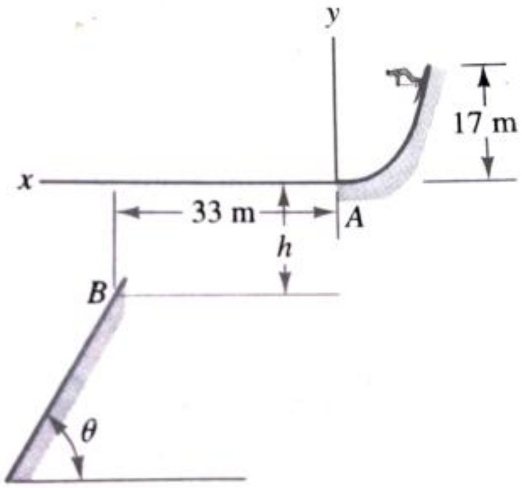
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
DEPARTMENT OF MECHANICAL ENGINEERING

September 2021 : Ph.D. Qualifying Examination : DES-2

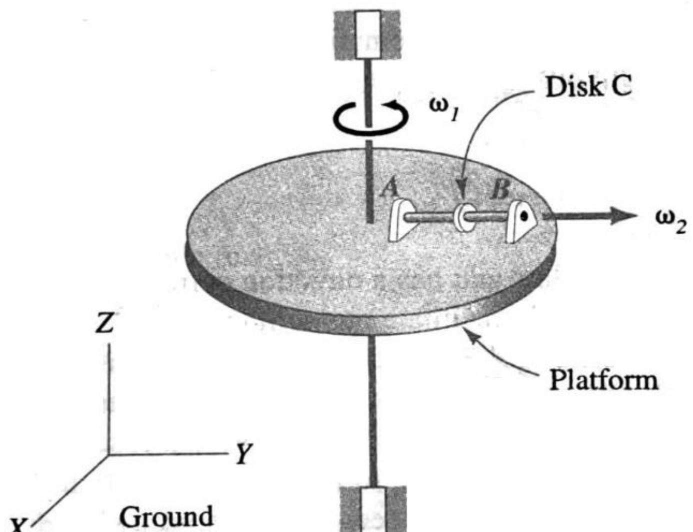
There are 10 questions. All questions are compulsory.

Q1. [5 marks]
A particle $\boldsymbol{A}$ with a mass of 1 kg has an initial velocity of $\boldsymbol{V_0} = \mathbf{10}\boldsymbol{i} + \mathbf{6}\boldsymbol{j}$ m/s. After particle $\boldsymbol{A}$ strikes particle $\boldsymbol{B}$, the velocity becomes $\boldsymbol{V} = \mathbf{16}\boldsymbol{i} - \mathbf{3}\boldsymbol{j} + \mathbf{4}\boldsymbol{k}$ m/s. If the time of encounter is 10 ms, what average force was exerted on the particle $\boldsymbol{A}$? What is the change of linear momentum of particle $\boldsymbol{B}$?

Q2. [10 marks]
A ski jumper moves down the ramp aided only by gravity as shown in the figure. If the skier moves 33 m in the horizontal direction and is to land smoothly at $\boldsymbol{B}$, what must be the angle $\boldsymbol{\theta}$ for the landing incline? Neglect friction. Also determine $\boldsymbol{h}$.

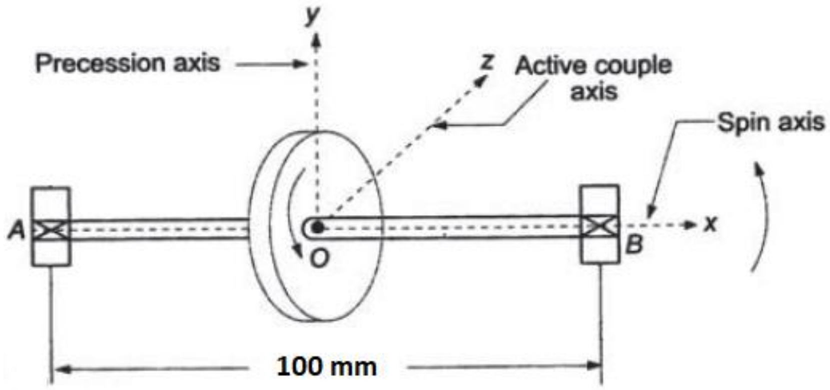


Q3. [10 marks]
A disc $\boldsymbol{C}$ is mounted on a shaft **AB** as shown in the figure. The shaft and disc rotate with a constant angular speed $\boldsymbol{\omega_2}$ of 10 rad/s relative to the platform to which bearings $\boldsymbol{A}$ and $\boldsymbol{B}$ are attached. Meanwhile, the platform rotates at a constant angular speed $\boldsymbol{\omega_1}$ of 5 rad/s relative to the ground reference $\boldsymbol{XYZ}$. What is the angular velocity vector $\boldsymbol{\omega}$ for the disc $\boldsymbol{C}$ relative to $\boldsymbol{XYZ}$? Also determine $\dot{\omega}$ and $\ddot{\omega}$ with respect to $\boldsymbol{XYZ}$.



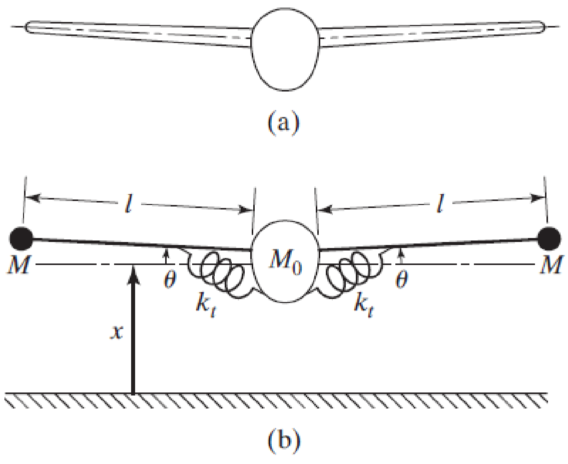
Q4. [5 marks]

A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about its axle with a constant speed of 1000 RPM while the axle precesses uniformly about the vertical axis at 60 RPM. The directions of rotation are as shown in the figure. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.

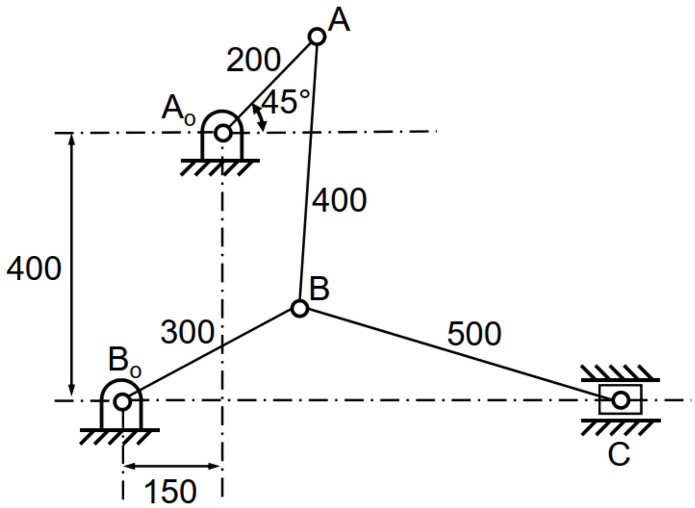


Q5. [10 marks]

When an airplane (see figure) undergoes symmetric vibrations, the fuselage can be idealized as a concentrated central mass $\boldsymbol{M_0}$ and the wings can be modelled as rigid bars carrying end masses $\boldsymbol{M}$, as shown in the figure. The flexibility between the wings and fuselage can be represented by two torsional springs of stiffness $\boldsymbol{k_t}$ each. (a) Derive the equations of motion of the airplane, using Lagrange's equations with $\boldsymbol{x}$ and $\boldsymbol{\theta}$ as generalized coordinates. (b) Find the natural frequencies and mode shapes of the airplane.

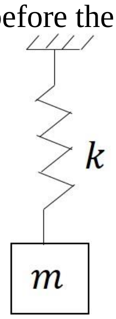


Q6. [10 marks]
In the mechanism shown below, AoA rotates anticlockwise at 210 rpm and 60 radians/s^2. All lengths are in mm. Find the velocity of the mid-point of BC by the graphical method. If any data seems missing, make an appropriate assumption and state that. If any data seems superfluous, state that and ignore that data. Label all points of the velocity polygon and state what the points (or the lines) stand for. Show the calculation necessary to arrive at the different components of the velocity polygon. There is no need to explain the method of construction. (You will be judged primarily on the approach to the construction of the velocity polygon, not on the correctness of the result.)



Q7. [20 marks]
Using the data and figure of Question 6, find the acceleration of the mid-point of BC through the graphical method. If you are not sure of your results of Question 6, then make appropriate assumptions of the results, state them and then proceed with the acceleration polygon. Label all points of the acceleration polygon and state what the points (or the lines) stand for. Show the calculation necessary to arrive at the different components of the acceleration polygon. There is no need to explain the method of construction. (You will be judged primarily on the approach to the construction of the acceleration polygon, not on the correctness of the result.)

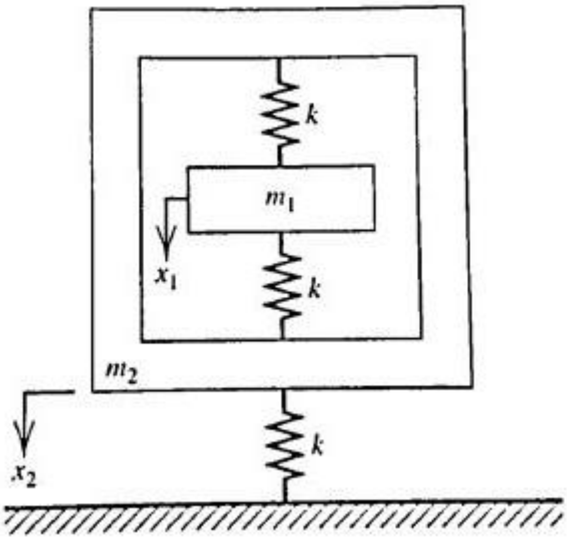
Q8. [5 marks]
Referring to the figure, if the mass hangs freely (under gravity), it stretches the spring by a distance h. Now, suppose the mass is held such that the spring is initially un-stretched and then suddenly released, determine the maximum stretch in the spring in terms of h before the mass begins to return up.



Q9. [10 marks]
Determine the response of an un-damped single degree of freedom system with mass m and spring constant k to an impulse P_0 at t = 0, with non-zero initial displacement x_0 and initial velocity v_0. [No marks if you set the initial conditions to zero.]

Q10. [15 marks]

For the system shown in the figure, determine the natural frequencies and mode shapes. Use $m_1=m_2=m$.



--xx--xx-- End of Question Paper --xx--xx--