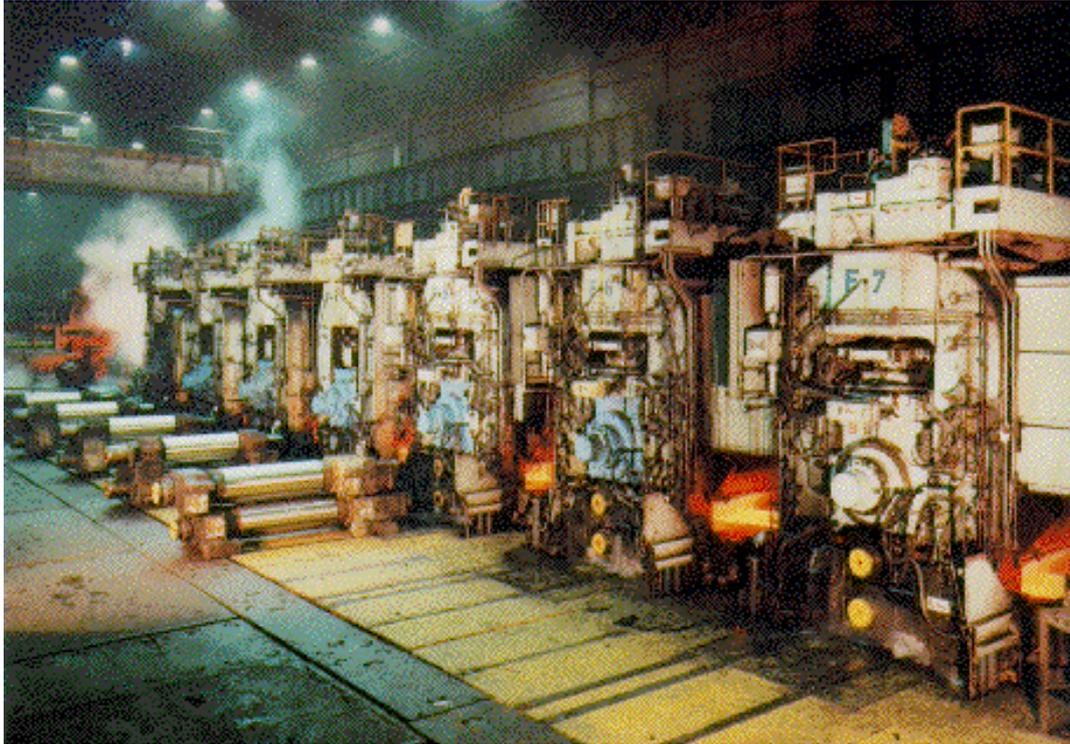
# Equipment



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Equipment

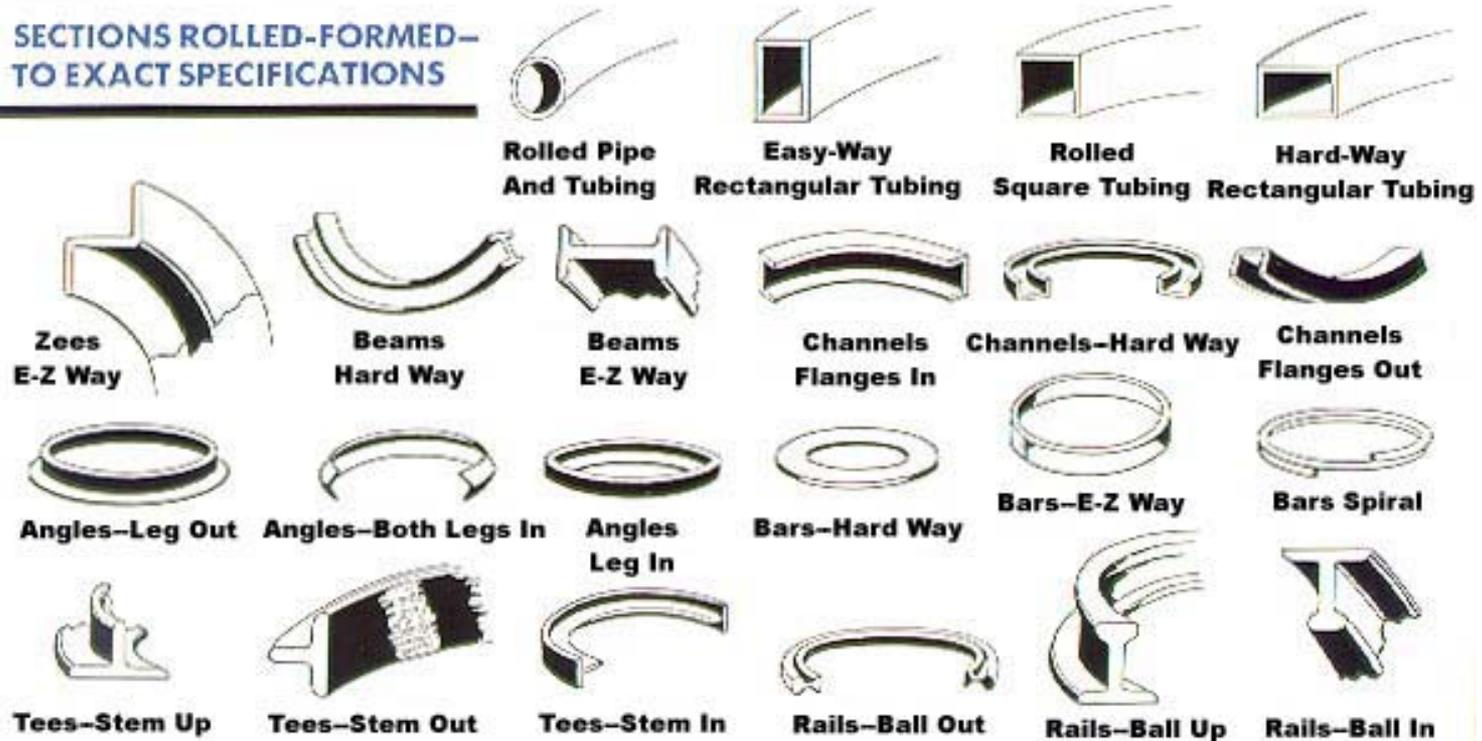


Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Products

**SECTIONS ROLLED-FORMED-  
TO EXACT SPECIFICATIONS**



# Products

- Shapes
  - I-beams, railroad tracks
- Sections
  - door frames, gutters
- Flat plates
- Rings
- Screws



# Products

- A greater volume of metal is rolled than processed by any other means.



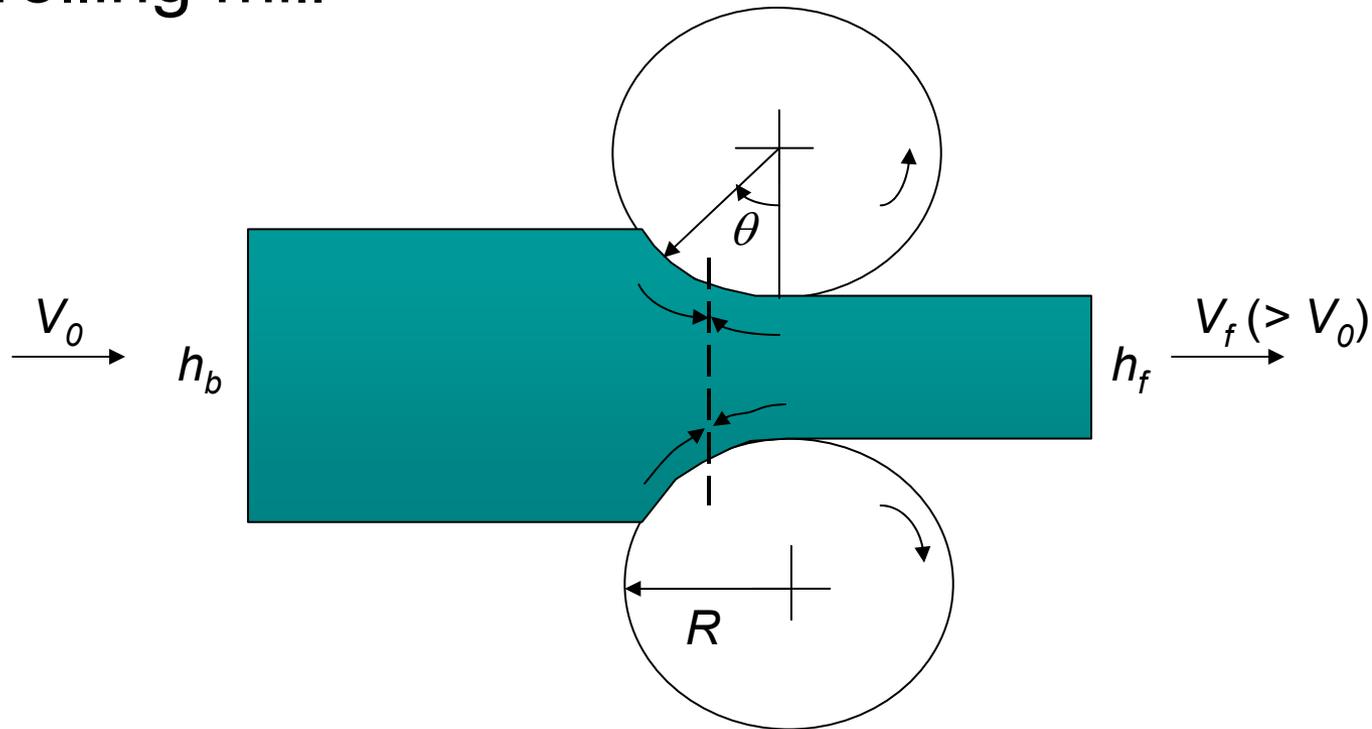
# Rolling Analysis

- Objectives
  - Find distribution of roll pressure
  - Calculate roll separation force (“rolling force”) and torque
  - Processing Limits
  - Calculate rolling power



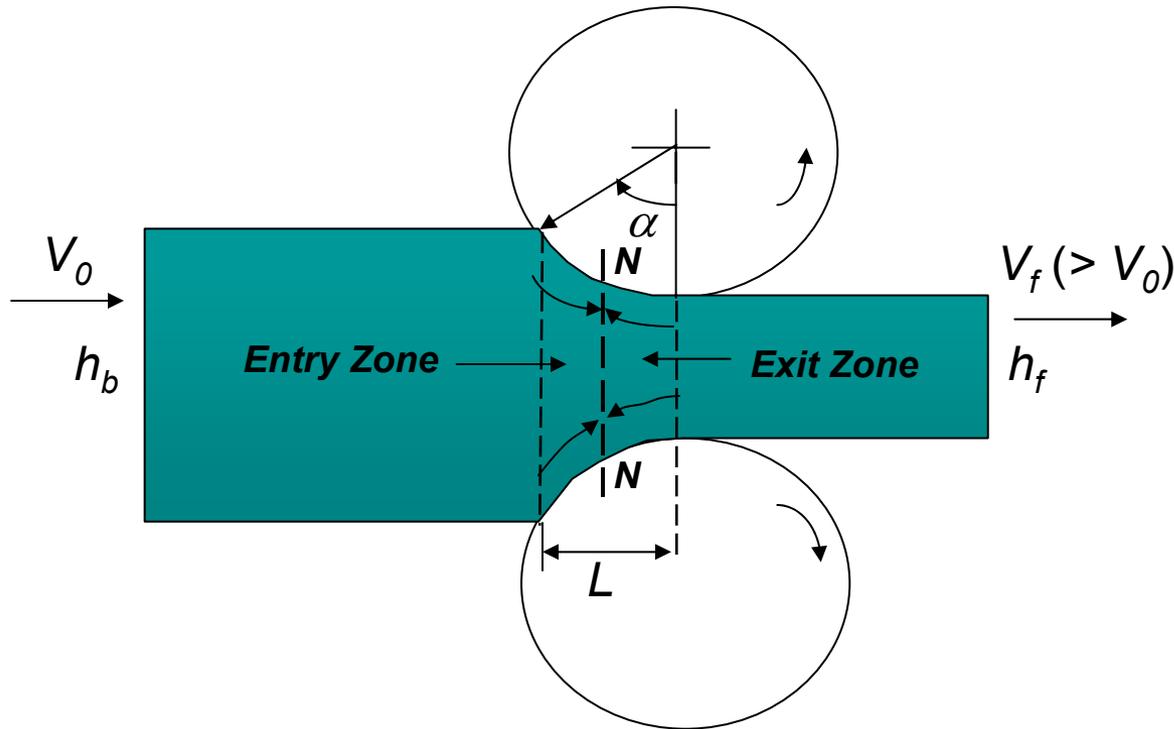
# Flat Rolling Analysis

- Consider rolling of a flat plate in a 2-high rolling mill



*Width of plate  $w$  is large  $\rightarrow$  plane strain*

# Flat Rolling Analysis

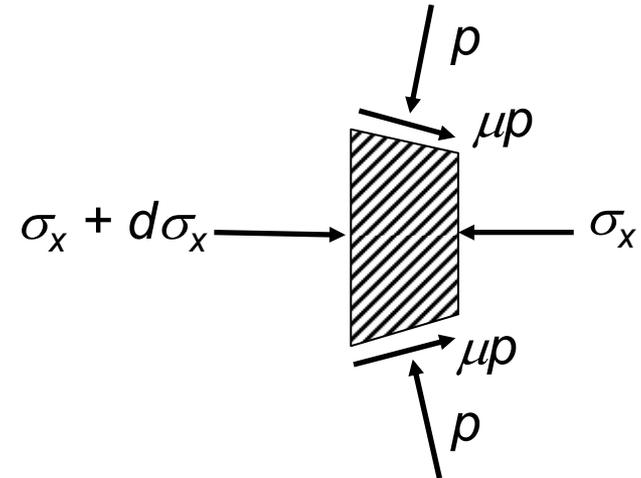
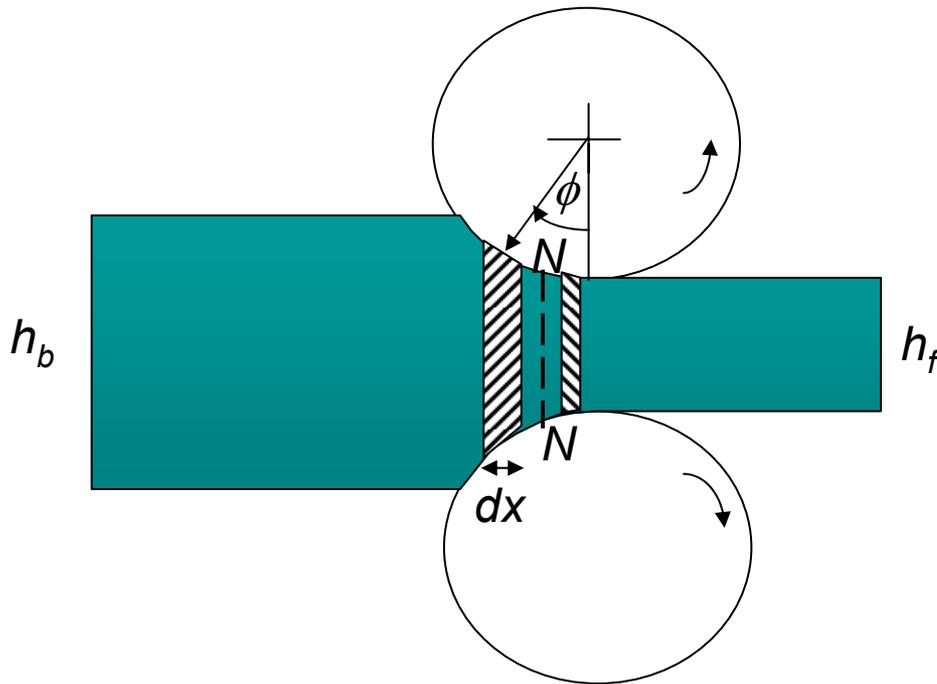


- Friction plays a critical role in enabling rolling  $\rightarrow \mu \geq \tan \alpha$   
cannot roll without friction; for rolling to occur
- Reversal of frictional forces at neutral plane (NN)

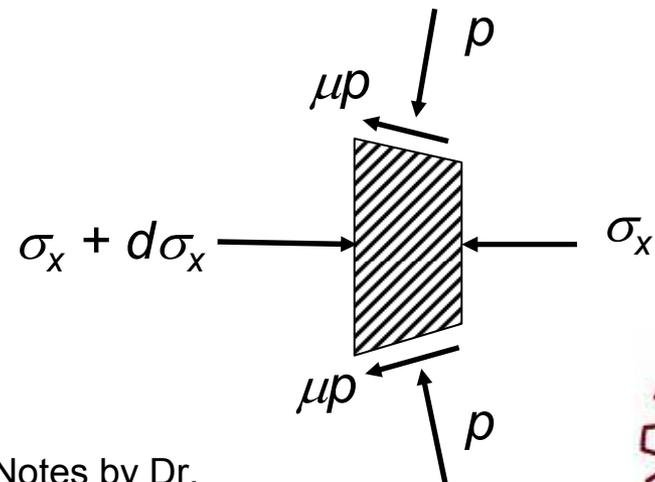


# Flat Rolling Analysis

## Stresses on Slab in Entry Zone



## Stresses on Slab in Exit Zone



# Equilibrium

- Applying equilibrium in x (top entry, bottom exit)

$$(\sigma_x + d\sigma_x) \cdot (h + dh) - 2pR \cdot d\phi \cdot \sin \phi \pm 2\mu pR \cdot d\phi \cdot \cos \phi - \sigma_x h = 0$$

Simplifying and ignoring HOTs

$$\frac{d(\sigma_x h)}{d\phi} = 2pR \cdot (\sin \phi \mp \mu \cos \phi)$$



# Simplifying

- Since  $\alpha \ll 1$ , then  $\sin\phi = \phi$ ,  $\cos\phi = 1$

$$\frac{d(\sigma_x h)}{d\phi} = 2pR \cdot (\phi \mp \mu)$$

- Plane strain, von Mises

$$p - \sigma_x = 1.15 \cdot Y_{flow} \equiv Y'_{flow}$$



# Differentiating

- Substituting

$$\frac{d[(p - Y'_{flow}) \cdot h]}{d\phi} = 2pR \cdot (\phi \mp \mu)$$

- or

$$\frac{d}{d\phi} \left[ Y'_{flow} \cdot \left( \frac{p}{Y'_{flow}} - 1 \right) \cdot h \right] = 2pR \cdot (\phi \mp \mu)$$



# Differentiating

$$Y'_{flow} \cdot h \cdot \frac{d}{d\phi} \left( \frac{p}{Y'_{flow}} \right) + \left( \frac{p}{Y'_{flow}} - 1 \right) \cdot \frac{d}{d\phi} (Y'_{flow} \cdot h) = 2pR \cdot (\phi \mp \mu)$$

Rearranging, the variation  $Y'_{flow} \cdot h$  with respect to  $\phi$  is small compared to the variation  $p/Y'_{flow}$  with respect to  $\phi$  so the second term is ignored

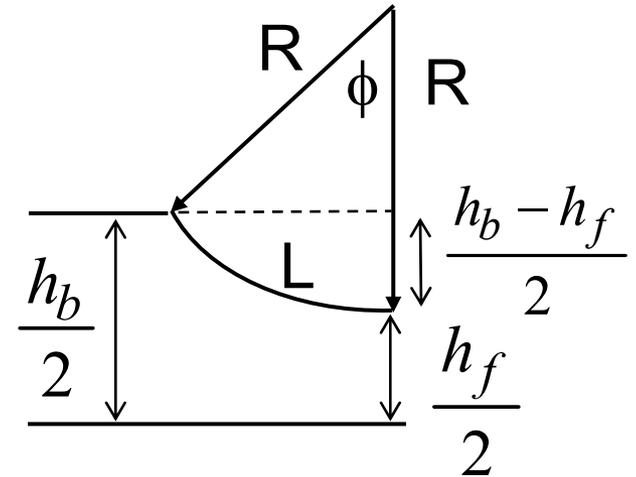
$$\frac{\frac{d}{d\phi} \left( \frac{p}{Y'_{flow}} \right)}{\frac{p}{Y'_{flow}}} = \frac{2R}{h} (\phi \mp \mu)$$



# Thickness

$$h = h_f + 2R \cdot (1 - \cos \phi)$$

from the definition  
of a circular segment



or, after using a Taylor's series expansion, for small  $\phi$

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

0

$$h = h_f + R \cdot \phi^2$$



# Substituting and integrating

$$\int \frac{d\left(\frac{p}{Y'_{flow}}\right)}{\frac{p}{Y'_{flow}}} = \int \frac{2R}{h_f + R \cdot \phi^2} (\phi \mp \mu) \cdot d\phi$$

$$\text{In[1]:= } \int \frac{2 R (\phi - \mu)}{h f + R \phi^2} d\phi$$

$$\text{Out[1]= } 2 R \left( -\frac{\mu \text{ArcTan}\left[\frac{\sqrt{R} \phi}{\sqrt{h f}}\right]}{\sqrt{h f} \sqrt{R}} + \frac{\text{Log}[h f + R \phi^2]}{2 R} \right)$$

$$\ln \frac{p}{Y'_f} = \ln \frac{h}{R} \mp 2\mu \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right) + \ln C$$



# Eliminating $\ln()$

$$p = C \cdot Y'_{flow} \cdot \frac{h}{R} \exp(\mp \mu H)$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



# Entry region

- at  $\phi = \alpha$ ,  $H = H_b$ ,

$$p = C \cdot Y'_{flow} \cdot \frac{h}{R} \exp(-\mu H)$$

$$C = \frac{R}{h_b} \exp(\mu H_b) \quad p = Y'_{flow} \frac{h}{h_b} \exp(\mu[H_b - H])$$

$$p = (Y'_{flow} - \sigma_{xb}) \frac{h}{h_b} \exp(\mu[H_b - H]) \quad \text{With back tension} = (Y'_{flow} - \sigma_{xb})$$

$$H_b = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \alpha \sqrt{\frac{R}{h_f}} \right)$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



# Exit region

at  $\phi = 0$ ,  $H = H_f = 0$ ,

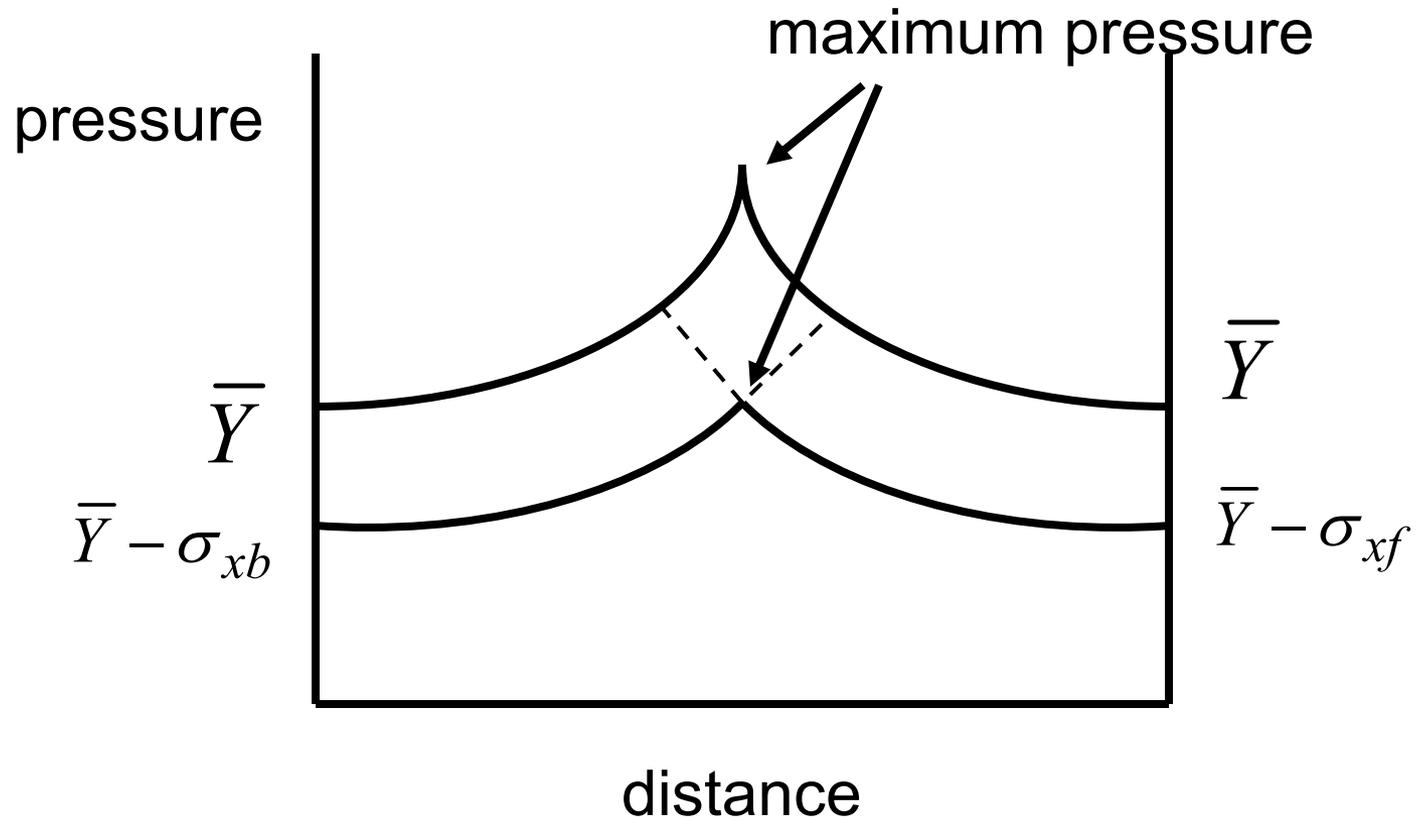
$$C = \frac{R}{h_f} \quad p = (Y'_{flow}) \frac{h}{h_f} \exp(\mu H)$$

$$p = (Y'_{flow} - \sigma_{xf}) \frac{h}{h_f} \exp(\mu H) \quad \text{With forward tension}$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$

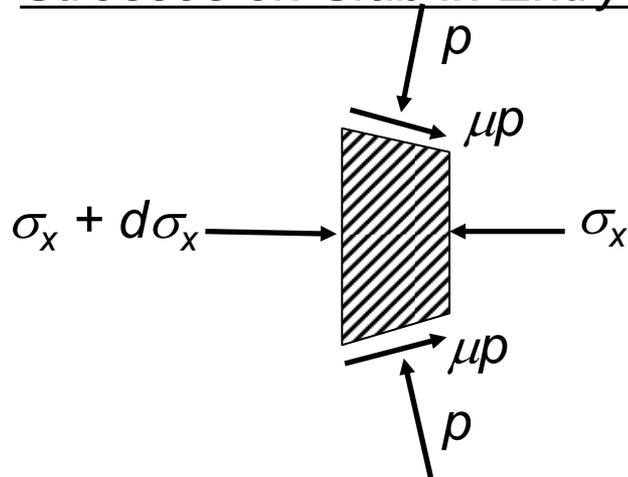


# Effect of back and front tension

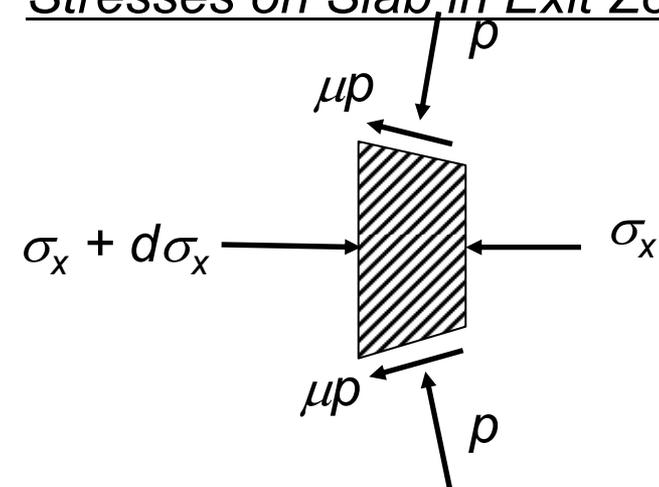


# Flat Rolling Analysis Results – without front and back tension

## Stresses on Slab in Entry Zone



## Stresses on Slab in Exit Zone



Using slab analysis we can derive roll pressure distributions for the entry and exit zones as:  $h_0$  and  $h_b$  are the same thing

$$p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_0} e^{\mu(H_0 - H)}$$

Entry Zone

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \sqrt{\frac{R}{h_f}} \phi \right)$$

$$H_0 = H @ \phi = \alpha$$

$$p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_f} e^{\mu H}$$

Exit Zone



# Average rolling pressure – per unit width

$$p_{ave,entry} = -\frac{1}{R(\alpha - \phi_n)} \int_{\alpha}^{\phi_n} p_{entry} R d\phi; \quad p_{ave,exit} = \frac{1}{R\phi_n} \int_0^{\phi_n} p_{exit} R d\phi$$



# Rolling force

- $F = p_{\text{ave,entry}} \times \text{Area}_{\text{entry}} + p_{\text{ave,exit}} \times \text{Area}_{\text{exit}}$



# Force

- An alternative method

$$F = \int_{\phi_n}^{\alpha} w \cdot p_{entry} \cdot R \cdot d\phi + \int_0^{\phi_n} w \cdot p_{exit} \cdot R \cdot d\phi$$

- again, very difficult to do.



# Force - approximation

$$F / \text{roller} = L w p_{ave}$$

$$L \approx \sqrt{R\Delta h}$$

$$\Delta h = h_b - h_f$$

$$p_{ave} = f\left(\frac{h_{ave}}{L}\right)$$



# Derivation of “L”

circular segment

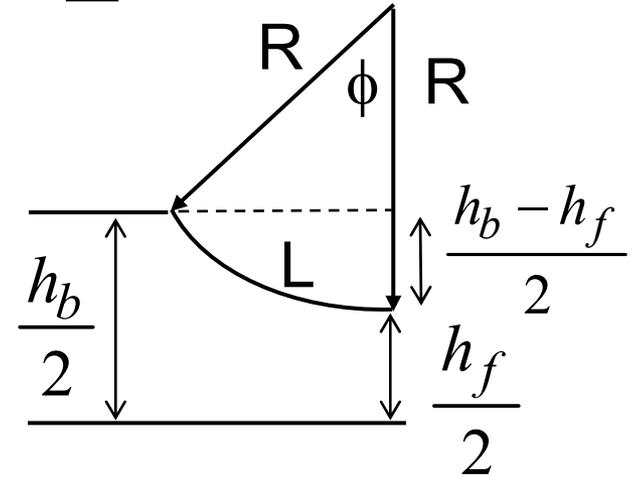
$$h = h_f + 2R \cdot (1 - \cos \phi)$$

Taylor's expansion

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

$$h = h_f + R \cdot \phi^2$$

$$R \cdot \phi = L$$



# Derivation of “L”

setting  $h = h_b$  at  $\phi = \alpha$ , substituting, and rearranging

$$h_b - h_f = \Delta h = R \cdot \left( \frac{L}{R} \right)^2$$

or

$$L = \sqrt{R \cdot \Delta h}$$



# Approximation based on forging plane strain – von Mises

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$

average flow stress:  
due to shape of element



# Small rolls or small reductions

$$\Delta = \frac{h_{ave}}{L} \gg 1$$

- friction is not significant ( $\mu \rightarrow 0$ )

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$

0

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow}$$



# Large rolls or large reductions

$$\Delta \equiv \frac{h_{ave}}{L} \ll 1$$

- Friction is significant (forging approximation)

$$P_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$



# Force approximation: low friction

$$\Delta \equiv \frac{h_{ave}}{L} \gg 1$$

$$F_{/roller} = 1.15 \cdot Lw\bar{Y}_{flow}$$



# Force approximation: high friction

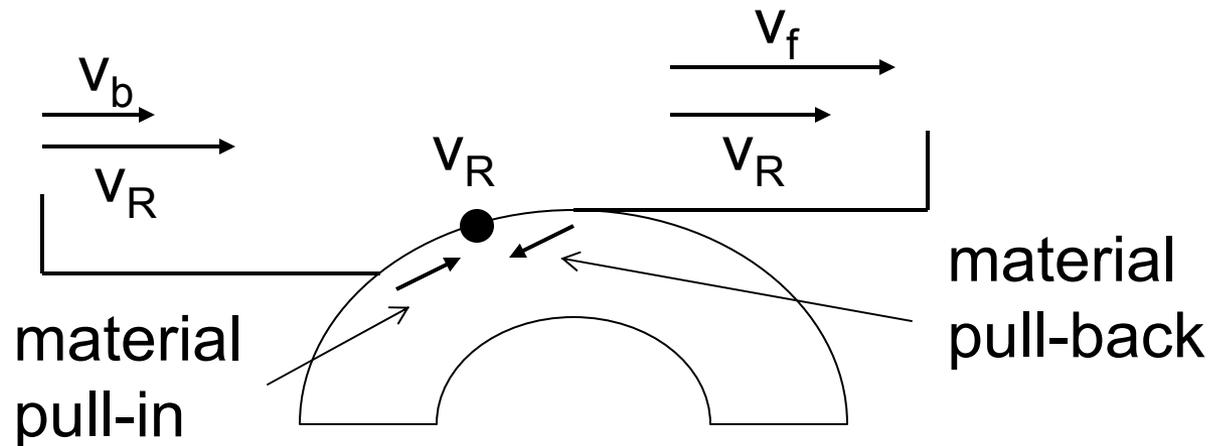
$$\Delta \equiv \frac{h_{ave}}{L} \lll 1$$

$$F_{roller} = 1.15 \cdot Lw\bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$



# Zero slip (neutral) point

- Entrance: material is pulled into the nip
  - roller is moving faster than material
- Exit: material is pulled back into nip
  - roller is moving slower than material



# System equilibrium

- Frictional forces between roller and material must be in balance.
  - or material will be torn apart
- Hence, the zero point must be where the two pressure equations are equal.

$$\frac{h_b}{h_f} = \frac{\exp(\mu H_b)}{\exp(2\mu H_n)} = \exp(\mu(H_b - 2H_n))$$



# Neutral point

$$H_n = \frac{1}{2} \left( H_b - \frac{1}{\mu} \ln \frac{h_b}{h_f} \right)$$

$$\phi_n = \sqrt{\frac{h_f}{R}} \tan \left( \frac{H_n}{2} \sqrt{\frac{h_f}{R}} \right)$$





# Power

$$\text{Power / roller} = T\omega = F_{\text{roller}}L\omega / 2$$

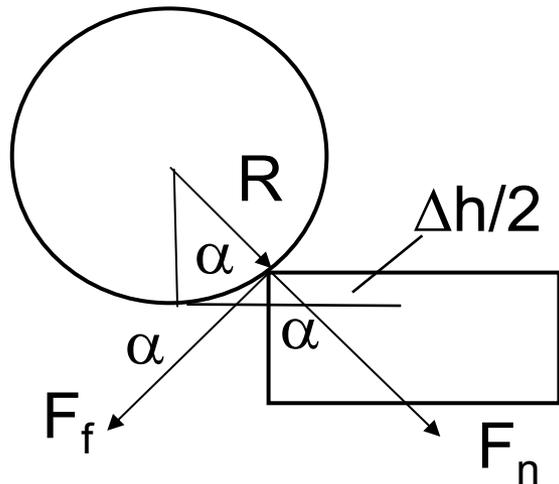
$$\omega = 2\pi N$$

$$N = [\text{rev/min}]$$



# Processing limits

- The material will be drawn into the nip if the horizontal component of the friction force ( $F_f$ ) is larger, or at least equal to the opposing horizontal component of the normal force ( $F_n$ ).



$$F_f \cos \alpha \geq F_n \sin \alpha$$

$$F_f = \mu \cdot F_n$$

$$\tan \alpha = \mu$$

$\mu$  = friction coefficient



# Processing limits

Also

$$\cos \alpha = \frac{R - \frac{\Delta h}{2}}{R} = 1 - \frac{\Delta h}{2R}$$

and  $\Delta h \ll R$   $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$

$$\sin \alpha = \sqrt{1 - 1 + \frac{\Delta h}{2R} - \left(\frac{\Delta h}{2R}\right)^2} \quad \sin \alpha \approx \sqrt{\frac{\Delta h}{R}}$$

$$\tan \alpha = \sqrt{\frac{\frac{\Delta h}{R}}{1 - \frac{\Delta h}{R} + \left(\frac{\Delta h}{2R}\right)^2}} \cong \sqrt{\frac{\Delta h}{R - \Delta h}} \approx \sqrt{\frac{\Delta h}{R}}$$



# Processing limits

So, approximately

$$(\tan \alpha)^2 = \mu^2 = \frac{\Delta h}{R}$$

Hence, maximum draft

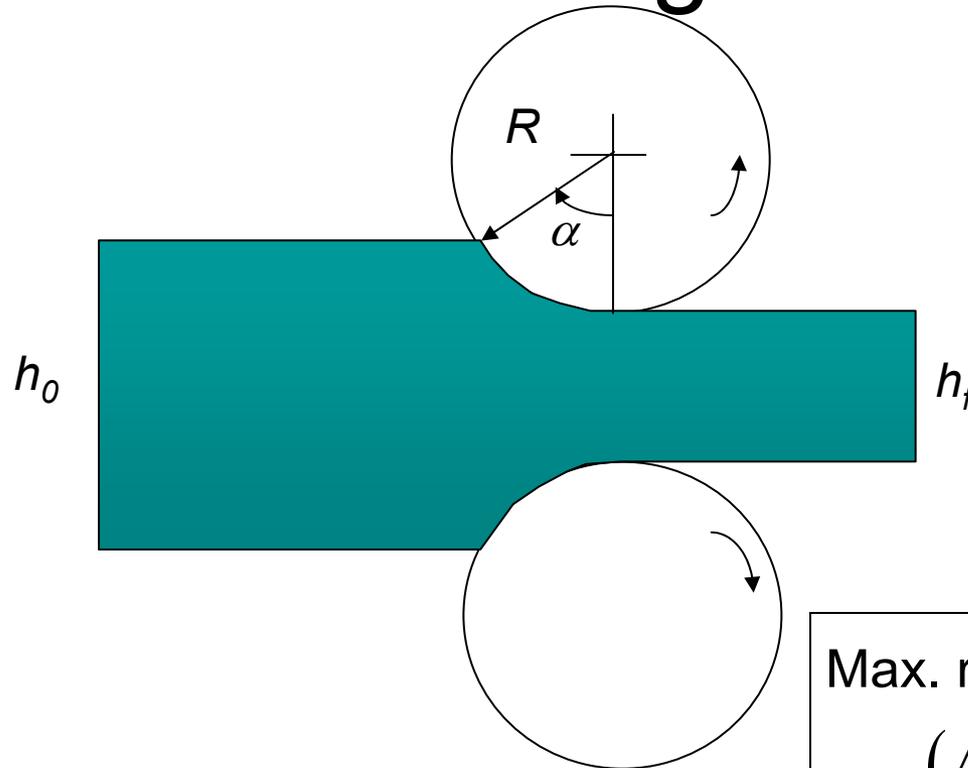
$$\Delta h_{\max} = \mu^2 R$$

Maximum angle of acceptance

$$\phi_{\max} = \alpha = \tan^{-1} \mu$$



# Processing Limits



Max. reduction in thickness

$$(\Delta h)_{\max} = \mu^2 R$$

Max. angle of acceptance

$$\phi_{\max} = \alpha = \tan^{-1} \mu$$



# Cold rolling

(below recrystallization point)  
strain hardening, plane strain – von  
Mises

$$2\tau_{flow} = 1.15 \cdot \bar{Y}_{flow} = 1.15 \cdot \frac{K\varepsilon^n}{n+1}$$

average flow stress:  
due to shape of element



# Hot rolling – (above recrystallization point) strain rate effect, plane strain - von Mises

- Average strain rate

$$\dot{\bar{\epsilon}} = \frac{\bar{\epsilon}}{t} = \frac{V_R}{L} \ln \left( \frac{h_b}{h_f} \right)$$

$$2\tau_{flow} = 1.15 \cdot \bar{Y}_{flow} = 1.15 \cdot C \cdot \dot{\bar{\epsilon}}^m$$



average flow stress:  
due to shape of element



# Example 1.1

- Cold roll a 5% Sn-bronze
- Calculate force on roller
- Calculate power
- Plot pressure in nip (no back or forward tension)



# Example 1.2

- $w = 10 \text{ mm}$
- $h_b = 2 \text{ mm}$
- height reduction = 30% ( $h_f = 0.7 h_b$ )  
–  $h_f = 1.4 \text{ mm}$
- $R = 75 \text{ mm}$
- $v_R = 0.8 \text{ m/s}$
- mineral oil lubricant ( $\mu = 0.1$ )
- $K = 720 \text{ MPa}$ ,  $n = 0.46$



# Example 1.3

- Maximum draft:

$$\begin{aligned}\Delta h_{\max} &= \mu^2 R \\ &= (0.1)^2 \cdot 75 = 0.75 \text{ mm}\end{aligned}$$

$$\begin{aligned}\Delta h_{\text{actual}} &= h_b - h_f = 2 - 1.4 \\ &= 0.6 \text{ mm}\end{aligned}$$



# Example 1.4

- Maximum angle of acceptance

$$\phi_{\max} = \tan^{-1} \mu = \tan^{-1}(0.1) = 0.1 \text{ radian}$$

$$\alpha = \sqrt{\frac{(h_b - h_f)}{R}} = \sqrt{\frac{(2 - 1.4)}{75}}$$
$$= 0.089 \text{ rad} = 5.12^\circ$$



# Example 1.5

- Roller force:  $F = L w p_{ave}$
- $L = (R\Delta h)^{0.5} = [75 \times (2-1.4)]^{0.5}$   
 $= 6.7 \text{ mm}$
- $w = 10 \text{ mm}$
- $h_{ave} = (h_b + h_f) / 2 = 1.7 \text{ mm}$   
 $h_{ave} / L = 1.7 / 6.7 = 0.25 < 1$   
 $\therefore$  friction is important

$$F/roller = 1.15 \cdot Lw\bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$



# Example 1.6

$$\varepsilon_f = \left| \ln \left( \frac{h_f}{h_b} \right) \right| = \left| \ln \left( \frac{1.4}{2} \right) \right| = 0.36$$

$$\begin{aligned} 2\tau_{flow} &= 1.15 \cdot \bar{Y} = 1.15 \cdot \frac{K\varepsilon_f^n}{n+1} \\ &= 1.15 \cdot \frac{720 \cdot (0.36)^{0.46}}{1.46} = 354 \text{ MPa} \end{aligned}$$



# Example 1.7

$$\begin{aligned} F/roller &= 1.15 \cdot Lw \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right) \\ &= 6.7 \times 10^{-3} \cdot 10 \times 10^{-3} \cdot 354 \times 10^6 \\ &\quad \times \left( 1 + \frac{0.1 \times 6.7}{2 \times 1.7} \right) \\ &= 28,392 \text{ N} = 3.2 \text{ tons} \end{aligned}$$



# Example 1.8

$$\text{Power (kW) / roller} = T \times \omega = \frac{F \cdot L \cdot V_R}{2 \cdot R}$$

$$\begin{aligned} \text{Power (kW) / roll} &= \frac{28,392 \cdot 6.7 \times 10^{-3} \cdot 0.8}{2 \cdot 0.075} \\ &= 1.01 \text{ kW / roll} = 1.35 \text{ hp} \end{aligned}$$



# Example 1.9

- Entrance

$$p = \left( Y'_{flow} - \sigma_{xb} \right) \frac{h}{h_b} \exp(\mu(H_b - H))$$

- Exit

$$p = \left( Y'_{flow} - \sigma_{xf} \right) \frac{h}{h_f} \exp(\mu(H))$$



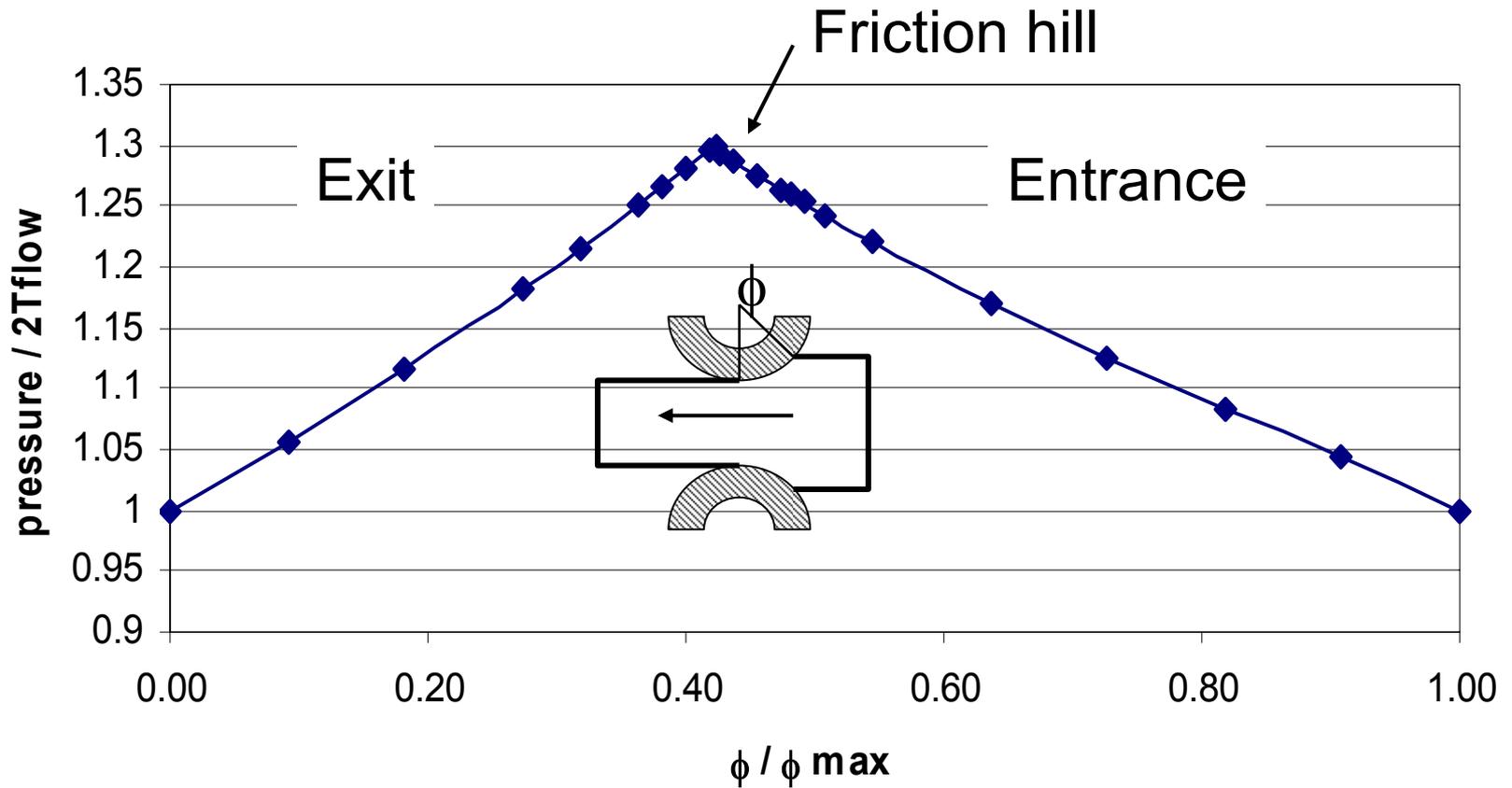
# Example 1.10

$$\phi = \sqrt{\frac{(h - h_f)}{R}}$$

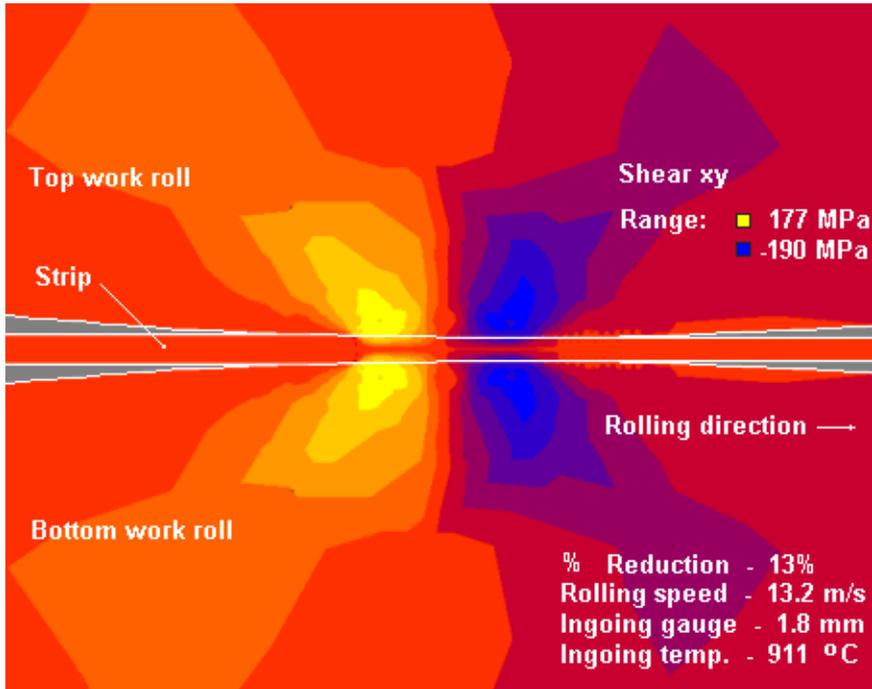
$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



# Example 1.11

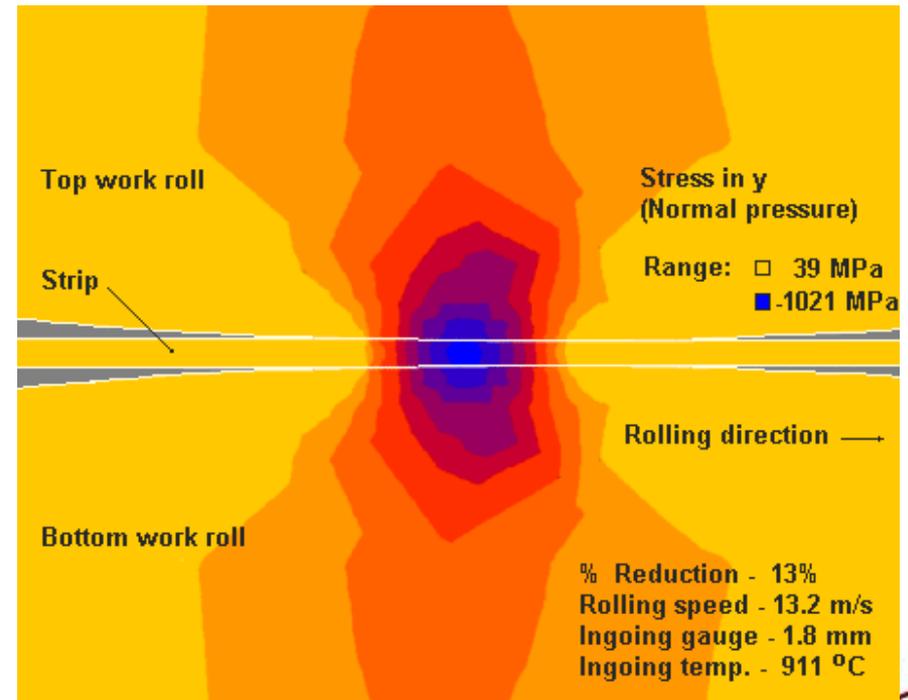


# Rolling

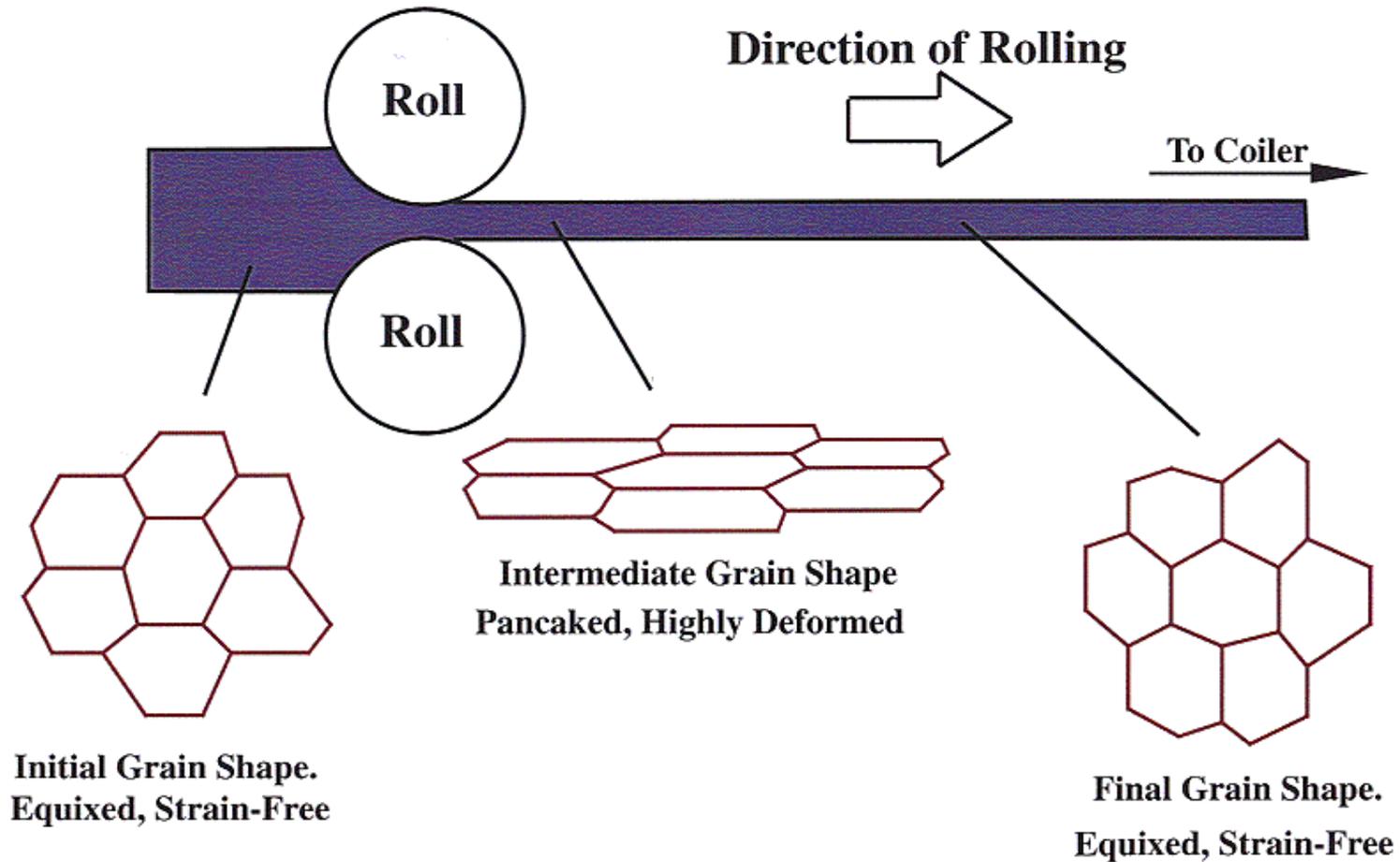


Shear stress

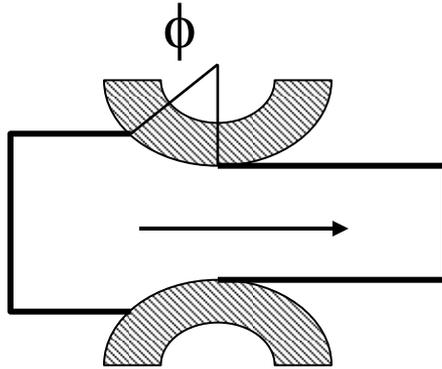
## Normal Stress



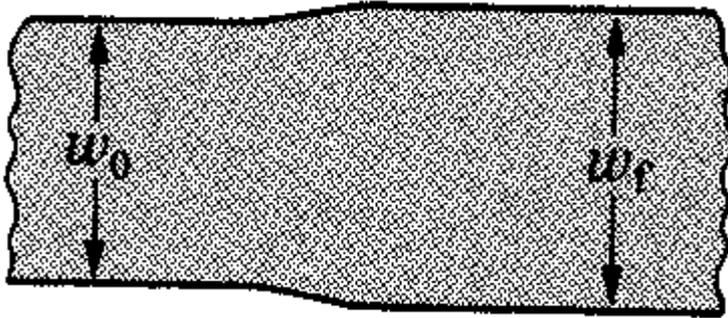
# EFFECT OF FINISH HOT-ROLLING ON THE STRIP SHAPE AND THE AUSTENITE GRAIN STRUCTURE.



# Widening of material



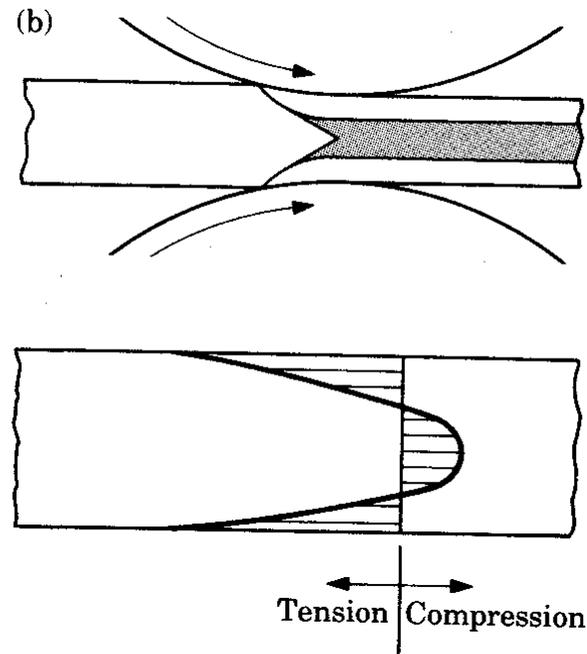
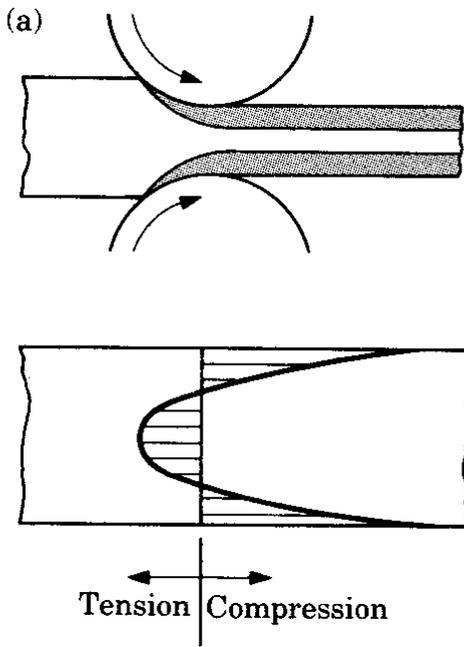
Side view



Top view

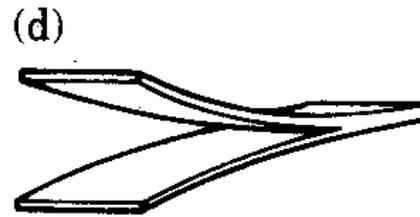
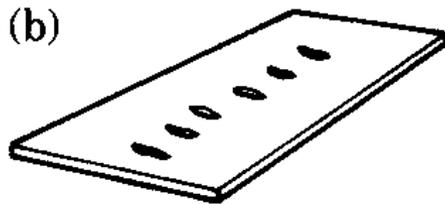
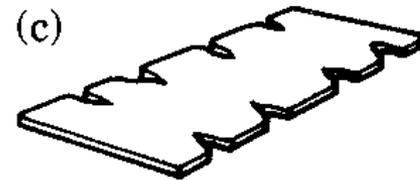
# Residual stresses - due to frictional constraints

- a) small rolls or small reduction (ignore friction)
- b) large rolls or large reduction (include friction)



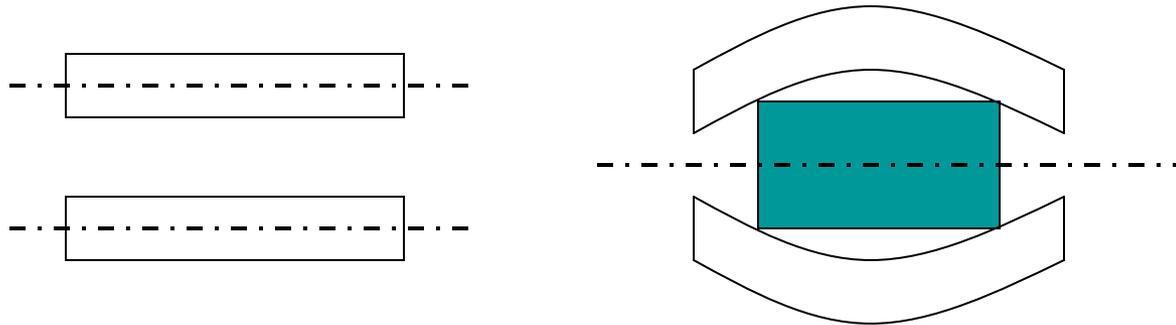
# Defects

- a) wavy edges
  - roll deflection
- b) zipper cracks
  - low ductility
- c) edge cracks
  - barreling
- d) alligatoring
  - piping, inhomogeneity

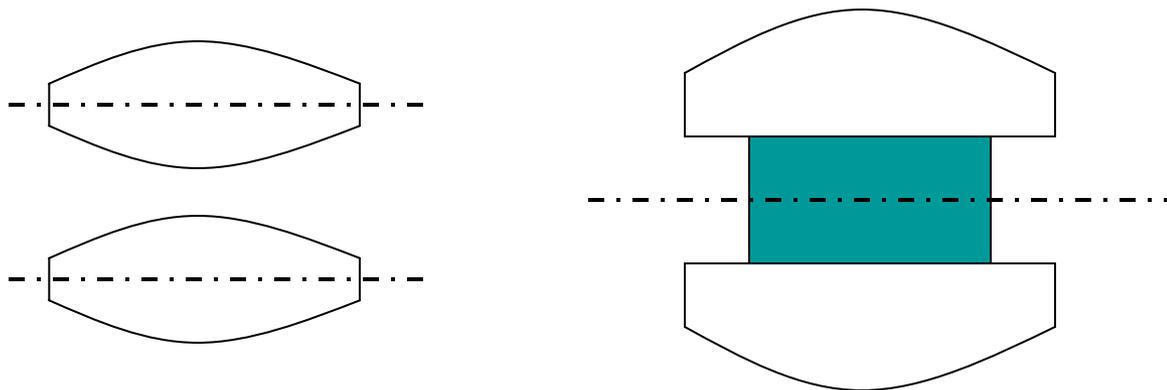


# Roll deflection

Rolls can deflect under load

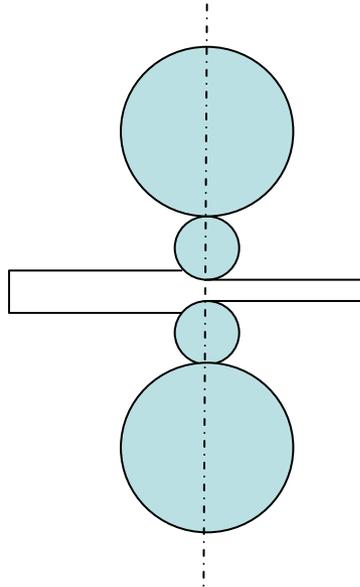


Rolls can be crowned



# Roll deflection

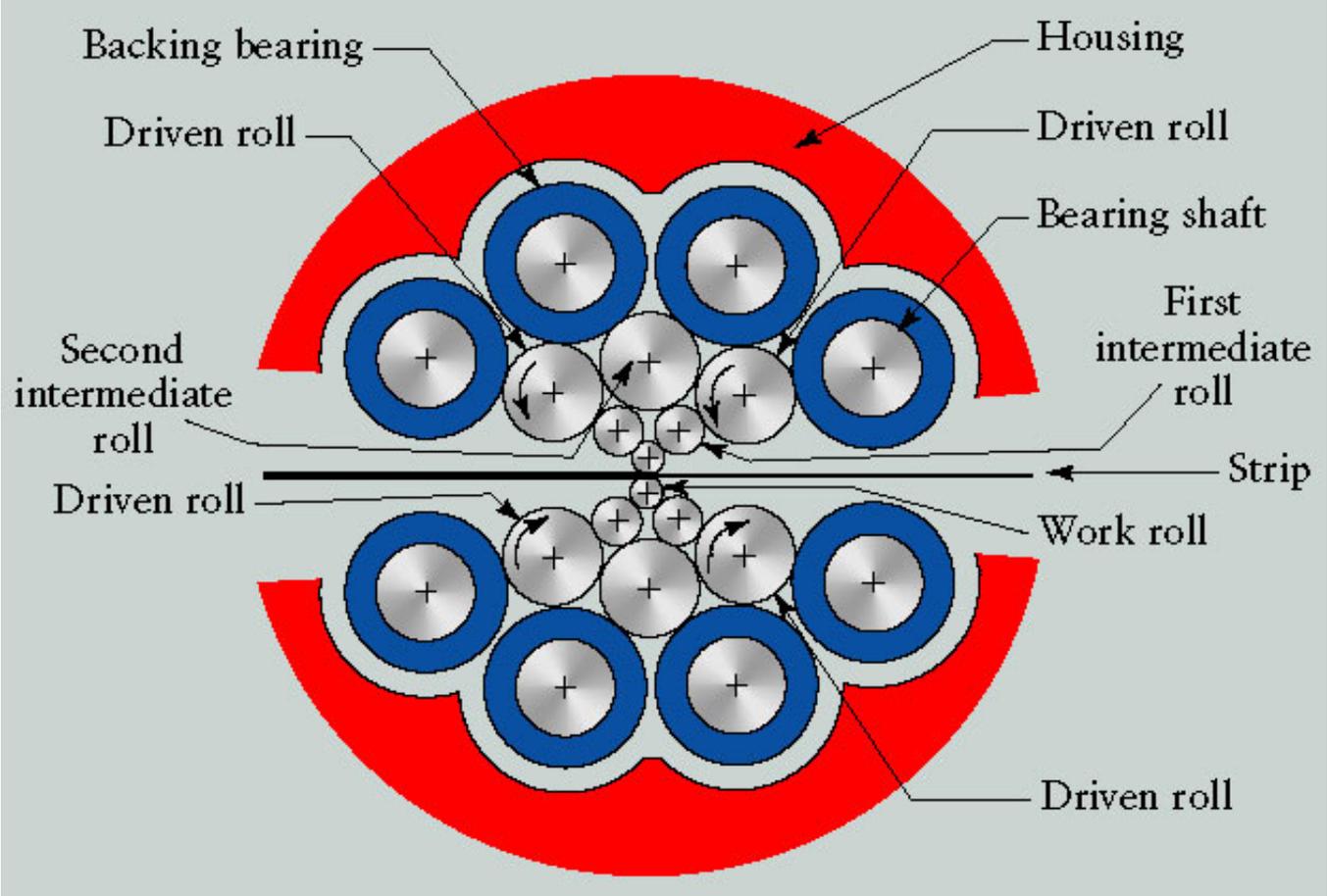
Rolls can be stacked for stiffness



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Method to reduce roll deflection



Prof. Ramesh Singh, Notes by Dr. Singh/ Dr. Colton



# Summary

- Process
- Equipment
- Products
- Mechanical Analysis
- Defects



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton

