Qualifying Examination
Feb 2022

DES2 Kinematics & Dynamics

General instructions:

- Write clearly and legibly, showing clearly the important steps in the solution, including any assumptions and approximations.
- No questions will be entertained. If something is not clear, make suitable assumptions and proceed.
- This exam is "closed books, closed notes."
- Sketches of response: a sketch is only an approximate plot, but should clearly show the key features such as the starting point, end point, relevant time periods, etc.
- Laplace transform method is not allowed!
- There are a total of six questions.
- Total points = 60. Total time allowed = 3 hours.

Problem 1 [12] Consider a thin massless string of length ***L*** that is passing through a very small frictionless hole on a friction less table. Masses ***m*** and ***M*** are attached on the two ends of the string such that ***m*** is able to move on the frictionless table and ***M*** is hanging below the table as shown in the figure below. Assume that the string is always taught and ***M*** can only move up and down with gravity acting downwards. The length of the string that is on the table at any given instant is denoted by the variable ***r(t)*** and the variable angle $\boldsymbol{\theta(t)}$ denotes the angle that the string on the table makes with a fixed reference line on the table as shown.

a. **[3]** Using Lagrange's/Energy Method, derive the equations of motion governing this system. Clearly state your choice of generalized variables for this system. Note: No credit will be given if Lagrange's/Energy method is not used.

b. **[2]** Derive the conditions under which the mass ***m*** can keep undergoing a circular motion on the table at a constant angular velocity.

c. **[7]** Assume that the mass ***m*** is undergoing this circular motion and you give a small perturbation in the variable ***r(t)***. Show that this system will undergo an oscillation under this small perturbation in **r(t)** by linearizing your equations of motion about this circular motion and find the frequency of this oscillation.

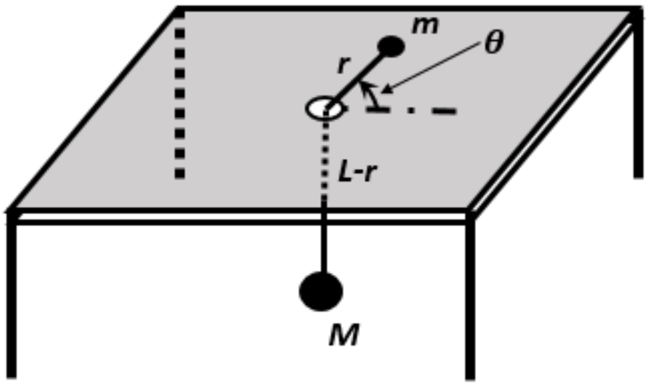


Figure 1

Problem 2 [8] The figure below shows a four-bar mechanism comprising of four links (numbered 1 through 4) and four revolute joints with Link 1 as the grounded link. Lengths of Link 1 as well as Link 2 is 6 cm each, Link 3 is $2\sqrt{2}$ cm and that of Link 4 is 8 cm. **(a) [2]** Is any link in this mechanism capable of making a full revolution? If yes, which link can make a full revolution. **(b) [6]** If Link 3 makes an angle of $\theta_3 = 45^{\circ}$ with the horizontal as shown in the figure below, **"analytically"** compute all compatible values of angles θ_2 and θ_4 as defined in the figure below in degrees.

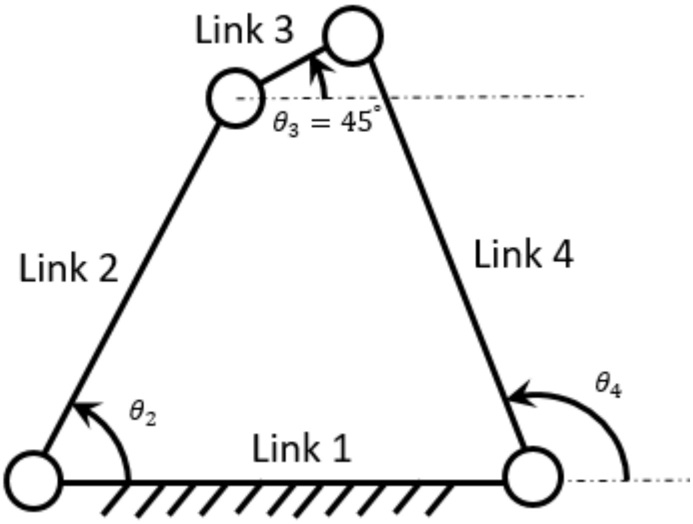


Figure 2

Problem 3 [10] A thin rod **AB** having mass ***4m*** and length ***4a*** is lying stationary on a horizontal frictionless table. Its center-of-mass is at its geometric center **C** and its mass moment-of-inertia about **C** is ***I*** (please do not use any formulae in place of ***I***). A point mass ***m*** impacts the rod at a distance ***a*** from **C** with a velocity ***u*** perpendicular to the rod just before the impact as shown below and sticks to the rod after the impact. An ***xy***-frame of reference is shown below. Compute the ***x*** and ***y*** component of the velocity of end **A** of the rod right after impact.

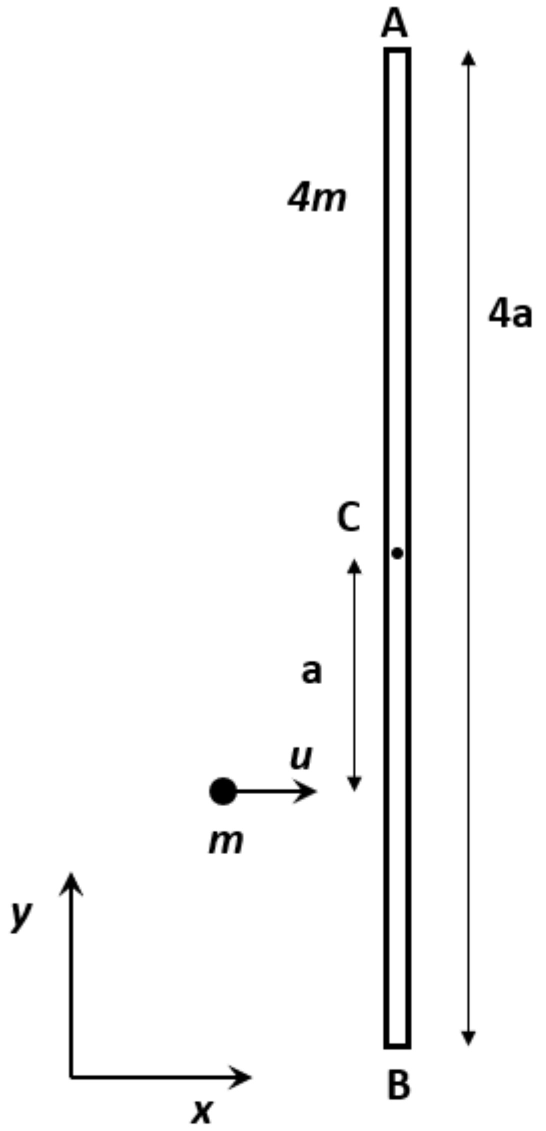


Figure 3

Problem 4 [15]

An accelerometer is a measurement device that can be used to measure (as the name suggests) the acceleration of a moving body. A typical accelerometer consists of an outer casing; inside the casing is a mass *'m'* that is attached to the outer casing by means of a spring of stiffness *'k'* and a damper of constant *'c'*. The casing is attached (say, by gluing or bolting) to the body whose motion is to be measured. When the body undergoes a displacement *'e(t)'*, the casing too moves and this motion causes the mass *'m'* inside to undergo a motion *x(t)*. (The acceleration of mass *'m'*, i.e., $\ddot{x}(t)$ is detected through some means such as electro-magnetism or the piezo-electric effect, but which is not important here).

A. **[5]** Consider that the accelerometer is attached to a rocket such that the motion of the mass 'm' (i.e., x(t)) is along the 'long' axis of the rocket.

i. Derive the equation of motion for x(t) when the rocket undergoes a given displacement e(t) along its 'long' axis. Clearly describe your coordinate system, sign convention and free body diagram.

ii. The mass 'm' is given an initial velocity v_0. (e(t) can be taken to be zero.) Solve the above equation of motion for the following two cases of the damping coefficient: $-1 < \zeta < 0$ and $\zeta = 1$. Sketch the responses for these cases.
(**Note**: while a sketch is only an approximate plot, it should clearly show key features such as the starting point, end point, relevant time periods, etc.)

B. **[3]** The rocket initially lies horizontally, and so the motion of the mass 'm' can also only be along the horizontal direction. The rocket is now turned by 90° such that it is now vertical. Describe clearly with steps (but without actually solving) how you would solve for the response x(t) when this occurs for the following two cases:
   i. when the 90° turn occurs instantaneously, and
   ii. when it is rotated at a uniform rate such that the 90° turn occurs over a time T_0.

   Sketch the expected responses for both cases. Consider $\zeta = 1$.

C. **[5]** Consider that the rocket is now falling vertically under the effect of gravity, such that its long axis is also vertical. The rocket's mass >> m, and hence, it's motion depends only on gravity and not on the motion of the much smaller mass 'm'. Ignore damping. Using Lagrange's method, derive and solve for the acceleration $\ddot{x}(t)$ at any instant of time 't'. Clearly define the coordinates and show the expressions for kinetic and potential energy. (Only half points if force-balance is used!)

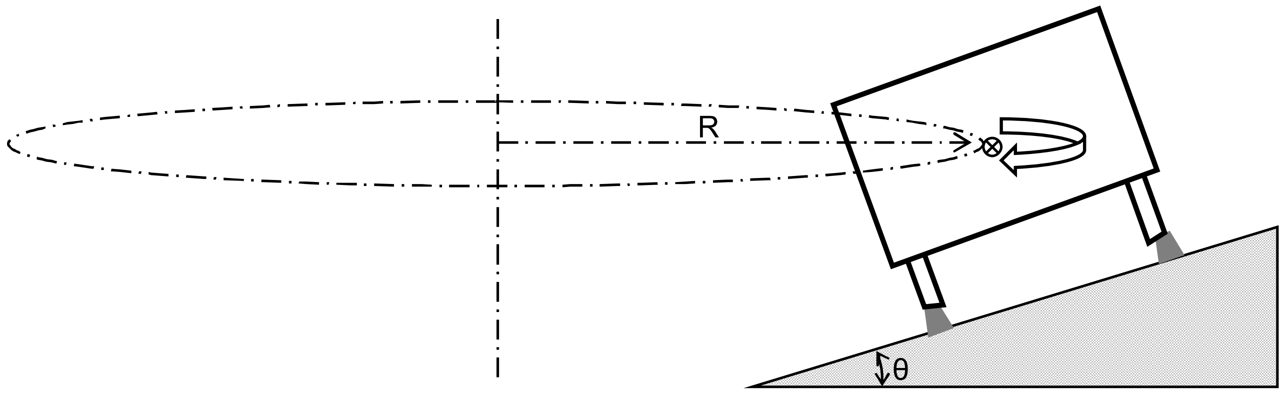
D. **[2]** The rocket now impacts the ground in the desert such that its body embeds in the sand and instantaneously comes to a stop. Describe (without actually solving) how you would solve for the response x(t) when this impact occurs. Sketch the expected response. Ignore damping.

Problem 5 [10]

A railway locomotive of mass 'm' is travelling at a speed *'v(t)'* (that is not constant) on a curved track with centre at 'O' and of radius 'R'. The track is 'banked', i.e. tilted, at an angle θ to the horizontal.

a. Obtain the instantaneous position vector **r**(t) of the centre of mass with respect to the centre *'O'*, and its next two time derivatives. Illustrate with a diagram the directions of these vectors.
b. In terms of the vector **r**(t), derive the direction and magnitude of the resultant of the reaction forces between the wheels of the locomotive and the track. [**Note**: You can assume the locomotive to be a cuboid with a set of wheels at each corner, though you are not required to derive the reaction forces at each wheel.]
c. Consider that the locomotive is now powered by a gas turbine of mass moment of inertia J that rotates at a constant angular speed Ω. How do the reaction forces change? Illustrate using vector diagrams. Consider the two scenarios: the first, where the turbine is placed longitudinally (i.e., in the direction of motion), and the second where it is placed transversely (i.e., horizontal, but perpendicular to the direction of motion).

[**Note**: In all of the above, clearly show the coordinate system used.]



Problem 6 [5] Consider a disc of radius 'R' that is mounted to rotate about the point O that is located at a distance 'e' from its geometric centre C. The disc is in contact with a spring-loaded 'follower', that is constrained to move in the vertical direction, as shown in the diagram. There is sufficient restoring force/preload in the spring such that the disc and the follower never lose contact with each other. Derive an expression relating the angular rotation speed of the disc and the vertical velocity of the follower. Note that the axis of motion of the follower does not pass through either of the points O or C.