**Kinematics and Dynamics, PhD Qualifying Exam, Jan-2019**
**Department of Mechanical Engineering, IIT Bombay**
**Max Points 100, Duration 3 hrs**

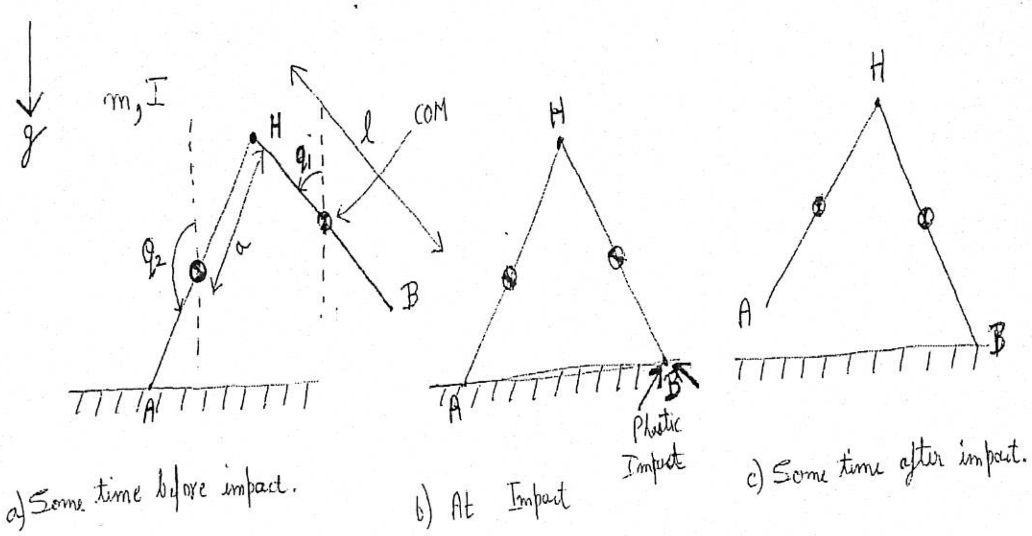
Instructions:

- Examination is closed book, closed notes.
- Use of calculator is allowed.
- Solve all seven questions.
- Clearly mention any assumptions you make.

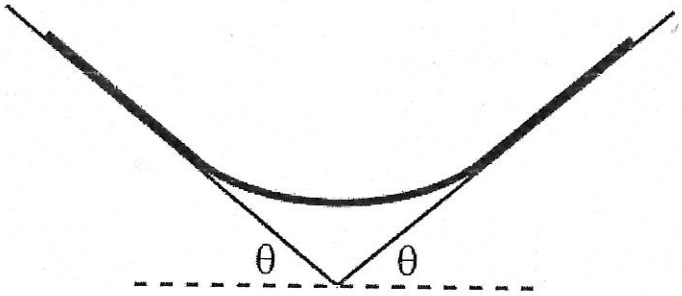
1. **[20 points]** Consider two links (HA & HB shown below) connected via a simple revolute joint at H. Assume that the links can move only in a two dimensional vertical plane (gravity acting downwards) where the axis of the revolute joint is perpendicular to this plane of movement. Assume the links to be rigid and very thin (having very small cross sectional areas). Also assume that the two links are identical, each having mass '*m*', length '*l*', distance of center-of-mass from H '*a*', and mass moment of inertia about center-of-mass '*I*'. Also $q_1$ & $q_2$ denote absolute angles that these two links make with the vertical measured positive in the counter-clockwise direction from the vertical as shown below.

   Initially, point A is placed firmly on the ground and point B is in air in a configuration somewhat similar to the one shown in the left-most figure below (the exact angles in this configuration are not important). Assume that the links move such that eventually point B impacts the ground plastically and point A lifts off simultaneously. Assume that during the motion prior to this instantaneous impact, point A does not slip on ground and link HA is able to tip about point A and after this plastic impact point B keeps sticking to the ground without slipping but link HB is able to rotate about point B. Also assume that there are no impulsive forces generated at point A due to the impact.

   You are told that just prior to the instantaneous impact the angles and angular velocities of the two links are given by $\boldsymbol{q}^- = [q_1^-, q_2^-]'$ and $\dot{\boldsymbol{q}}^- = [\dot{q}_1^-, \dot{q}_2^-]'$ (the boldface variable are column vectors). The angles and angular velocities right after the instantaneous plastic impact are denoted by $\boldsymbol{q}^+ = [q_1^+, q_2^+]'$ and $\dot{\boldsymbol{q}}^+ = [\dot{q}_1^+, \dot{q}_2^+]'$. Find a relationship between the pre- and post- impact angles and angular velocities.

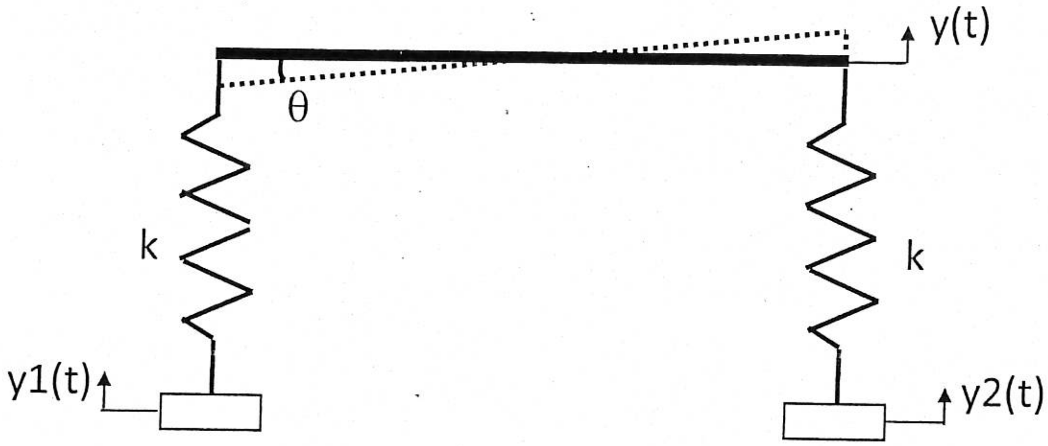


2. **[10 points]** A rope rests on two platforms which are both inclined at an angle θ as shown below. You are allowed to choose this angle. The rope has uniform mass density, and its coefficient of friction with the platform is 1. The system has left-right symmetry. What is the largest possible fraction of the rope that does not touch the platforms? What angle θ allows this maximum value?

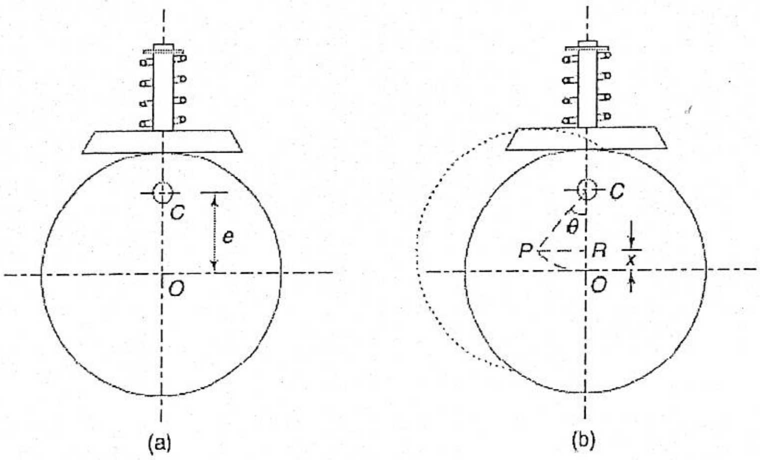



3. **[13 pts]** Consider the system, comprising of a uniform rigid rod of mass m supported by two springs of stiffness k, as shown below.

   a. Derive the nonlinear equations governing the motion of the rod (do not assume small angles). You may assume that the springs remain vertical throughout the motion of the rod.
   b. Assuming small angles, linearize the equations derived in part a, and write the resulting equations in matrix-vector form.
   c. Determine the natural frequencies and corresponding mode shapes of the system from part b. Sketch the mode shapes.
   d. Considering $y1(t) = Y1\sin(\omega t)$ and $y2(t) = Y2\sin(\omega t)$, derive the expression for the linear and angular displacement of the rod using the linearized equations (assume zero initial conditions).



4. **[12 pts]** Consider the frictionless rigid eccentric cam with a flat face follower of mass m, as shown. At time t=0, the follower is at its lowest position as shown, with the spring (of stiffness k) compressed such that the spring force is P. The cam rotates counter-clockwise with a constant angular velocity ω.

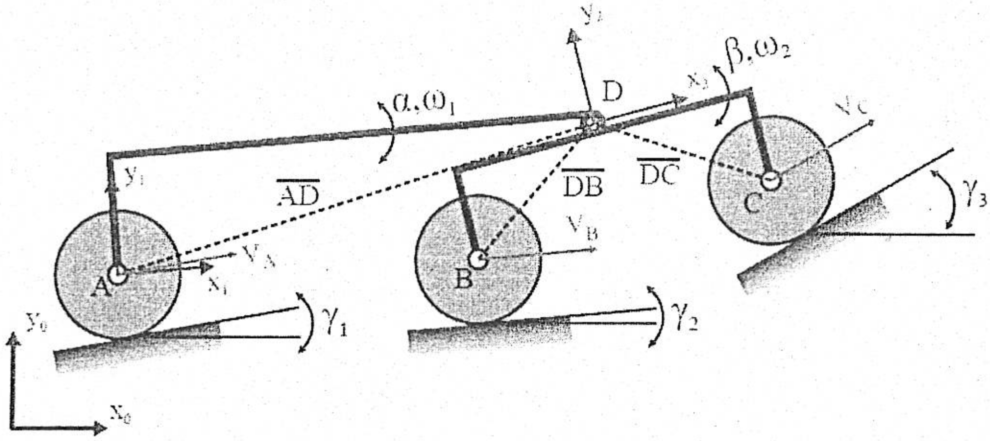



   a. Assuming the zero position of the follower to be at its lowest position, derive the expression for the vertical position x(t) of the follower in terms of the eccentricity e. Determine the velocity and acceleration of the follower as functions of time.
   b. Draw free body diagrams for both the follower and the cam, clearly indicating all the forces and torques acting on them.
   c. Determine the vertical force F(t) exerted by the cam on the follower. How does this force vary with time? Sketch the variation of the force over one cycle of rotation of the cam.
   d. When is F(t), derived in part (c) above, lowest? Determine the maximum value of w such that the follower maintains contact with the cam throughout the cycle.
   e. Derive the expression of torque T(t) that should be applied to the shaft of the cam. Sketch the variation of T(t) over the cycle.

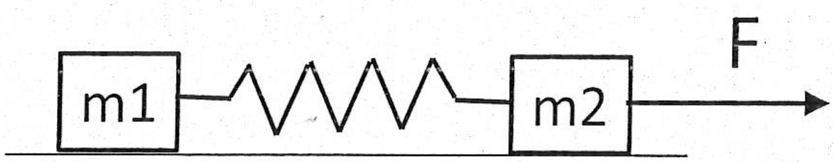
5. **[10 pts]**
   a. **[3 pts]** A four-bar mechanism comprising only of revolute joints, is to be constructed using links of lengths 3, 4, 1, and 5 units. How many crank-rocker mechanisms are possible using these links? Determine the time ratio for the forward and backward strokes for any one of these crank-rocker mechanisms.
   b. **[2 pts]** A truck moves on a horizontal track with a constant forward velocity v. A disk of radius R rolls on the bed of the truck without slipping (in the horizontal direction). If the angular velocity of the disk is ω, determine the location of the instantaneous center of the rotation and the velocity of the center of the disk.
   c. **[5 pts]** The figure below shows a rocker-bogie, comprising of the rigid links AD and BDC with the three wheels, as shown (there is a pin joint at D). Such a vehicle can traverse uneven terrain. In the figure, α and β represent the instantaneous angular positions of links AD and BDC, while ω1 and ω2 represent the angular velocities of the links.

      Let the velocity of point A at a particular instant be $V_A$. At the instant shown, the positions of points A, B, C and D are known. Provide a set of equations that can be solved for velocities $V_B$ and $V_C$ (of points B and C, respectively), and the angular velocities of the two rigid links. Express your equations using vectors $\overline{AD}$, $\overline{DB}$ and $\overline{DC}$.

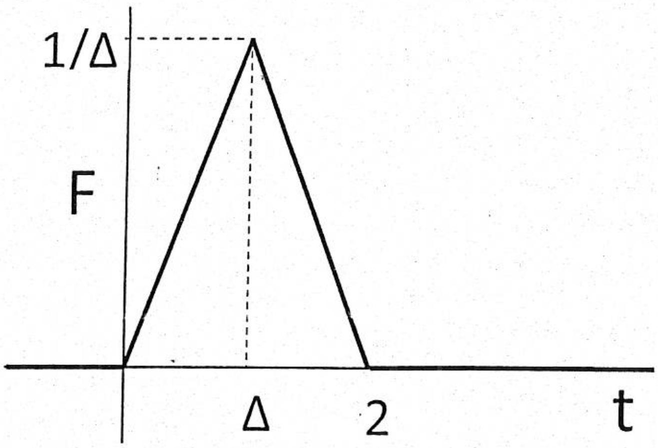
      Use of appropriate illustrations to explain your method is expected and encouraged.




6. **[15 pts]** Consider two masses m1 (= 0.1kg) and m2 (= 0.1kg) resting on a friction-less floor. The masses are connected through a spring with a spring constant 5 N/m. Mass m2 is acted upon by a force, F, on as shown below.




a. Write down the equations of motion of the system. In particular, obtain the equation that defines the evolution of, x(t), representing elongation/compression of the spring.
b. From the equations in part a, obtain the natural frequency of oscillation of the system
c. Suppose F(t) is as shown in the figure. Solve for x(t)




d. In part c, suppose Δ tends to zero, what would x(t) converge to? How would you interpret the result you have obtained.

7. **[20 marks]** Consider a satellite of mass 500 kg going around the earth on a geo-stationary (ie with respect to a person on the earth, the satellite appears stationary) orbit. Assume the following: mass of the earth = $5.97 \times 10^{24}$ kg ; gravitational constant = $6.67 \times 10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$

   a. Show that the geo-stationary orbit necessary has to be circular. Obtain the radius of the geo-stationary orbit and the linear speed of the satellite.

   b. Suppose now that a fragment of mass 100kg fires and separates itself, tangentially, at a speed of 800 m/s relative to the rest of the satellite. Further assume that the separation causes an increase in speed of the rest of the satellite.

      i. Carefully sketch a picture that illustrates the future trajectory of the rest of the satellite in as much detail as possible. (Hint: Consider, amongst others, the following questions: Will the rest of the satellite continue to orbit the earth? If so, would it be a new orbit? Would the new orbit be circular? If no, what shape would it be?)

      ii. What is the work done, if any, by the fragment on the rest of the satellite?

      iii. What is the maximum distance, from the centre of the earth, the rest of the satellite would get to post separation from the fragment?