Theoretical and experimental investigations on Stirling-type pulse tube cryocoolers with U-type configuration to achieve temperature below 20 K

AD Badgujar and MD Atrey

Abstract
To achieve lower temperatures is a subject of recent research and development activities in the field of pulse tube cryocoolers. To reach a temperature of 20 K, multi-staging is necessary in Stirling-type pulse tube cryocoolers. In the present work, Sage software is used to design a three-stage gas-coupled as well as thermal-coupled pulse tube cryocooler. A single-stage and a two-stage pulse tube cryocoolers are developed, tested and are coupled by a thermal link to build up a three-stage thermal-coupled pulse tube cryocooler. The lowest temperature of 19.61 K is obtained with a cooling capacity of 220 mW at 30 K at the third stage operating at 17 bar charge pressure and 68 Hz frequency. The phase-shifting mechanism used is a double inlet valve at the third stage while the inertance tube is used for the other stages.

Keywords
Multi-staging, Stirling-type pulse tube cryocooler, U-type, double inlet valve

Date received: 15 March 2014; accepted: 6 June 2014

Introduction
Pulse tube cryocooler (PTC) is a device used to generate and maintain very low temperatures, below 80 K. The applications of PTC have spread in the areas like space, medical, superconducting device, and fundamental physics research. The research interest in this field has grown up to achieve much lower temperature which may be obtained by multi-staging of the PTCs. The multi-stage PTC provides a small refrigeration effect at low temperature levels. These stages can either be gas coupled or thermal coupled. In the gas-coupled PTC, the gas travels from one stage to the other and transfers the cooling effect; all the stages are operated at the same frequency and charge pressure and a single compressor is used. This makes the optimization of PTC a bit difficult. In the thermal-coupled PTC, the stages can be operated independently using independent compressors. The Gifford Mc-Mahon (G-M) type PTC operating at a very low frequency of 1–2 Hz, can easily achieve a temperature of 2.5 K with use of rare earth material and lead in the regenerator, with an input power of several kilowatts. However, the use of this type of PTC is limited to ground applications only. The Stirling-type multi-stage PTC, where the compressor is directly coupled to the cold head, is being investigated extensively for the last few years. Chan et al. achieved a no-load temperature of 28 K with a thermal-coupled PTC. Nast et al. achieved 19.8 K with 0.5 W heat lift at 35 K on a two-stage PTC. Yan et al. reached a lowest temperature of 14.2 K with thermal-coupled PTC operating at 32 Hz. Yang and Thummes developed thermal-coupled PTC, which achieves a no-load temperature of 12.8 K, with 200 W input power to each stage, by using lead-coated meshes at the second-stage regenerator. Dietrich and Thummes reported a minimum temperature of 13.7 K with 4.6 kW input power, which is very high and equivalent to the power requirement of Gifford Mc-Mohan PTCs. The G-M type PTC is not so efficient due to the presence of valves. Recently, temperature below 5 K is obtained by Qui et al. using HoCu2 powder as a regenerator material below 20 K.
and the third stage operating at 29.9 Hz. An input power of 850 W is given to the first and the second stages. Most of the researchers have used rare earth material or lead as the second- or third-stage regenerator material due to their high heat capacity at lower temperature. In many cases, usage of rare earth material and lead needs to be avoided due to health as well as cost considerations.

PTC with U configuration involves a change in the direction of gas flow as it proceeds from the regenerator to the pulse tube. The U-bend has an adverse effect on pulse tube cooling action due to the dead volume at the cold end, pressure drop, and undesirable mixing of gas at the cold end of the pulse tube. In order to understand the role of flow straighteners in PTC, a theoretical and experimental investigation on single-stage U-type PTC has been carried out by Atrey and Badgujar. They also reported work on the design and development of Stirling-type two-stage PTC with a double inlet (DI) valve as phase shifter.

In the present work, no rare earth material and lead are used to achieve a no-load temperature below 20 K. A three-stage gas-coupled PTC and a three-stage thermal-coupled PTC are designed and developed using DI valve as a phase shifter.

**Working of PTC**

Figure 1 shows the schematic of the PTC with U configuration where regenerator and pulse tube are parallel to each other. The pressure wave generator (PWG) generates the pressure waves in the system, which pressurizes and depressurizes the Helium gas in the system. During pressurization, the after-cooler (AC) takes care of the heat of compression. The gas is cooled in the regenerator (REG) due to heat transfer from the gas to the regenerator mesh. During depressurization, the gas in the pulse tube expands to a lower temperature than the temperature at which it enters the pulse tube (PT). This cold gas, while passing through the regenerator, becomes warm due to heat transfer from the regenerator matrix to the gas. The temperature at the hot end of the pulse tube and at the after-cooler is maintained at room temperature by using water. The phase-shifting mechanism (PSM) is used to optimize the phase shift between the mass flow rate and pressure in the pulse tube, which is essential for improving the performance of the PTC.

**Theoretical analysis**

Theoretical analysis of the PTC is carried out using Sage software. Sage is a simulation software which can be used to design a PTC. It has drag and drop visual interface where a user can assemble complete cryocooler from standard components, such as pistons, cylinders, heat exchangers, etc. Different design and operating parameters also can be optimized using this software. The PTC models are designed using Sage software.

**Sage simulation for PTC**

Models are created by interconnecting different components, which principally are a set of equations solved using various numerical methods. To understand the modeling with Sage, the regenerator model is explained and shown in Figure 2 as an example. The regenerative heat exchanger consists of stainless steel (SS) meshes with a mesh size of 400. The tubular canister (renamed as Regenerator (REG)) forms the parent component, with the length of tube, wall thickness, inside diameter, and material defined for the same. Woven screen matrix (a child component of “Regenerator”) models regenerator mesh inside the regenerator tube. The wire thickness, material, and porosity of mesh are defined in this component. The matrix gas, rigorous surface, and distributed conductor (all child components of ‘Regenerator’) model the heat exchange between SS meshes and helium gas. In the matrix gas component, mass flow connections (mGT 27, 15) and in the rigorous surface component, heat flow connections (Qstdy 32, 36, 68, 70) are introduced as shown in Figure 2. The connections of matrix gas components (mGT 27, 15) and rigorous surface component (Qstdy 32, 36, 68, 70) are exported to parent component level. The connection numbers are generated by the software in the sequence of the connection made. These connections are the set of equations, for mass and heat interaction between the main components. In this way, the entire PTC is modeled. Pulse tube is a special class of model which considers molecular conduction, turbulent conduction, free convection, boundary convection, and streaming convection. It transfers the PV work of expansion from the cold end to the warm end, thereby extracting heat out of the cold end. Similarly, the other components like after-cooler (AC), cold end heat exchanger (CHX), etc., are modeled; each of them may or may not have subcomponents.
Optimization may be carried out to study the effect of various parameters.

Sage models oscillating gas flows (laminar, turbulent, and laminar–turbulent transition) in ducts and cylinder spaces. The equations used are designed specifically for one-dimensional internal flows with space- and time-variable flow area. The equations are as follows:

**Continuity**
\[
\frac{\partial \rho A}{\partial t} + \frac{\partial \rho u A}{\partial x} = 0
\]

**Momentum**
\[
\frac{\partial \rho u A}{\partial t} + \frac{\partial \rho u^2 A}{\partial x} + \frac{\partial P A}{\partial x} = F - FA
\]

**Energy**
\[
\frac{\partial \rho e A}{\partial t} + \rho \frac{\partial u A}{\partial t} + \frac{\partial u P A}{\partial x} + q = Q_e
\]

where \( A \) is the flow area and \( e \) is the mass-specific total gas energy and \( u \) is flow velocity. These gas dynamic equations determine three implicit solution variables \( \rho, \rho u A, \) and \( \rho e \). The \( F \) in the momentum equation is the viscous terms and the \( Q_e \) in the energy equation is the heat flow per unit length, through the negative \( z \) surface, due to film heat transfer. The equations are discretised over a grid of points \( (x_i, t_j) \). Upon this grid, the above equations are solved implicitly.

Staggered-grid formulation is used to solve the partial differential equations accurately. Based on the Sage model, the multi-stage gas coupled and thermal-coupled PTCs are designed and fabricated. Experimental results are compared with the theoretical predictions.

**Experimental setup**

An inertance tube is used as a PSM for single-stage PTC as shown in Figure 3(a). An inertance tube is a 1–3 m long capillary tube having a diameter of 2–3 mm and it opens in a reservoir. A DI valve with inertance tube and reservoir is used as a PSM at the second stage of two-stage PTC as shown in Figure 3(b). With DI valve at the second stage, a fraction of gas flows from the compressor to the hot end of the pulse tube bypassing the first- and second-stage regenerators, depending on the opening of the valve. The DI valve for single stage is found to be non-productive. The thin-walled pulse tubes and the regenerators are made out of SS 304. The regenerators consist of SS 304 meshes with a mesh size of 400. Based on the Sage modeling, the dimensions of various components of the PTCs are obtained and are given in Table 1.

Figure 4 shows the schematic of the experimental setup developed for testing the PTCs. Linear compressor or PWG of Chart Inc. make is used to generate oscillating pressure waves in the PTC.
recuperative heat exchangers, made of copper, are designed and are used as after-coolers (to cool the gas at the outlet of the compressors) and to cool the gas at the hot ends of respective pulse tubes. A Swagelok make (SS-4MG-MH) valve is used as DI valve. ENDEVCO make piezoresistive transducers are used for pressure measurement while silicon diode is used for temperature measurement below 50 K, and PT100 is used for measurement of temperature above 50 K in multi-stage PTC. The minimum temperature is measured using Lakeshore make temperature indicator with silicon diode sensors. The sensors are calibrated and give an accuracy of ±6 mK at 20 K. The cold head is fitted in the vacuum jacket and all leads related to temperature and heat load measurements are taken out using sealed feedthroughs.

**Experimental procedure**

After assembling the PTC, before starting the experiments which involve charging of the gas, following procedure is carried out.

(a) The PTC is cleaned by evacuating the entire system to $10^{-6}$ mbar.

(b) The system is purged 3–4 times using Helium gas before charging it to a required pressure.

(c) The vacuum jacket is pumped to create vacuum levels of $10^{-6}$ mbar.

(d) The input power is given using variable frequency drive by setting the desired operating frequency. The input power is increased in the steps of 100 W during the cooldown process.

(e) The cooldown time and the temperatures are noted when the minimum temperature variation of ±0.01 K is obtained.

**Result and discussions**

An experimental investigation is carried out to study the effect of various operating and design parameters. To optimize for no-load temperature, the helium charge pressure and the frequency of operation are varied in a certain range. The DI valve setting (number of turns) and the lengths of inertance tubes are also optimized. In the next sections, theoretical and experimental results of different PTCs are discussed in detail for achieving temperatures below 20 K. During the experimental investigations, the tests are repeated several times and the results are reproducible within the temperature variation of ±0.05 K.

### Table 1. Dimensions of single-stage and two-stage PTC.

<table>
<thead>
<tr>
<th>Sr. no.</th>
<th>PTC unit</th>
<th>Stages</th>
<th>Regenerator</th>
<th>Pulse tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single-stage PTC</td>
<td>–</td>
<td>28 × 54 × 0.15</td>
<td>12.2 × 74 × 0.15</td>
</tr>
<tr>
<td>2</td>
<td>Two-stage gas coupled PTC</td>
<td>1st stage</td>
<td>16 × 55 × 0.15</td>
<td>8 × 60 × 0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd stage</td>
<td>9 × 60 × 0.15</td>
<td>4 × 140 × 0.15</td>
</tr>
</tbody>
</table>

$D \times l \times t$ (Inside diameter × length × thickness in mm).

PTC: pulse tube cryocooler.
Single-stage PTC

Experiments are carried out on a single-stage U-type PTC in order to understand the sensitivity of different parameters affecting the performance of the PTC. Figure 5(a) shows the photograph of the cold head of the single-stage PTC while Figure 5(b) shows the cold head of two-stage PTC. Figure 6(a) shows the cooldown curve for the single-stage PTC for different charging pressures. The lowest temperature recorded is 54.7 K for 15 bar charge pressure and 50 Hz operating frequency. The input power for the compressor is 300 W while the cooldown time in this case is around 60 min. The cooldown time is defined as the time beyond which the variation in minimum temperature is ±0.01 K only.

The refrigeration effect is measured at various pressures and temperatures. Figure 6(b) gives variation in refrigerating effect at different temperatures for different charging pressures. A heater of Nichrome wire, having a resistance of 25 Ω, is used as dummy load on the CHX. A suitable DC power supply is used for this purpose. It may be observed that the variation in refrigeration effect with respect to temperature is approximately linear for a given charge pressure. A refrigeration effect of 6.1 W at 80 K is measured for 17 bar charge pressure.

Figure 6(b) shows that the refrigeration effect at a particular temperature increases with the increase in charge pressure due to increased mass flow rate at the cold end. However, the no-load temperature remains more or less unaffected, due to the fact that the no-load temperature depends on the specific heat of the regenerator material, SS meshes in this case.

The refrigeration effect at 17 bar charge pressure is compared with the theoretical results predicted by the Sage software as shown in Figure 7. Sage predicts a minimum temperature of 46.82 K with 8.56 W of refrigeration effect at 80 K with 300 W of input power. The results predicted by Sage are in good approximation with the experimental results.

The theoretical predictions are somewhat higher than the experimental results, which may be attributed to the unaccounted losses, e.g. radiation and conduction through lead wires and feedthroughs.

Two-stage gas-coupled PTC

To achieve a temperature below 20 K, a two-stage gas-coupled PTC is designed and developed using Sage model. Figure 5(b) shows the two-stage gas-coupled PTC cold head. In the two-stage PTC, the cooling effect produced by the first stage is transferred to the second stage by the helium gas as it enters the second-stage regenerator. This low-temperature gas expands at the cold end of the second-stage pulse tube, thus decreasing the temperature at the second stage. The first-stage refrigeration effect is also utilized to cool the radiation shield covering the second stage so as to shield the radiation, the radiation coming on the second stage from the ambient.

Performance of two-stage PTC. Figure 8 shows the cooldown curve for the two-stage gas-coupled PTC, having a charge pressure of 17 bar, with optimized DI valve setting. The frequency is optimized for the two-stage PTC and is found to be 68 Hz. The minimum temperatures recorded are 89.52 K and 22.67 K for the first and second stage, respectively. The PTC takes 120 min to reach the minimum temperature with an input power of 300 W. A refrigeration effect of 180 mW at 30 K is measured at the second stage for

Figure 5. Cold heads of single-stage and two-stage PTC (a) single-stage cold head and (b) two-stage cold head.

Figure 6. Performance of single-stage PTC. (a) cooldown curve and (b) refrigeration effect.
operating pressure of 17 bar with input power of 300 W.

The refrigeration effect at 17 bar charge pressure is compared with the theoretical results predicted by the Sage software as shown in Figure 9. Sage predicts a minimum temperature of 21.2 K with 221 mW of refrigeration effect at 30 K with 300 W of input power. The theoretical results thus are found to be close to the experimental results.

**Effect of frequency.** It is important to run the PWG at its natural or “resonance” frequency. Operating away from this frequency results in the losses. The resonance frequency changes with the change in volume of the system and gas pressure for the same PWG. The volume of the two-stage PTC is less than the volume of single-stage PTC; so the resonance frequency for two-stage PTC is higher than the single-stage PTC. Also, the regenerator effectiveness depends on thermal penetration depth, which changes with the frequency. Therefore, frequency is an important parameter to be optimized for achieving the lowest temperature. For the two-stage PTC, the operating frequency is found to be 68 Hz as shown in Figure 10 while the lowest temperature obtained is 22.67 K. At this temperature, the losses due to the regenerator ineffectiveness are low and the power input is minimum resulting in a better performance of the cryocooler. The variation is found to be repeatable and the Silicon diode sensors are calibrated to ±6 mK accuracy.

**Three-stage gas-coupled PTC**

Based on the conceptual understanding of multi-staging, a three-stage gas-coupled PTC is designed and developed. The schematic of the same is shown in Figure 11. It consists of three regenerators in series with three corresponding pulse tubes connected to respective regenerators. Each pulse tube has its own CHX, hot end heat exchanger, and an independent PSM which can be an inertance tube and reservoir or inertance tube, reservoir with DI valve.

Figure 12 shows the cooldown curve for three-stage gas-coupled PTC, operating at the charge pressure and frequency of 17 bar and 60 Hz, respectively. The minimum temperatures recorded are 80.15 K and 48.07 K for second and third stage, respectively. However, the first-stage temperature is 209.12 K, which is relatively high. The PTC takes 120 min to reach to the minimum temperature with input power of 300 W.

The Sage software predicted the no-load temperature of 18.53 K, with an operating frequency of 50 Hz.

---

![Figure 7. Refrigeration effect of single-stage PTC.](image1)

![Figure 9. Refrigeration effect for two-stage PTC.](image2)

![Figure 8. Cooldown curve for a two-stage gas-coupled PTC.](image3)

![Figure 10. Effect of frequency for a two-stage gas-coupled PTC.](image4)
and 214 W input PV power with a predicted pressure drop of 2.06 bar. It was noted during the experiments on two-stage PTC, that the temperature of the gas is on the higher side at the entrance to the regenerator due to inefficient cooling of the gas in the after-cooler. In order to overcome this problem, the design of the PTC, in this case, was modified. In view of this, the after-coolers and hot end heat exchangers are provided with independent cooling arrangements. However, this resulted in increased length of U-bends at the cold ends for all the stages. Although the design was modified for providing better heat rejection in the after-cooler and at the hot ends of pulse tubes, these modifications lead to an increase in the dead volumes at the cold end of all the stages due to increased length of U-bends. This resulted in lowering of pressure ratio and increased pressure drop, degrading the performance of the PTC. Due to underestimation of the effect of incremental change in length of the U-bends, the required performance could not be achieved.

**Pressure drop**

The pressure variation during the cycle is recorded on oscilloscope at the steady state condition. Table 2 gives the pressure ratios \( \frac{P_{\text{max}}}{P_{\text{min}}} \) at the inlet to the regenerator and at the outlet of the pulse tubes both by Sage model and by experiments. The pressure ratio indicates the magnitude of the refrigeration effect that could be produced at low temperature. The difference between the pressure ratio obtained at the inlet to the regenerator and at the outlet of the pulse tube indicates the pressure drop in the regenerator. It may be observed that the actual pressure ratio at the outlet of the pulse tube for the single stage (PT1) is 1.24. The pressure ratio for two-stage PTC is 1.20 and 1.09 at the outlet of the first stage pulse tube (PT1) and the second stage pulse tube (PT2), respectively. However, the pressure ratio measured at the third-stage pulse tube (PT3) in case of three-stage gas coupled PTC is just 1.03. This low pressure ratio indicates enormous pressure drop across the length of the regenerators. As can be seen from the Table 2, the results obtained by Sage software are approximately the same as the experimentally obtained results.

Figure 13 shows the pressure pulse obtained at the inlet to the regenerator and at the outlet of the pulse tube. A maximum pressure drop \( \Delta P \) of 0.29 bar (Figure 11(a)) is measured for single-stage PTC. The pressure drop measured for the two-stage PTC is

![Figure 11. Three-stage gas-coupled PTC.](image)

![Figure 12. Cooldown curve for three-stage gas-coupled PTC.](image)

<table>
<thead>
<tr>
<th>Pressure ratio</th>
<th>Inlet to regenerator</th>
<th>PT1</th>
<th>PT2</th>
<th>PT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-stage exp.</td>
<td>1.28</td>
<td>1.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-stage Sage</td>
<td>1.25</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-stage exp.</td>
<td>1.31</td>
<td>1.20</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>Two-stage Sage</td>
<td>1.28</td>
<td>1.22</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Three-stage GC exp.</td>
<td>1.38</td>
<td>1.19</td>
<td>1.08</td>
<td>1.03</td>
</tr>
<tr>
<td>Three-stage GC Sage</td>
<td>1.31</td>
<td>1.24</td>
<td>1.13</td>
<td>1.05</td>
</tr>
</tbody>
</table>

GC: gas coupled.
1.23 bar and 2.03 bar (Figure 11(b)) for the first stage and the second stage, respectively. Pressure drop in the three-stage gas coupled PTC is still higher due to the increased length of the regenerator. The increased length of the regenerator needed more number of meshes in the regenerator and thus resulted in higher pressure drop. The pressure ratio at the outlet of the third stage of the pulse tube is as small as 1.03, meaning that there is hardly any cooling effect available at lower temperature. The total pressure drop measured is 2.45 bar for the third stage (Figure 11(c)), while the pressure drop measured for the first stage and the second stage are 1.28 and 2.08 bar, respectively.

It may therefore be concluded that three-stage gas-coupled PTC may not yield lower temperature with the available compressor. The lowest temperature obtained is only 48.07 K as against design value below 20 K. This is due to the fact that some modifications are required to be done at the hot end of the pulse tubes and the after-cooler in order to improve the heat transfer to circulating water. It is therefore
decided to go for the thermal coupled PTC where in two compressors can be employed.

**Three-stage thermal-coupled PTC**

A three-stage thermal-coupled PTC is designed and developed as shown in Figure 14. This is done by thermally coupling the single-stage PTC with the two-stage PTC, by a thermal link made of high-conductivity copper.

The two thermal-coupled units are independent of each other and use independent PWG. As a result, their performances can be independently optimized. The design and operating parameters are optimized and the refrigeration effect at different temperatures is measured independently. The following sections present the detailed experimental investigations on three-stage thermal-coupled PTC. The theoretical predictions given by Sage are compared with the experimental results.

**Cooldown curve.** Figure 15 shows the cooldown curve of the three-stage thermal-coupled PTC. In this thermal-coupled arrangement, the two-stage PTC operates at the charge pressure and frequency of 17 bar and 68 Hz, respectively, while the single-stage PTC (which precools the two-stage PTC) operates at a charge pressure and frequency of 19 bar and 50 Hz, respectively. After optimizing the length of inertance tube and DI valve opening, a minimum temperature of 19.61 K is recorded at the third stage. The temperatures across the thermal bridge (TB) (Figure 13) are 82.27 K and 67.51 K. The temperature difference across the TB is attributed to contact resistance. The PTC takes 120 min to reach the steady state temperature with an input power of 300 W, for each compressor.

**Refrigeration effect.** The charge pressure of two-stage PTC, which is precooled by single-stage PTC, is varied from 13 bar to 21 bar to measure the refrigeration effect at different temperatures. Figure 16 shows the comparison of predicted and experimentally obtained no-load temperature for different pressure while Figure 17 shows the comparison of refrigeration effect at different temperatures obtained at 17 bar. The theoretical results are found to be in good approximation with the experimental results. Sage predicts lower no-load temperatures because the model does not take in to consideration the losses due to radiation, the heat in-leak from the lead wires and other unaccounted losses.

As shown in Figure 17, a refrigeration effect of 220 mW at 30 K is obtained at the third stage for an operating pressure of 17 bar (two-stage PTC) with a compressor input power of 300 W to each compressor. The results predicted by Sage model show the similar trends and are in reasonable agreement with the experimental results.

**Conclusions**

The investigations are carried out on the Stirling-type single stage, two-stage, and three-stage PTC with a U-type configuration for various design and operating conditions. The theoretical results predicted by Sage software are found to be close to the experimental results.

The single-stage PTC and the two-stage PTC achieved the minimum temperature of 54.7 K and 22.67 K, respectively. The refrigeration effects obtained with these PTCs are 6.1 W at 80 K and 180 mW at 30 K, respectively, with an input power of 300 W with the charge pressure of 17 bar.

To achieve a temperature below 20 K, the three-stage gas coupled and the thermal-coupled PTCs are designed and developed. The gas-coupled PTC could not give good results due to the modifications made for better heat transfer at the hot end of the pulse tubes.

In the case of thermal-coupled PTC, a minimum temperature of 19.61 K is obtained at the third stage, which is the second stage of two-stage PTC. A refrigerating effect of 220 mW at 30 K is obtained for the same PTC.

**Funding**

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.
Conflict of interest
None declared.

References