DES 1.

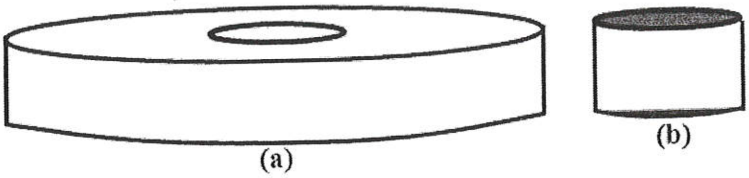


Figure 1

1. (10 marks) A cork of inner radius a shown in Figure 1(b) is to be used to seal a thin hollow disc of inner radius a_0 and outer radius b_0 shown in Figure 1(a). The elastic modulus of cork and disc material is E_c and E_d respectively. Assume that the Poisson's ratio is same and equal to zero for both the materials.

(a) What is the necessary condition for the cork to act as stopper? **[1]**

(b) If the condition is satisfied, will there be any stress in cork and disc material? **[1]**

(c) If yes, then calculate the stresses in cork. Clearly state the conditions for the validity of the solution. **[5]**

(d) If the limiting hoop stress for cork material is S^*, find out if there are any restriction on dimensions of cork and disc. **[3]**

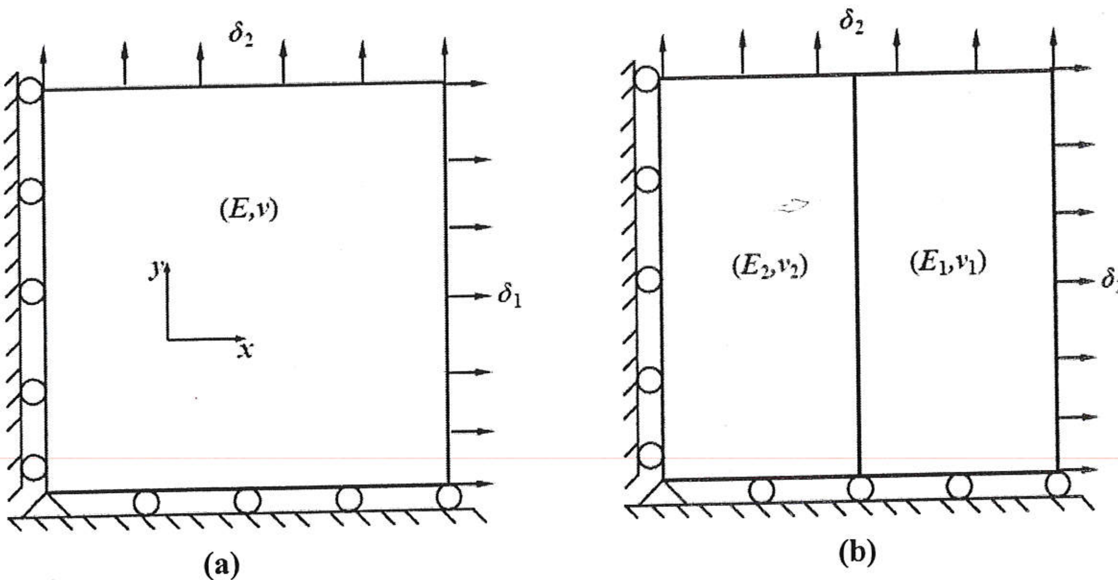


Figure 2

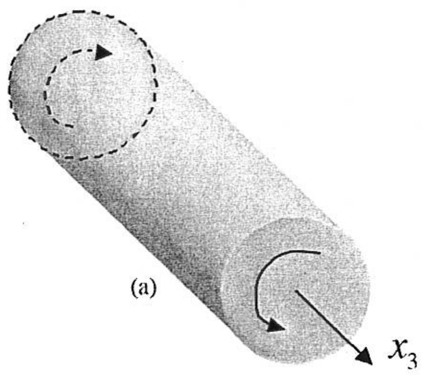
2. (10 marks)

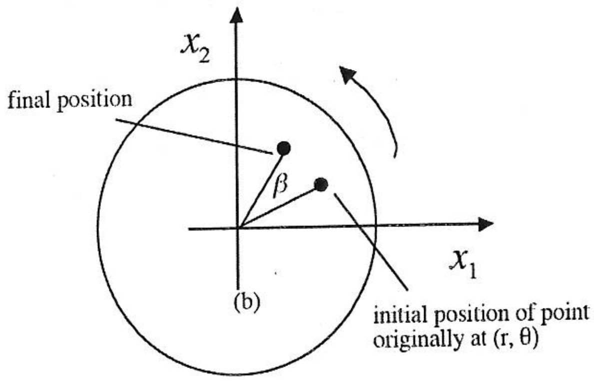
a) Figure 2(a) shows a thin square plate of edge length a, whose two adjacent sides are subjected to displacement δ_1 and δ_2 in x and y direction respectively. Find out the stress field in the plate. **[3]**

b) In Figure 2(b), the homogeneous plate of Figure 1(a) is replaced by thin plate made up of two plates having dimensions $a/2$ x a and made up of dissimilar material perfectly joined along the common side. If the two adjacent sides of the plate are subjected to displacement δ_1 and δ_2 in x and y direction respectively then find out the stress field in the whole plate. **[7]**

3. *(5 + 5 = 10 Marks)* Answer the following two questions:

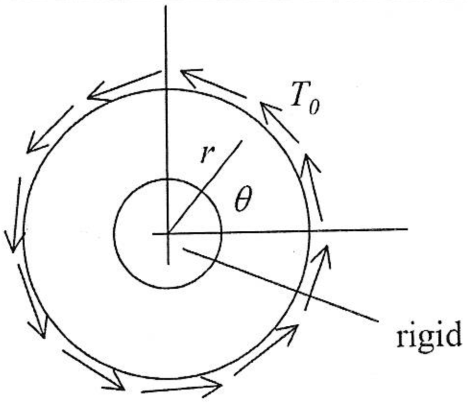
(1) A torque is applied to the ends of a circular engine shaft causing it to twist around its axis (see figure below). As a result, each circular cross-section rotates about the x_3 axis, while remaining in its original plane perpendicular to x_3, through an angle β (see figure below) that is linearly proportional to the distance of the section from the origin (i.e. $\beta = \alpha x_3$). Also assume that $u_3 = 0$ everywhere. (a) Find the u_1 and u_2 displacement field components as a function of position and β. (b) Assuming that $\beta << 1$ (i.e. small displacement gradients), find the infinitesimal strain tensor as a function of position and α. (c) Find the volume dilatational induced by this strain field, and the mean and deviatoric parts of the strain field. Note: You may find the following relations useful $\cos(a+b) = \cos a \cos b - \sin a \sin b$ $\sin(a+b) = \sin a \cos b + \cos a \sin b$



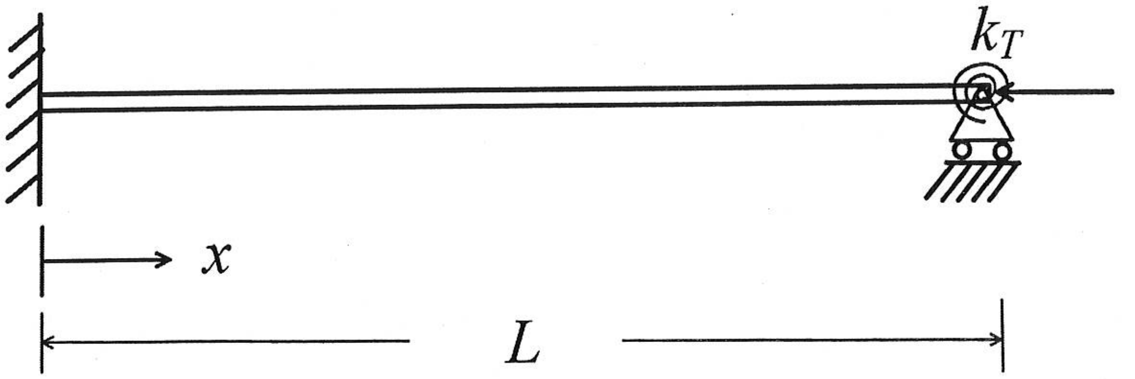


(2) Show that the principal values, S_n, of the stress deviator $S(S_{ij} = \sigma_{ij} - (1/3)\sigma_{kk}\delta_{ij})$ are given by the equation $S_n^3 - J_2 S_n - J_3 = 0; n = 1,2,3$ where, $J_2 = (1/3)(I_1^2 - 3I_2]$; $J_3 = (1/3)S_{ij}S_{jk}S_{ki}$, and I_i are the invariants of the original stress tensor σ.

4. *(10 Marks)* A circular disk of inner radius a and outer radius b is fixed to a rigid bar at the inner boundary. A uniform shear traction T_0 is applied at the outer boundary of the disk. Determine the stresses $\sigma_{rr}, \sigma_{\theta\theta}$ and $\sigma_{r\theta}$, and the displacements u_r and u_θ of the disk. Hint: start by assuming a functional form for the stresses as $\sigma_{rr} = \sigma_{\theta\theta} = 0$ and $\sigma_{r\theta} = f(r)$, i.e., function of only r.



5. (10 marks) The figure below shows a long slender column of length L and flexural rigidity EI subject to an axial load P. The column is fixed into the wall at one end and connected to a pinned joint through a torsional spring with stiffnes k_T as shown. Use the method of minimum potential energy to calculate the approximate buckling load of this column.



6. (10 marks) The cross-section of a steel ($E = 200$ GPa) cantilever beam AC is 60 mm × 60 mm. Before the load P was applied, a gap $\delta_0 = 0.5$ mm existed between the cantilever beam AC and the rigid support at B. Use force/moment equilibrium methods to answer the following questions exactly (a) What is the magnitude of P required to just close the gap? (b) What is the magnitude of P for which the vertical deflection at C is 1 mm?