Indian Institute of Technology Bombay

Department of Mechanical Engineering

PhD Qualifying Examination, January 23, 2020, 2:30 PM to 5:30 PM

Des-2 (**Kinematics and Dynamics**)

General Instructions

(a) This is a closed book, closed notes examination

(b) Answers should be written clearly and legibly

©Show all important steps and clearly highlight assumptions and approximations

(d) Answer all the six questions

Maximum marks: 100

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1. Consider a point *P* on a spinning top that has a position vector **r** with respect to a fixed coordinate system ('basis') whose origin is at the point of contact of the top with the ground.

a. Derive the expression for the position vector **r**, assuming that the point of contact does not move.

b. What are the magnitude and direction of the force acting on a segment of the top (of mass 'm') located at the point *P*.

c. The segment (of part (b) above) detaches from the rest of the top at the time instant 't'=0. Determine the subsequent relative motion of the detached segment relative to the point *P* on the top.

d. What is the motion of the top, if we relax the condition of part (a), and allow the point of contact to move (as it happens with a 'real' top)?

[15 marks]

2. A joyride at an entertainment park comprises a platform fixed to an axis that is oriented at a constant angle θ to the vertical, and which precesses about the vertical at a constant speed Ω. (Note: the platform itself does not rotate about its axis.)

a. If a typical rider can tolerate an acceleration a_{max} without feeling motion sickness, find the expression relating a_{max} to the maximum speed Ω that the ride can have.

b. What is the reaction force felt by the rider from the seat to which he is strapped.

c. What is the angular momentum of the platform with respect to its centre of mass? What is its angular momentum with respect to the fixed pivot point 'O'?

[15 marks]

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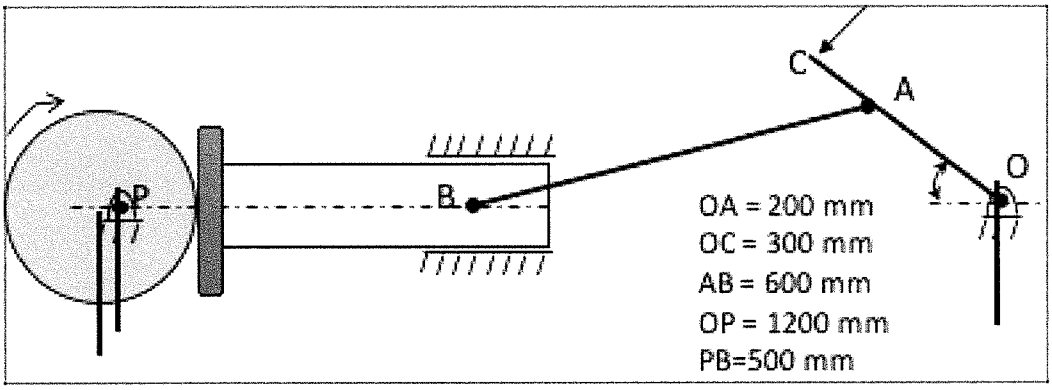
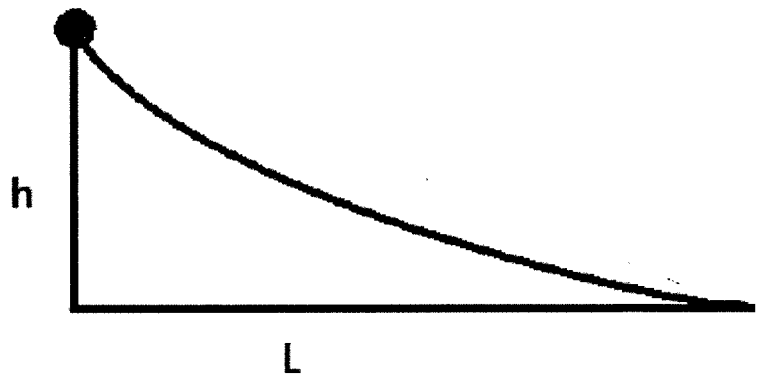
3. A circular cam with flat face follower is connected to a slider crank mechanism as shown in the figure. Radius of the cam profile is 200 mm while the off center distance of its shaft is 75 mm. Figure shows the cam for the least displacement of the follower: consider cam rotation θ = 0 for this position. Assume the follower does not leave contact with the cam.

a) Derive a mathematical expression for displacement x of the follower point B as a function of θ. Further find velocity and acceleration of B in terms of θ and its higher derivatives. **[7 marks]**
b) By using graphical or analytical method, find the angle of link OA in the given position **[3 marks]**
c) If the cam is rotating at an uniform angular speed of 100 rpm, determine velocity and total acceleration of point C and angular acceleration of link OA at θ = 120 deg **[10 marks]**
d) At θ = 120 deg if force F=10 N is applied at C perpendicular to link OC, determine torque to be applied on the cam to maintain equilibrium. Assume frictionless motion with negligible mass of linkages. **[4 marks]**
e) Minimum and maximum angle of the link OA during one complete rotation of cam **[3 marks]**



Cam driven slider crank mechanism

4. Point mass moves down height **h** and horizontal distance **L** purely under gravity. It is guided frictionless by a parabolic curve origin being at the point of start. Find velocity at any point on the curve and obtain expression for time taken by point mass to reach ground in this case. Compare this with time taken if the curve would have been straight line instead of parabola. **[8 marks]**



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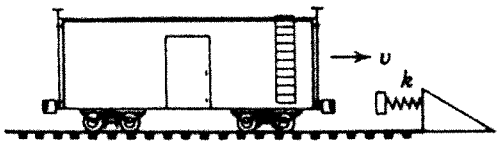
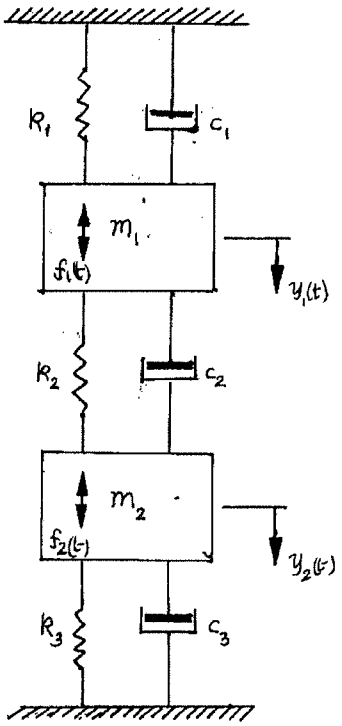
5. A railroad car with a mass of 19000 kg and rolling along a level track is brought to rest by a bumper spring. The spring is deformed by 250 mm and has a modulus of 250 kN/m.
(a) With what velocity did the car strike?
(b) If a dashpot of damping coefficient 30 kN-s/m is later added in parallel to the spring, what will be the spring deformation with the above strike velocity? **[10 marks]**



6. Consider the two-degree of freedom spring-mass-damper system shown in the figure. Assume k_1=10 kN/m, k_2=30 kN/m, k_3=20 kN/m and m_1=2 kg, m_2=1 kg.



(a) Write down the mass, stiffness and damping matrices of the given system.

(b) Determine the undamped natural frequencies of the system and the corresponding mode shapes [assuming that the first degree of freedom moves unit distance (peak value) during free vibration]

(c) Determine the damping coefficients c_1, c_2 and c_3 such that the damping factors of the uncoupled system are respectively 0.09 and 0.12.

(d) Write down the uncoupled system of equations due to excitation forces $f_1(t)=30 \cos 200\, t$ and $f2(t)=0$.

(e) Determine the forced response of the two masses due to excitation forces of part(d).

[25 marks]

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