Comprehensive Exam

Section: Kinematics and Dynamics

Day: XXX | Duration: 3 hours

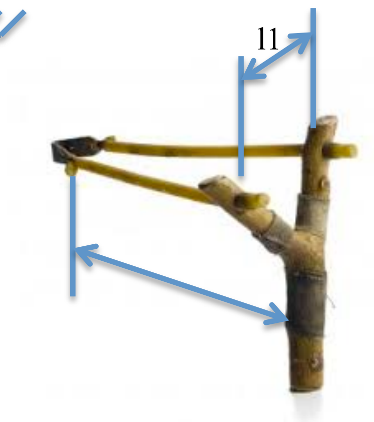
Date: XXX | Max Marks: 50

Passing Marks:

Open book exam.

1. (10 marks) Consider a sprinkler disk of radius R, rotating with a constant angular velocity W about a central axis perpendicular to the plane of the disk. Due to mechanical imperfections, the disk also wobbles in a sinusoidal fashion, at a frequency w, with and amplitude A, about a transverse axis passing through the center. A caterpillar silting on the sprinkler starts crawling, with a velocity v relative to the disk, towards the center as the disk rotates.
   I. Determine the velocity and acceleration of the caterpillar with respect to the ground if:
      (a) The starting location of the caterpillar is on the axis of wobble (at a distance d from the center)
      (b) The starting location of the caterpillar is on the axis perpendicular to the axis of wobble (at a distance d from the center)
   II. What will be the time taken by the caterpillar to reach the center?
   State all the assumptions made.

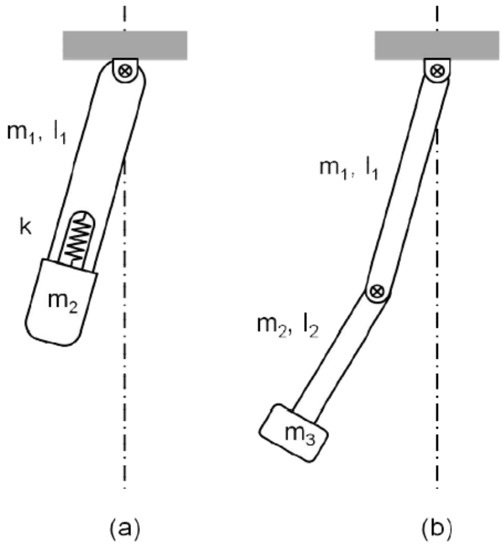
2. (5 marks) An experiment is conceived to measure mass moment of inertia of a rigid circular bar having unknown dimensions by hanging it by two inextensible strings of negligible mass and length L attached at its two ends as shown in the figure below. The rod is set into oscillations by giving it a small angular motion in horizontal plane and the time period of oscillations is measured. Derive the expression for mass moment of inertia about a transvers axis passing through the center.



3. (5 marks) A slingshot is shown in the figure above. Each of the rubber arms have stiffness k. Distance between two fixed points of rubber is l_1 and free (rubbers unstretched) distance from handle to the stone is l_2. Assuming stone to be point mass with negligible radius derive the expression for force on the stone in any stretched position of stone holder. If the stone holder is stretched by distance δl from free position, determine the velocity of stone when it leaves the slingshot.

4. (10 marks) Derive equations of motion for the following systems without making any assumptions on small displacements (the acceleration due to gravity g acts vertically):
   a. The system of sub-figure (a) comprises a bar of mass m_1 and length l_1 that is pivoted about a frictionless joint. A mass m_2 is attached to the end of the bar by means of a spring of stiffness k.
   b. The compound pendulum of (b) comprises two rods (masses and lengths as indicated) connected to each other by means of a frictionless joint. An additional mass m_3 is attached at the end as shown.

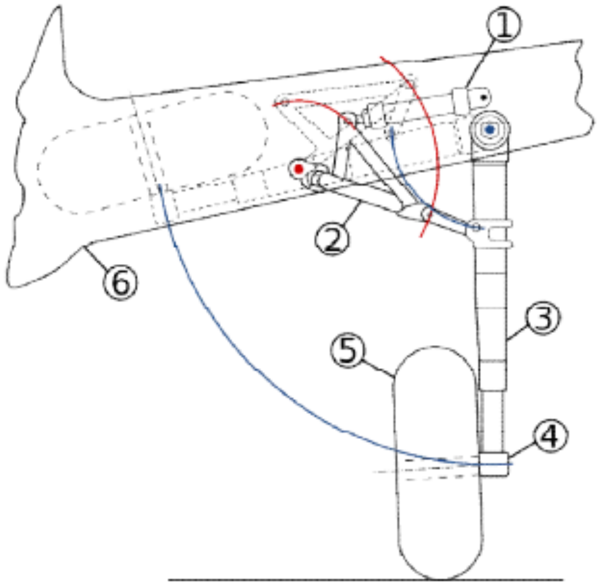
   Use both Newton's laws as well as Lagrange's equations.



5. (10 marks) A container of mass 'M' carrying sensitive scientific instruments is to be dropped on the moon. The container first descends with a parachute, and when it has reached close to the ground, 'retro-rockets' (oriented vertically, and with their nozzles facing downward) are fired. The firing of the rockets (which generate a total thrust force 'F') retards the vertical speed of descent of the container before it impacts the surface. The container's impact is cushioned by an inflated tube/cushion attached to its bottom surface, which can be treated as a spring of stiffness 'k'. The cushion has a plate of mass 'm' attached to its bottom surface to protect it from rupturing. Assume that the impact is instantaneous, and the coefficient of restitution of the impact is 'e'. Take M = 10m.
   a. What is the limit on the thrust force generated by the rockets?
   b. What are the natural frequencies and mode shapes of the system. Physically interpret the significance of the mode shapes.

6. Figure below shows the schematic of an aircraft landing gear mechanism in two configurations: deployed (solid lines) and retracted (dashed lines). Parts 1 and 6 are the hydraulic actuator and the aircraft body, respectively.
   a. Draw a kinematic diagram of the mechanism. Label the links, and indicate the corresponding parts on the schematic.
   b. What are the number and type of joints used in this mechanism?
   c. Calculate the number of degrees of freedom for this mechanism.
   d. Determine the kinematic (position, velocity, and acceleration)

relationships that describe the input-output behavior of the system. Clearly label your diagram, and list any assumptions made.



Schematic for an aircraft landing gear mechanism (Source: en.wikipedia.org)

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