## Manufacturing Processes II Qualifiers - January, 2023

1. In an orthogonal machining process, if the shear stress experienced by the material along the shear plane during machining is a function of the normal stress, i.e., $\tau_{\mathrm{s}}=\tau_{0}+\mathrm{k} \sigma_{\mathrm{s}}$, where $\tau_{0}$ and k are constants, and $\sigma_{s}$ is the normal stress on the shear plane; then show that the cutting force $\left(\mathrm{F}_{\mathrm{c}}\right)$ can be expressed as

$$
F_{c}=\frac{\tau_{0} t_{0} w \cos (\beta-\alpha)}{\sin (\phi) \cos (\phi+\beta-\alpha)[1-k \tan (\phi+\beta-\alpha)]}
$$

where, $\mathrm{t}_{0}$ is the uncut chip thickness, w is the chip width, $\beta$ is the friction angle, $\alpha$ is the rake angle, and $\phi$ is the shear plane angle.
[8 Marks]
2. In orthogonal machining, as seen in Fig. 1, relief or clearance angle is the angle between the flank surface and the cutting velocity $\bar{V}$. A positive relief angle is maintained to avoid rubbing of the flank surface of the tool with the newly machined surface of the workpiece.
(a) If the relief angle is made zero, and the coefficient of friction between the flank and the workpiece is $\mu$ (same as that between the chip and rake face), derive the expression $\phi$ in terms of the rake angle $\alpha$, and coefficient of friction $\mu$. Clearly state your assumptions and show all your steps. (Hint: Assume Ernst and Merchant's hypothesis to be valid. The hypothesis is that the shear plane is located at an angle $\phi$ such as that the cutting force, i.e., total force in the direction of cutting velocity, is minimized)
[4 Marks]


Fig. 1 : Question 2
(b) For the orthogonal cutting operation described in [A], the following parameters are given:

The workpiece is a block of 304 stainless steel with a shear strength of $500 \mathrm{~N} / \mathrm{mm}^{2}$. Both the rake angle and relief angle are $0^{\circ}$ each. Undeformed chip thickness is 1 mm , and the width of the cut per pass is 5 mm . The tool moves with a cutting speed $(\mathrm{V})$ of $1 \mathrm{~m} / \mathrm{s}$. The coefficient of friction between the workpiece material and the tool is 0.3 . If the power transmission efficiency of the machine is $80 \%$, what is the minimum power requirement for the machine to carry out this operation? What fraction of total power is dissipated as friction?
[6 Marks]
3. A cylindrical rod of diameter 50 mm and length 200 mm is machined using turning operation. A single point cutting tool with side rake angle and end rake angle of $0^{\circ}$ and $45^{\circ}$ was used. The machining was done with a certain side cutting edge angle, with a depth of cut of 1 mm and spindle speed of 300 rpm . The cutting forces along the $\mathrm{x}, \mathrm{y}$, and z directions were measured using a dynamometer as 200 N , 200 N , and 1000 N , respectively. The cutting action is along the z-direction, and the feed is along the y-direction. The cut chip thickness was measured as 1 mm . Estimate the total energy consumed in machining the block for a length of 100 mm in a single pass. Assume Ernst and Merchant relation to be valid.
[10 Marks]
4. A rectangular slot of cross-section $2 \mathrm{~mm} \times 2 \mathrm{~mm}$ is machined using a slab milling operation on a mild steel block of dimensions $10 \mathrm{~mm} \times 10 \mathrm{~mm} \times 10 \mathrm{~mm}$. The cutter is of 8 mm diameter and 2 mm width. It has six straight cutting edges with radial rake angles of $0^{\circ}$ and rotates at 60 rpm . The slot was machined in a single pass with a feed velocity of $120 \mathrm{~mm} / \mathrm{min}$. The coefficient of friction is 0.5 and the ultimate shear strength of the workpiece material is $400 \mathrm{~N} / \mathrm{mm}^{2}$. Calculate the torque acting on the arbor with time and plot it. Ignore the first cut. Note that the assumption of very small depth of cut as compared to the diameter of the cutter is invalid here.
[10 Marks]
5. A machine shop needs to manufacture cylindrical parts of Aluminum with diameter $\mathrm{D}=100 \mathrm{~mm}$, length $\mathrm{L}=300 \mathrm{~mm}$, and surface finish $\mathrm{Ra} \leq 5$ micron. They produce these parts using a single-pass turning operation with HSS tool with back rake angle $\alpha=10 \mathrm{deg}$, side cutting edge angle $\gamma_{\mathrm{s}}=15 \mathrm{deg}$, and end cutting edge angle $\gamma_{\mathrm{e}}=5 \mathrm{deg}$. When a tool wears out, it takes $\mathrm{t}_{\mathrm{c}}=2$ min/edge to replace the tool edge and costs $\mathrm{k}_{\mathrm{t}}=100 \mathrm{Rs}$ /edge. The setup time is $\mathrm{t}_{\mathrm{s}}=1 \mathrm{~min}$ per part. The machine shop has a capacity utilization rate of $\mathrm{k}_{0}=100 \mathrm{Rs} / \mathrm{min}$ that includes machine cost, labor cost, and other overheads. The cost rate associated with the actual machining time, including cost of electricity, cutting fluid, etc., is $\mathrm{k}_{\mathrm{m}}=$ $20 \mathrm{Rs} / \mathrm{min}$. All other costs and overheads can be neglected.
a. If the machine shop sets a price of $300 \mathrm{Rs} /$ part, what would be the maximum profit per part?
b. What would be the rate of production in parts/hr under maximum profit conditions? Approximate your answer to the nearest integer.
[2 Marks]
Clearly show all your steps to receive full credit. Use following formulae to arrive at your answer,
i. Unit manufacturing cost $u=u_{o}+u_{m}+u_{t}=k_{o} t+k_{m} t_{m}+k_{t} n_{t}$, where $n_{t}$ is the number of cutting edges used per part. t is the total machining time per part $\left(\mathrm{t}=\mathrm{t}_{\mathrm{s}}+\mathrm{t}_{\mathrm{m}}+\mathrm{t}_{\mathrm{c}} \mathrm{n}_{\mathrm{t}}\right)$, $\mathrm{t}_{\mathrm{s}}$ is the setup time per part, $\mathrm{t}_{\mathrm{m}}$ is the actual machining time per part, $\mathrm{t}_{\mathrm{c}}$ is the time required for each tool change, and T is the tool life.
ii. Taylor's extended tool life equation is given as $\mathrm{Vf}^{0.2} \mathrm{~T}^{0.1}=3 \times 10^{5}$, where V is the cutting speed in $\mathrm{mm} / \mathrm{min}$, f is the feed in $\mathrm{mm} / \mathrm{rev}$, and T is the tool life in min.
iii. Surface roughness in turning, $R_{a}=\frac{f}{4\left(\tan \gamma_{s}+\cot \gamma_{e}\right)}$, where f is the feed in $\mathrm{mm} / \mathrm{rev}$.
6. During electrochemical machining of iron plate, for a voltage of 10 V (with an over voltage of 1.5 V ), the feed rate is optimized such that the interelectrode equilibrium gap is approximately 0.125 mm . The electrolyte used for the experiment has a specific conductance of electrolyte $=0.2 \Omega^{-1} \mathrm{~cm}^{-1}$.
a) During the electrolysis process, if iron dissolves as ferrous $\left(\mathrm{Fe}^{2+}\right)$ hydroxide, calculate the feed rate.
b) Calculate \% change in the feed rate if iron dissolves as ferric $\left(\mathrm{Fe}^{3+}\right)$ hydroxide.

With the above setting, the ECM is used to make a hole of a diameter 100 mm in iron plate of 5 mm thickness. The tool used as an ECM electrode has a length of 40 mm and a diameter of 5 mm .
c) Develop a processing strategy in order to achieve the desired geometry.
d) What will be approximate time to complete the job?
e) Discuss what will happen if initial interelectrode gap is larger or smaller than 0.125 mm .
(For iron: gram atomic weight $=55.85 \mathrm{~g}$, density $=7.85 \mathrm{~g} / \mathrm{cm}^{3}$ )
[15 Marks]
7. An EDM process is being carried out using the process parameters: Peak current: 50 A , Duty cycle: $40 \%$ and Pulse ON time: $40 \mu \mathrm{~s}$. When the EDM parameters are changed as given below in (a) and (b), what will be the impact on Material Removal Rate, Surface Finish and Heat Affected Zone (HAZ) in each case?

Explain with proper pulse and discharge diagrams. Assume pulses to be rectangular.
a) Duty cycle: $40 \%$ and Pulse ON time: $20 \mu \mathrm{~s}$
b) Duty cycle: $80 \%$ and Pulse ON time: $40 \mu \mathrm{~s}$
[6 Marks]
8. A titanium tool of diameter $D=10 \mathrm{~mm}$ is used to machine a copper workpiece using ultrasonic machining. See the table (next page) for material properties. The tool vibrates with a frequency of 30 kHz , an amplitude $a=25 \mu \mathrm{~m}$ (from the mean position), and the mean gap ( $h$ ) between the tool and the workpiece is $50 \mu \mathrm{~m}$. The abrasive slurry contains SiC particles of diameter, $d=30 \mu \mathrm{~m}$ mixed in water at a concentration of $\eta=30 \%$ by volume.
a) Describe the mechanisms involved in the material removal.
b) Derive expressions for workpiece material removal rate (MRR) and tool wear rate (TWR). Clearly justify the assumptions involved.
c) Calculate the ratio of MRR to TWR.

| Material | Density $(\mathrm{g} / \mathrm{cc})$ | Flow stress $(\mathrm{MPa})$ |
| :--- | :--- | :--- |
| Copper | 8.96 | 100 |
| Titanium | 4.5 | 450 |
| SiC | 3.21 | -- |

9. A journal and a bearing have a basic size of 200 mm each.

For the bearing: Fundamental deviation $=0$ micron, and Tolerance $=46 \mathrm{micron}$.
For the journal: Fundamental deviation $=-820$ micron, and Tolerance $=115$ micron.
a) Find the dimensions of the journal and the bearing, assuming unilateral system of tolerances.
b) Determine the allowance and type of fit.
c) Which system of fit (Hole-basis or shaft basis) has been adopted?
[9 Marks]
10. A 200.00 mm sine bar is used to inspect an angle on a part.

The angle has a dimension of $35.0 \pm 1.8^{\circ}$.
The sine bar rolls have a diameter of 30.0 mm . A set of gage blocks is available that can form any height from 10.0000 to 199.9975 mm in increments of 0.0025 mm .
Determine
a) The height of the gage block stack to inspect the minimum value of the angle
b) The height of the gage block stack to inspect the maximum value of the angle, and
c) The smallest range of angle that can be setup at the nominal angle size.

All the inspections are performed on a surface plate.
[10 Marks]
11. Consider a repairable system that can exist in one of three states namely, $\mathrm{H}, \mathrm{M}, \mathrm{S}$. It can be assumed that the state transitions happen according to a Markov process. The state transition probability matrix is as follows

$P=$| $H$ |
| :---: |
| $M$ |
| $S$ |\(\left[\begin{array}{ccc}0.7 \& 0.2 \& 0.1 <br>

0.2 \& 0.6 \& 0.2 <br>
0.6 \& 0.2 \& 0.2\end{array}\right]\)

The current state probability matrix is

$$
\begin{array}{ccc}
\mathrm{H} & \mathrm{M} & \mathrm{~S} \\
\mathrm{~S}_{0}=\left[\begin{array}{c}
0.7
\end{array} 0.3\right. & 0]
\end{array}
$$

a) What is the probability of finding the system in state ' $S$ ' after two time steps give the current state probability matrix $S_{0}$ ?
[3 Marks]
b) Simulate two future state transitions given current state probability matrix $\mathrm{S}_{0}$. Show and explain the calculations properly.
[4 Marks]
c) Show how will you simulate two state transitions under the condition that if the system is found in state ' $M$ ', then a corrective action is taken. However, the effect of the corrective action is not deterministic. After a corrective action taken in state ' M ', the new state after the action will
continue to be ' M ' with a probability 0.3 and will become ' H ' with a probability 0.7 . Assume that the current state matrix is as follows

$$
\begin{array}{ccc}
H & M & S \\
S_{0}=\left[\begin{array}{ccc}
0.1 & 0.9 & 0
\end{array}\right]
\end{array}
$$

[6 Marks]
d) Explain with your own example, how to estimate the probabilities for any state transition matrix from data.
[3 Marks]
e) Explain when a higher order state transition matrix needs to be used and how to construct such a matrix and estimate the transition probabilities. Use your own example.
12. Prove or disprove the following arguments. Please provide a proof if you wish to prove the argument or a counterexample to disprove the argument.
a) Consider the problem of minimizing a strictly concave function over a polyhedron. Then, the optimal solution lies at an extreme point of the polytope.
b) Consider a road consisting of $n$ line segments. There are $m$ lamp posts placed at fixed locations on this road. If lamps of power $p_{1, \ldots, p_{m}}$ are installed on these posts, the illumination of segment $j$ is given by

$$
\sum_{i=1}^{m} a_{i j} p_{i}, j=1, \ldots, n
$$

where the $a_{i j}$ 's are known constants. The $p_{i}$ 's are unknown decision variables.
i. Write a Linear Program to determine powers subject to $0 \leq p_{i} \leq 1$ so that the illumination of each segment is "close" to $I_{\text {des }}$, a given desired illumination level.
[2 Marks]
Suppose that we now add the following additional constraint, given an integer $r$ : No more than half the total power ${ }^{\frac{1}{2} \sum_{i=1}^{m} p_{i}}$ must be contained in any subset of lamps of size $r$. In other words, power must not be concentrated among a few lamps! One way to model this constraint is to write the following set of linear inequalities

$$
\begin{equation*}
\sum_{i \in S} p_{i} \leq \frac{1}{2} \sum_{i=1}^{m} p_{i}, \text { for all } S \subseteq\{1, \ldots, n\},|S|=r \tag{3}
\end{equation*}
$$

Observe that there are $\binom{m}{r}$ such constraints, which is an exponential number. For instance, if $n=50$ and $r=15$, then there are more than $2 \times 10^{12}$ such constraints! Any linear programming solver will struggle with such a large number of constrains.
In what follows, you will model the concentration-of-power constraint using a small number of constraints and a small number of additional variables.
ii. Given $p \in \mathrm{R}^{m}$ and $r \in \mathrm{~N}$, characterize all optimal solutions of the linear program

$$
\begin{equation*}
\max \left\{\sum_{i=1}^{m} p_{i} y_{i}: \sum_{i=1}^{m} y_{i}=r, 0 \leq y_{i} \leq 1, i=1, \ldots, m\right\} \tag{3Marks}
\end{equation*}
$$

iii. Incorporate the concentration-of-power constraints (3) to the linear program of part (a) using only $m+1$ additional variables and $2 m+1$ linear constraints.

