Solid Mechanics Comprehensive Examination

Day: XXX
Date: XXX
Passing Marks: 30

Duration: 3 hours
Max Marks: 60

Note: This is an open notes/books exam.

1. Consider a long prismatic shaft of length L whose cross-section is an isosceles triangle with sides a, a, b with $b << a << L$. Determine the approximate torsional rigidity of the shaft using Prandtl's stress function approach. Assume that the shaft material is linearly elastic, isotropic and homogeneous. (**10 marks**)

2. Consider a long thin beam of length L and flexural rigidity EI. The beam is fixed into a rigid wall at one end $x = 0$. The other end $x = L$ is attached to a pinned roller support through a torsional spring with stiffness k. An axial compressive load P is applied at the end $x = L$. Use the principal of stationary potential energy to determine the approximate critical load P^* at which said beam will buckle. Assume that the beam material is linearly elastic, isotropic and homogeneous. (**10 marks**)

3. A thick circular cylinder of inner and outer radii, a and b, respectively, is subjected to radially varying thermal loading. The radial variation in temperature from $r = a$ to $r = b$ is given by the linear relation, $T(r) = T_0 + T_1 r$. Assuming plane strain conditions, find the stresses (σ_{rr} and $\sigma_{\theta\theta}$) and strains (ε_{rr} and $\varepsilon_{\theta\theta}$) in the cylinder as a function of r. In cylindrical coordinates the equilibrium, strain-displacement and constitutive equations (including thermal strains) are: (**10 marks**)

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\varepsilon_{rr} = \frac{du_r}{dr}$$

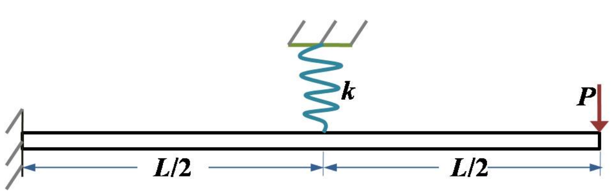
$$\varepsilon_{\theta\theta} = \frac{u_r}{r}$$

$$\varepsilon_{rr} = \frac{1}{E}[\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})] + \alpha T$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}[\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})] + \alpha T$$

$$\varepsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})] + \alpha T$$

4. The figure shows a beam of bending stiffness EI subjected to an end load P and attached with a linear spring of stiffness k. Before the application of load, the beam is straight and spring is unstretched. (**10 marks**)



   (a) Draw the free body diagram showing all the forces-moment acting on the beam.

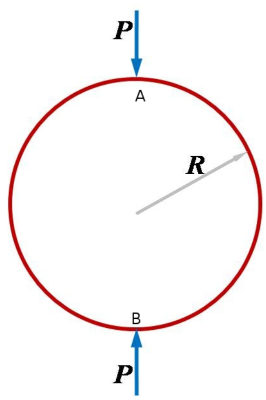
   (b) Is the problem statically determinate? Give reasons.

(c) Calculate the deflection under the load in terms of EI, L, k.

(d) Use answer to (c), to find the deflection under the load if spring is replaced by a pin joint.

5. The figure shows a linear elastic isotropic sphere of radius R, elastic modulus E and Poissons ratio ν subjected to two point loads P acting along the diameter AB. We want to calculate the volume change in sphere due to the applied loads. (**10 marks**)

(a) If ϵ_{ij} is the strain at any location in the sphere, what is the change is the volume of differential element at that location. Thus express the total change in volume in terms of the strain.



(b) If the loading in the figure is replaced by a uniform pressure p_0 everywhere on the surface of the body, calculate the stresses, strains in the sphere and displacement at location A and B.

(c) Apply Reciprocity theorem to relate the two loading situations. Thus combine answers to (a) and (b) to calculate volume change of sphere for the loading shown in the figure

6. Damage typically initiates on surfaces of components on planes of maximum shear strains or on planes of maximum normal stress. Consider a solid circular rod of radius $R = 5$ cm made up of steel with $E = 200$ GPa and $\nu = 0.25$. It is subjected to an axial load of 10 kN and a torque of 100 Nm. Find the orientation of the possible planes of damage initiation with respect to the axis of the rod. (**5 marks**)

7. In a flat thin stressed plate which is loaded in the x-y plane it is known that at a point $\sigma_x = 145$ MPa, τ_{xy} = 42 MPa and ϵ_z = -360 $\mu m/m$. Calculate σ_y, ϵ_x, ϵ_y and γ_{xy} if the Youngs modulus and the Poisson ratio of the plate material are 200 GPa and 0.25, respectively. (**5 marks**)