Indian Institute of Technology

Department of Mechanical Engineering

PhD Qualifying Examination, January 2023

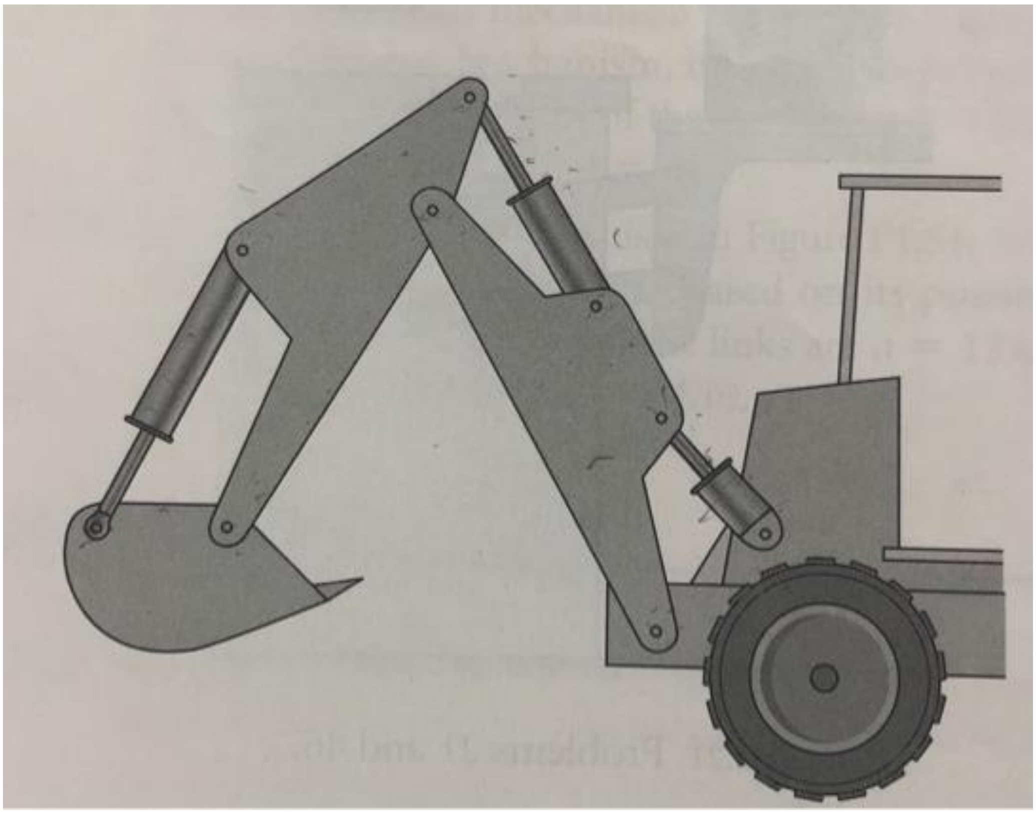
Design-2, Kinematics and Dynamics

General Instructions

(a) This is a closed book, closed notes examination
(b) Answers should be written clearly and legibly
(c) Show all important steps
(d) Clearly highlight assumptions, approximations and important answers of sub-questions
(e) Answer all the six questions that are printed on four pages on both sides

Maximum marks: **100**

Problem 1 [8 points] The image below shows the sketch of a backhoe. Identify and number the links, draw a kinematic diagram and calculate the degrees of freedom. Assume the body of the backhoe to be ground for this purpose.



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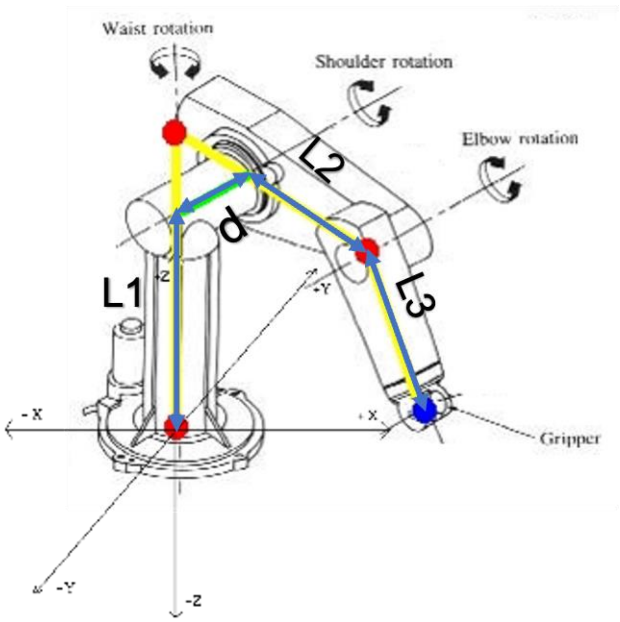
Problem 2 [20 points] The robot arm shown below holds a ball of mass **m** as shown below.



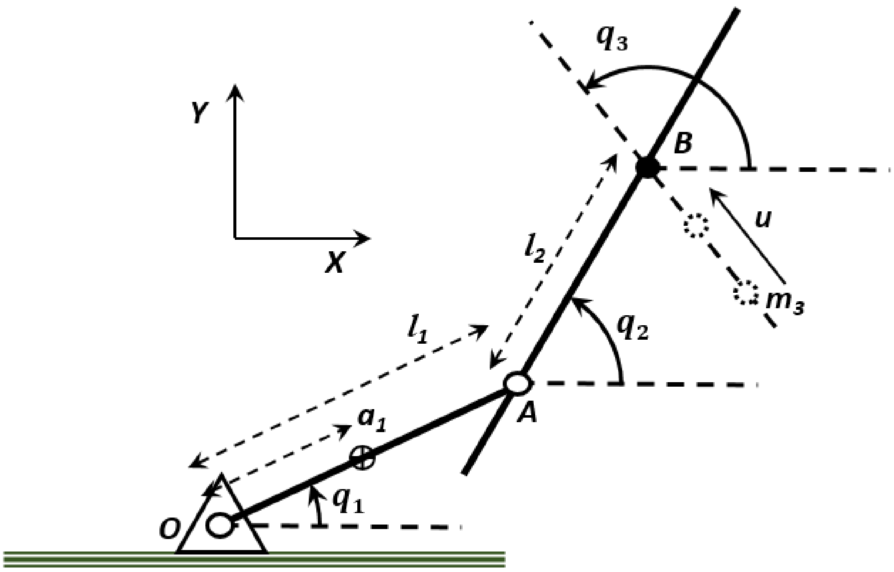
Determine the expressions for the following as a function of joint angles $\mathbf{q_1}$**,** $\mathbf{q_2}$ and $\mathbf{q_3}$, corresponding to the waist, shoulder and elbow rotations respectively. and their derivatives:

a) The position vector representing the location of the center of the ball. **Assume** that the arm is outstretched pointing in the +X-direction, with shoulder and elbow parallel to the ground, when $q_1 = q_2 = q_3 = 0$.
b) The velocity of the ball at any instant of time.
c) The net total, frictional and normal forces acting between the ball and the gripper assuming that the surfaces of the gripper holding the ball lie in a plane perpendicular to the XY-plane.

Problem 3 [5 points] Block A, with mass m_A, is constrained to move in a horizontal frictionless slot, and connected to block B, with mass m_B, through a uniform thin connecting rod of length L and mass M. The rod is hinged to both the blocks, and block B is constrained to move vertically. At some instant of time, block A if found to have a velocity **v**, while the connecting rod is inclined at an angle $\boldsymbol{\alpha}$ to the horizontal. Obtain the angular velocity of the rod at this instant, and velocity of block B.

Problem 4 [24 Points] Consider an open-chain, two-link mechanism shown below. This mechanism moves in the horizontal plane as its joint axes are vertical. Assume its joints to be frictionless. Its link-1 is connected to a fixed support via a pin joint at ***O*** and link-2 is connected to link-1 via a pin joint at ***A***. Initially, this mechanism is at rest in a configuration where the joint angles are $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ as shown below. A point mass ($\boldsymbol{m_3}$) moving at a speed ***u*** impacts the second link at point ***B*** perfectly plastically. Before the impact, this point mass was moving along a line that makes an angle $\boldsymbol{q_3}$ with the horizontal as shown in the figure below. In this problem, your goal is to solve for the absolute angular velocities of links-1 and 2 right after the impact. Derive the “two” linear simultaneous equations which can be solved to obtain these two unknown angular velocities. You do not have to solve these linear simultaneous equations but you should simplify the coefficients of the unknown angular velocities in these equations and should present them in the matrix form at the end.

Denote masses of links-1 and 2 to be m_1 and m_2 respectively. The center-of-mass of link-1 is at a distance a_1 from the first joint as shown below. Assume the center-of-mass of the link-2 to be at ***A***. Denote mass moments of inertia of links-1 and -2 about their respective centers-of-mass to be I_1 and I_2 respectively. Denote length ***OA*** as l_1 and ***AB*** as l_2 as shown below. Use any other quantity you need and provide its proper description and name. Also state any assumptions you make in solving this problem



Problem 5 [8 Points] A skinny rigid ring of mass ***M*** and radius ***R*** is lying on a smooth horizontal table. It is set into rotation about its own axis at an angular velocity ω such that its center is stationary on the table and it is undergoing a pure rotation with its axis of rotation perpendicular to the table. Calculate the tension in the ring assuming it uniform mass per unit length. Show all steps with proper reasoning.

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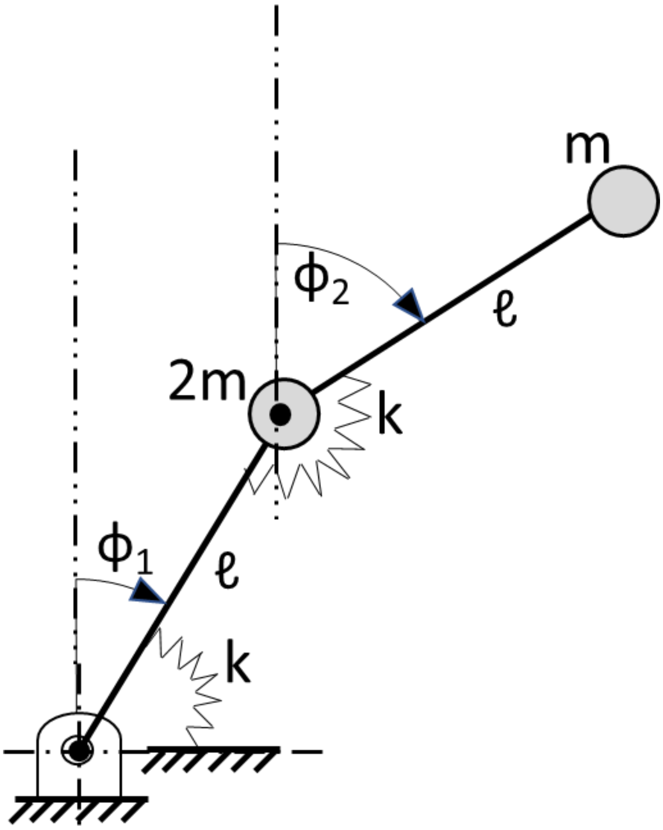

6. Two massless rods, each of length ℓ and attached to masses 2*m* and *m*, as shown in the figure move along a horizontal plane, restrained by linear torsional springs, each of stiffness k. The figure shows an instant at which the rods make angles, measured with respect to the vertical, as shown.

(a) Write down equations for kinetic and potential energies of the system. **[16]**
(b) Using part (a), write down the Lagrangian of the given system **[3]**
(c) Using part (b), obtain various derivatives of the Lagrangian that are required for formulating the differential equations of motion of the given system and linearize them for small angles of ϕ_1 and ϕ_2.**[5]**
(d) Using part ©, write down the differential equations of motion of the given system **[2]**
(e) If m=1 kg, k=10000 N-m/rad, and ℓ=0.250 m, determine the natural frequencies and modeshapes of the above system. **[9]**

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