Department of Mechanical Engineering, IIT Bombay
Ph. D. Qualifying Examination: Applied Mathematics
January 2018
Total points: 60, Time: 3 Hours
Minimum passing score: 24 points (40%)
Closed Book, Closed Notes Examination

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1. (4 marks) Find the equation of the tangent plane to the surface given by the following equation, at the point (2, 2, 3).

$$\vec{R}(u,v) = u\vec{\mathbf{i}} + 2v^2\vec{\mathbf{j}} + (u^2 + v)\vec{\mathbf{k}}$$

2. (6 marks) A 3-D vector field, *F*, for a dynamical system is given as

$$\vec{F}(x,y,z) = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + z^4\vec{\mathbf{k}}$$

Find $\iint_S \vec{\boldsymbol{F}}.\boldsymbol{dS}$ where the surface *S* is a part of the cone $z = \sqrt{x^2 + y^2}$ below the plane *z*=1, and is oriented downwards.

3. (5 marks+5 marks)

a) Consider a discrete time linear dynamic system

$$\vec{x}(t+1) = A\vec{x}(t) + \vec{\boldsymbol{B}}u(t), \quad t = 0, 1, \ldots$$

where, $\vec{x}(t) \in R^n$, and $u(t) \in R$, and A and B are matrices of appropriate size.

Given matrices A, B, and $\vec{x}(0) = \vec{x}_i$. Write $\vec{x}\,(t+1)$ in the form

$$\vec{x}(t+1) = C\vec{x}_i + D\,\vec{U}$$

$$\text{where, } \vec{U} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(t) \end{bmatrix}$$

You need to provide explicit expressions for matrices C and D, in terms of matrices A and B.

b) Consider the specific instance of the problem in part (a) with

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \vec{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and } \vec{x}_i = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Provide the minimum value of N, and a sequence of inputs

$u(0), u(1), \ldots, u(N-1)$ that results in $\vec{x}(N) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

4. (10 marks) The position of a body is governed by the differential equation:

$$m\frac{d^2x\,(t)}{dt} + k\,x(t) = -F\,\mathrm{sgn}\left(\frac{dx}{dt}\right)$$

where, sgn denotes the *signum* function. Solve for $x(t)$ given that $x(0) = x_0$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$. How does the amplitude of oscillation change with time? You may assume x_0 to be greater than 0. Note that $\operatorname{sgn}(z)$ is 1 if z is positive, -1 if z is negative and 0 otherwise.

5. (**8 marks+2 marks = 10 marks**)

a. Consider the ordinary differential equation

$$x(x-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0, \qquad (0 < x < \infty)$$

Find an appropriate solution for this equation at *x* = *0*. Give the radius of convergence, *R*, for the solution obtained.

b. Find the eigenvalue and eigenfunction (if any) for the boundary value problem, when $\lambda > 0$.

$$y'' + \lambda y = 0; \quad y'(0) = 0, \text{and } y'(2\pi) = 0$$

6. (**5 marks**) Obtain the general solution $u(x, y)$ of the partial differential equation $yu_x - u_{xy} = 0$, using a suitable substitution of a partial derivative.

7. (**2 marks+3 marks**) Consider a disc *D* of radius *R* = 1, and assume that the temperature distribution $u(r,\theta)$ on the disc is obtained by the solution of the Laplacian (Note: It is not required to solve the PDE).

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

with $u(1,\theta) = f(\theta)$ a given temperature distribution on the boundary ∂D.

(a) If $\max\left(f(\theta)\right) = T_1$ & $\min\left(f(\theta)\right) = T_2$, for $T_1 \neq T_2$. What are the maximum and minimum temperatures on the surface of the disc and why? What can you conclude when $T_1 = T_2 = 0$?

(b) Assuming the boundary temperature to be $f(\theta) = 3\theta\sin\theta$, determine the exact temperature at the center of the disc.

8. (**10 marks**) Consider a heated rod of length *L*, whose transient temperature distribution is represented by the equation $T_t - T_{xx} = 0$. The initial temperature distribution in the rod is linear and is given by $T(x) = x$, for $0 \le x \le L$. Using the Fourier series solution method, find an expression for the temperature distribution of the rod if both ends are insulated. Roughly sketch an approximate variation of the temperature at 3 distinct time instances.

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Useful information: Fourier series representation for a function *f (x)*:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

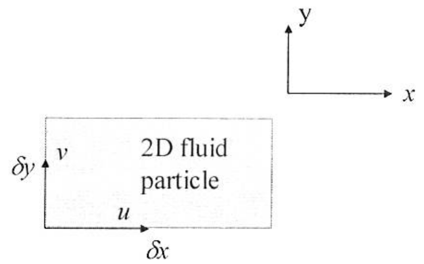
Ph.D. Qualifying Examination (Spring 2018)

Fluid Mechanics (TFE-1); Maximum marks: 100; Time: 3 hours

This is a **closed book, closed notes** examination. Answer all questions. Make suitable assumptions if required and state them clearly.

Problem 1: [8+5+5 Marks]

a) Consider a fluid (density ρ) flowing through an infinitesimal 2D Cartesian control volume (CV) of size δx and δy along x and y-axis, respectively. Consider velocity components as u and v at northwest corner of the CV, respectively. Derive expression of linear strain rates in x and y directions using Taylor series. Show that the volumetric strain rate of the particle is equal to the divergence of velocity vector.



b) Consider the free stream flow (velocity U_∞) over a surface (length L) with x axis along the surface and y axis being normal to the surface, with p_∞ as ambient pressure. Using the equation(s) given in Appendix, obtain non-dimensional form of y-component of the incompressible Navier-Stokes equation. Include gravity in your analysis. Identify the dimensionless number that you obtain after non-dimensionalization.

c) For a free-stream flow across a bluff body with flow separation, show the streamlines considering the body as square-cylinder, circular-cylinder and airfoil; of same frontal area. Discuss reasons of flow-separation and relative magnitude of the drag force for the various bodies.

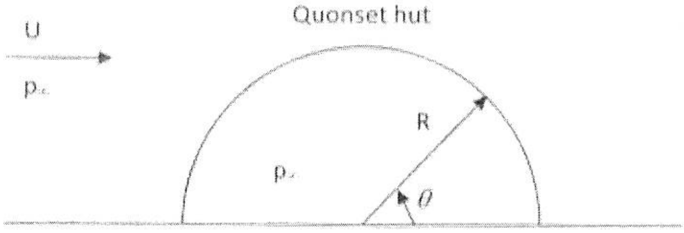
Problem 2: [15 Marks]

A PhD student at IIT Bombay can pedal his bike at V_B on a straight, level road with no wind. Ignore the rolling resistance of the bike. The drag area of the student and his bike is A. Student's mass is M and the bike mass is m. Student now encounters a head wind of V_W. (a) Develop an equation for the speed V at which student can pedal into the wind. Do not solve the equation.

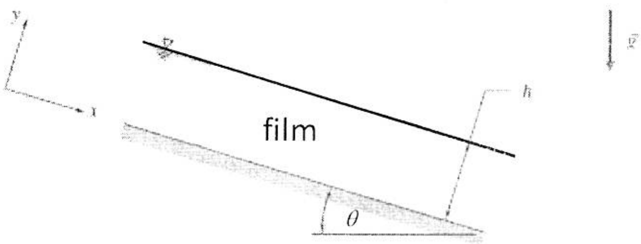
Problem 3: [20 Marks]

The flow over a Quonset hut may be approximated by the velocity distribution. $\vec{V} = U\left[1-\left(\frac{R}{r}\right)^2\right]\cos(\theta)\cdot\vec{e}_r - U\left[1+\left(\frac{R}{r}\right)^2\right]\sin(\theta)\cdot\vec{e}_\theta$ with $0 \le \theta \le \pi$, where r, θ, $\vec{e}_r$ and $\vec{e}_\theta$ are radial coordinate, azimuthal direction, unit vector along radial direction and unit vector along azimuthal direction, respectively. During a storm the wind speed reaches U at atmospheric pressure p_∞. The pressure inside the hut is p_∞. The hut has a radius R and length L (along z direction). (a) Prove that Bernoulli's equation can be used in this problem. (b) Ignore elevation differences and use

Bernoulli's equation to determine the net force tending to lift the hut off its foundation. The density of air inside and outside the hut is ρ.

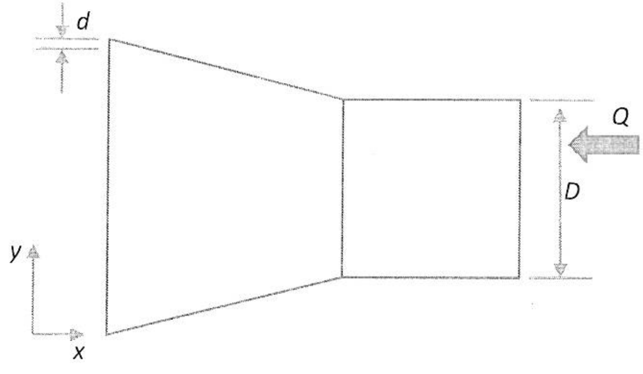


Problem 4: [20 Marks] An incompressible liquid film flows down an inclined plane surface at an angle θ. Assume, steady, fully-developed, laminar flow in two-dimensional Cartesian coordinates. Also assume film has a constant thickness, h. The film is non-isothermal, and the temperature distribution across the film is given by, $T(y) = T_0 + (T_w - T_0)\left(1 - \frac{y}{h}\right)$, where T_w and T_0 respectively, the wall and ambient temperatures. The fluid viscosity decreases with increasing temperature and is assumed to be described by, $\mu = \frac{\mu_0}{1+a(T-T_0)}$, with $a > 0$. Derive an expression for the velocity profile. Use equations given in the appendix, if needed.



Problem 5: [15 Marks]

Water enters a shower head through a circular tube with inside diameter of D. The water leaves in N streams, each of diameter, d. The volume flow rate is Q. Estimate the minimum water pressure needed at the inlet to the shower head, for which viscous losses and gravity effects are negligible. Also neglect any minor losses. Evaluate the force needed to hold the shower head onto the end of the circular tube.



Problem 6: [12 Marks]

Consider the free stream flow (velocity U_∞) over a surface (length L) with x axis along the surface and y axis being normal to the surface. Ignore gravity and obtain dimensionless, 2D, steady-state Navier Stokes equations with following dimensionless variables, $U = u/U_\infty$, $V = v/U_\infty$, $X = x/L$, $Y = y/L$. Considering u velocity scales as U_∞, x scales as L and y scales as δ (boundary layer thickness).

a) Comment on the relative magnitudes of inertia and viscous forces in the following regions: (i) near plate, (ii) inside boundary layer and (iii) outside boundary layer.

b) Using results of part (a, ii) and order of magnitude analysis, show that the non-dimensional boundary layer thickness ε (ε = δ/L) varies as inverse of square root of Re_L.

c) Using order of magnitude analysis on continuity equation, show that non-dimensional y-velocity v scales as the non-dimensional boundary layer thickness ε (ε = δ/L).

d) Using order of magnitude analysis on X-momentum equation, show the term $\frac{\partial^2 U}{\partial X^2}$ can be neglected in x-momentum equation.

e) Using order of magnitude analysis on Y-momentum equation, show that the pressure gradient in y-direction can be neglected.

f) Using results of parts c, d and e, simplify non-dimensional, 2D, steady-state Navier Stokes equations obtained earlier to Prandtl boundary layer equations.

Appendix

Continuity equation in Cartesian coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Navier Stokes Equations in Cartesian coordinates:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho f_x-\frac{\partial p}{\partial x}+\mu\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho f_y-\frac{\partial p}{\partial y}+\mu\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho f_z-\frac{\partial p}{\partial z}+\mu\left\{\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right\}$$

Bernoulli equation:

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{L,maj}+h_{l,min}$$

IIT Bombay : Department of Mechanical Engineering
2017–18 Semester 2 : PhD Qualifying Examination
Paper TFE2 : Heat Transfer : 2018 01 23 : 0930 – 1230

Please note:

1. Closed book examination. However, you may use a self-prepared A4 sheet (both sides may be used) for the examination. It must be in your own handwriting and must be handed in with the answer-book.
2. The duration of this examination is three (3) hours.
3. Total marks: 100. Passing marks: 40.
4. List the assumptions you make.
5. Be clear and neat. Marks also depend on the quality of your answers.

Q1. [15M] A long, solid rod, of radius R is cooled by a fluid. The heat transfer coefficient is h, and the fluid temperature is T_f. The rate of volumetric heat generation in the rod varies with radius as $q_g''' = a + br$, where a and b are positive constants and r is the radial distance.

(a) Determine the steady-state temperature distribution in the rod, and an expression for the maximum temperature.

(b) Derive an expression for the maximum temperature when the rod generates heat uniformly, and has the same amount of heat generation per unit length.

(c) Compare $T_{\max}$ for the two cases. Which one is higher? Why?

Q2. [15M] Consider the transient conduction in a long solid cylinder. Initially (at $t = 0$), the cylinder as well as the surrounding fluid are at a uniform temperature T_o. For $t > 0$ the curved surface of the cylinder is exposed to radiant flux of q_o'', all of which is absorbed. Simultaneously, that surface is cooled by the surrounding fluid (which remains at T_o) with a heat transfer coefficient h.

(a) Provide a qualitative sketch of the transient temperature profiles.

(b) Write down the governing equations and boundary conditions for solving $T(r, t)$.

(c) Can the solution be obtained by a direct application of the separation of variables method? Explain why.

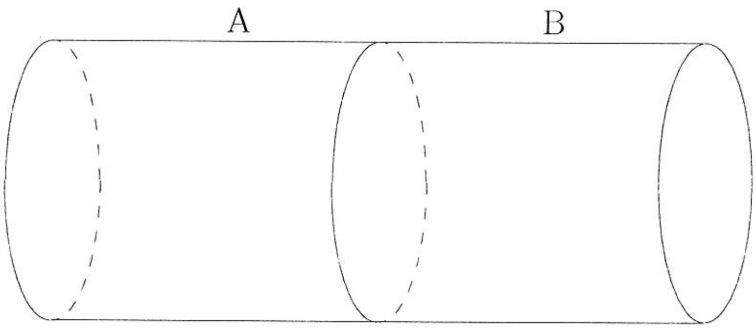
(c) If your answer to (b) is *yes*, then obtain that solution. If your answer is *no*, then use superposition to split the problem into sub-problems, such that each sub-problem is either an ODE or a PDE that can be solved by separation of variables. (You need not solve the sub-problems.)

(d) Do you recognise any sub-problem as a 'standard' situation? Explain.

Q3. [15M] The figure shows a cylindrical surface, 1 m in diameter and 2 m in length. The curved surface has two equal parts, A and B. Determine the shape factor for diffuse radiation from A to B.
It is given that the shape factor between two coaxial parallel circular discs of radii R_1 and R_2 and separated by L along the axis is:

$$F_{1-2} = \frac{1}{2X^2}\left[(1+X^2+Y^2) - \left\{(1+X^2+Y^2)^2 - 4X^2Y^2\right\}^{1/2}\right]$$

where $X = R_1/L$ and $Y = R_2/L$.



Q4. [15M] A long furnace has a cross-section in the form of an equilateral triangle, 2 m each side. Surface 1 ($\epsilon = 0.2$) is at a temperature of 1200 K. Surface 2 ($\epsilon = 0.2$) is at a temperature of 900 K. Surface 3 ($\epsilon = 0.5$) is a re-radiating surface. Determine the heat flux that needs to be supplied to Surface 1, and the temperature of Surface 3. Does the value of ϵ_3 affect your results? Why?

Q5. [20M] An experiment involved heating a tube of length L and diameter D through which a fluid flows with mass flux G_o, and inlet temperature $T_{m,i}$. Several heating patterns were applied to the walls of the tube and the temperatures of the fluid and were measured. One such heating pattern was that of a sinusoidal wave, *i.e.* the heat flux is $q''_w(x) = q''_o \sin(\pi x/L)$ where q''_w is heat flux at the wall, q''_o is heat flux coefficient and x is the axial distance. Derive expressions for (a) outlet fluid temperature, and (b) axial wall temperature distribution. Assume that the single-phase fluid flowing through the tube is turbulent, and the situation is fully developed.

(c) During one such experimental run with the sinusoidal heating pattern, the pump that maintained the fluid flow failed and the mass flux dropped continuously as $G = G_o\mathbf{exp}(-t/\tau)$, where t is time and τ a constant. As the mass flux fell, boiling occured and phase change was induced. Derive expressions, applicable during this period, for (c1) enthalpy of the two-phase fluid, (c2) equilibrium quality as function of x and t. Assume that the pressure, wall heat flux, inlet temperature, and specific heat remain unchanged during the transient.

Q6. [20M] Consider the constant property laminar forced flow of a liquid metal in a circular pipe. At the entrance, both the velocity and temperature profiles are flat and the boundary condition of a constant heat flux q''_w is applied. Since the Prandtl number of the liquid metal is low and of the order of 0.01, the velocity profile develops very slowly as compared to the temperature profile. As a first approximation, one may therefore assume the velocity profile to be flat. Make this approximation and show that the Nusselt number after the temperature profile is fully-developed, is constant and determine its value. You may wish to use the following equation to determine the temperature profile:

$$k\left[\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] = \rho c_p\left[V_z\frac{\partial T}{\partial z} + V_r\frac{\partial T}{\partial r}\right]$$

— Paper Ends —

PhD Qualifying Exam – Solid Mechanics DES 1

Maximum marks: 60, Passing marks: 24

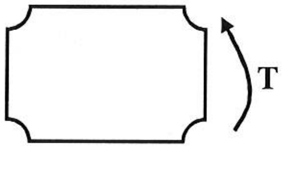
January 16 2018, Duration 3 hours

Instructions:

1. Exam is closed books and closed notes
2. There are **SIX** questions. Each question carries 10 marks
3. **Answer each question on a separate sheet of paper**
4. In any information in the paper is missing/ambiguous/not clear, please make and clearly state appropriate assumptions and solve the problem

1. (a) A shaft is made of a rectangular section ($2a\times 2b$, $a>b$) with rounded corners as shown. It transmits a torque T. Indicate the locations where the shear stress is maximum and zero.

(3 marks)



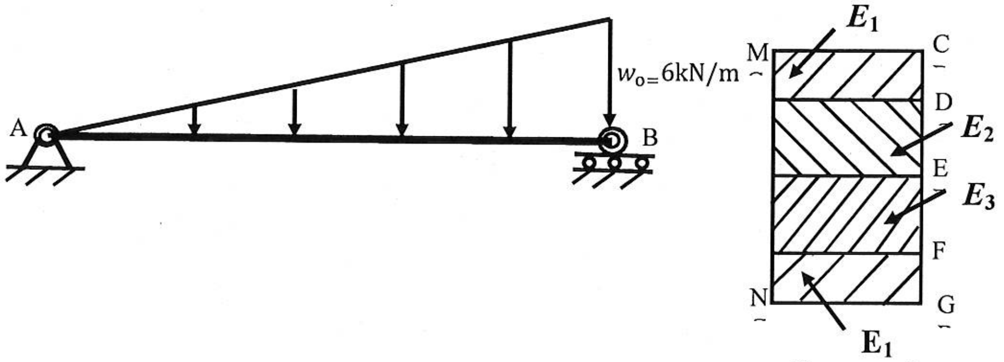
(b) The stresses at a point in a two dimensional body corresponding to a set of specified boundary tractions (S_x, S_y) and body forces (X, Y) are given by the following two sets of stresses:

(i) $\sigma_{x1}, \sigma_{y1}, \tau_{xy1}$ (ii) $\sigma_{x2}, \sigma_{y2}, \tau_{xy2}$

Establish whether the two sets are the same or different? **(7 marks)**

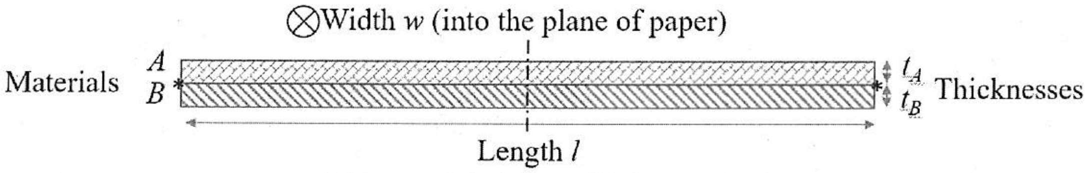
2. Determine the maximum tensile and compressive bending stress in the beam shown. The beam is made up by gluing four beams of three different materials. What is the basic assumption in your estimation of the stresses? Indicate it clearly. **(8 marks +2 marks)**

Given AB=12m, MC=NG=50mm, CD=FG=15mm, DE=EF=25mm, $E_3=E$, $E_2=1.5E$ and $E_1=2E$.



Beam section

3. You are given a bi-material strip: a pair of thin strips of materials A and B, stuck firmly together along their wide surfaces using an extremely thin but strong adhesive layer (i.e. the layers cannot disbond or delaminate from each other). Both materials are isotropic, perfectly elastic and homogenous, and of thickness t_A, t_B equal widths w, and equal lengths l, **(t, w<<l)** respectively, and with coefficients of thermal expansion α_A, α_B with $\alpha_A > \alpha_B$, and Young moduli E_A, E_B respectively. Assume material properties are temperature independent. At initial room temperature T_C the bi-material strip has a straight thin rectangular cuboidal shape as shown.

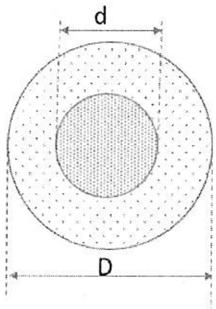


A bi-material strip, with $l >> w, t$

The bi-material is heated to high temperature T_H. There are no other external forces or constraints on the strip. As a result of non-uniform thermally driven material deformation the shape of the bi-material strip changes from the original straight thin rectangular cuboidal shape. Let $t_A = t_B = t$, & $E_A = E_B = E$.

a) What shape does the bi-material strip take upon heating. Draw a quick sketch. **(2 marks)**
b) Plot the stress state of the bi-material along its thickness (for a cross-section as shown towards center of the figure). **(2 marks)**
c) At T_C, the ends of the bi-material strip (shown by * on left and right of the figure above) are separated by a distance *l*. Due to deformation the linear Euclidean distance between the ends changes to l'. Derive this distance between ends l' in terms of given properties, dimensions and ΔT ($= T_H - T_C$), Also approximately plot the normalized distance between ends (l'/l) as a function of ΔT. What is the smallest and the largest Euclidean distance possible between the ends. **(6 marks)**

4. (a) A solid steel shaft of diameter D is severely twisted clockwise by applying a torque T_{init} such that only an inner diameter d elastic core remains on the inside. Assume the material is elastic-perfectly-plastic with shear modulus G and shear yield stress τ_Y. What residual stresses (across the circular cross section) and residual rotation (per unit shaft length) will remain upon release of the applied torque? **(6 marks)**



(b).On the same shaft with residual stresses from part (a), what is the maximum torque you can apply clockwise such that the whole of shaft behaves elastically (i.e. there is no additional plastic deformation)? On the same shaft with residual stresses from part (a), What is the maximum torque you can apply counter-clockwise such that the whole of shaft behaves elastically? **(4 marks)**

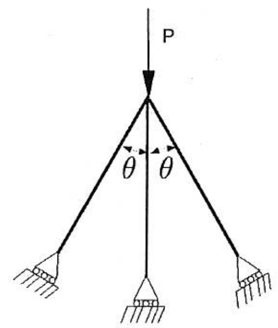
5. One is interested in determining the stresses in a solid thin turbine disk of radius R which is rotating with a constant angular velocity ω. The temperature distribution in the disk, measured above the room temperature, is a function of only the radial coordinate and is given by T(r). The outer edge of the disk is traction free. The disk is made up of linear elastic isotropic homogeneous material with density ρ, Young's modulus E, Poisson's ratio ν and coefficient of thermal expansion α. To proceed:

a. Identify the nonzero stress components and the nonzero strain components. **(1 mark)**
b. Write the appropriate boundary conditions. **(1 mark)**
c. Write the strain displacement relations and the stress strain relation. **(2 marks)**
d. Write the equilibrium equation in the radial direction. **(1 mark)**
e. Using the strain displacement relations, stress strain relation and the equilibrium equation in the radial direction show that the radial stress component, σ_{rr}, is governed by the following differential equation. **(3 marks)**

$$\frac{1}{r^2}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) + \alpha E\frac{dT}{dr} + (3+\nu)\omega^2\rho r = 0$$

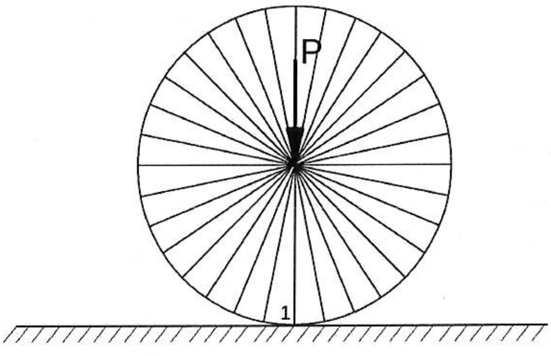
f. Integrating the above differential equation and using the boundary conditions, obtain an expression for the radial stress component. **(2 marks)**

6. (a)Consider a truss structure shown in the figure. The trusses can be assumed to pin-joined at both the ends. The length of each rod is L, its cross section is A and the Young's modulus is E. A vertical force P acts on the structure as shown. Find the downward deflection at the point of application of the force. **(4 marks)**



(b) Now consider a bicycle wheel shown in the figure. It is assumed that the rim of the wheel is rigid and the spokes (32 in number) are radial and are uniformly distributed over the circumference. You can also assume that each of the spokes is pin joined at both the end and

behaves as a truss member. The length of each spoke is L, its cross section is A, and the Young's modulus is E. A downward force acts at the center of the wheel as shown.



a. Estimate the downward deflection of the hub under the action of the force P. **(3 marks)**
b. Graphically show the variation of the force in spoke 1 (the spoke which is in contact with the ground) as the wheel completes one revolution. Describe the variation of the force. Find the maximum compressive and tensile load carried by the spoke. **(3 marks)**

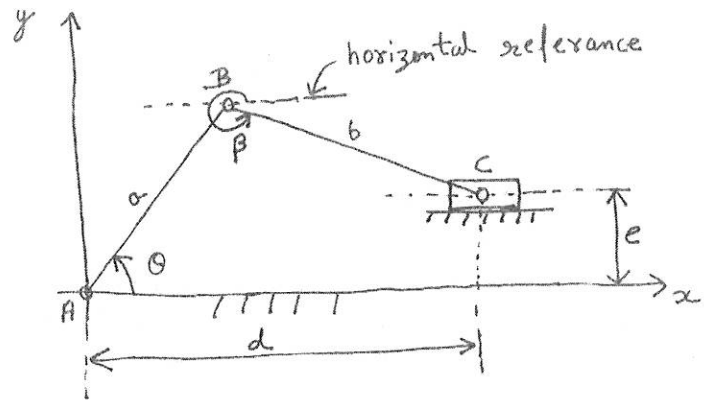
19th Jan 2018

DES2: Kinematics and Dynamics of Machines
Ph.D. Qualifying Examination

General instructions:

- Write clearly and legibly. Answers should be 'to-the-point', but should clearly show the important steps in the solution, including all assumptions and approximations.
- No queries will be entertained. If any information appears to be missing, make suitable assumptions and state them clearly. Clearly mark/highlight such assumptions, if any, by enclosing them in a box.
- Total = 100 **points**. Time = **3 hours**.

Q1 [12pts]: Analytically perform a complete (i) [3pts] Position Analysis, (ii) [4pts] Velocity Analysis, and (iii) [5pts] Acceleration Analysis for a slider-crank mechanism in which the stroke-line of the slider does not pass through the axis of rotation as shown below (use $a, b, d, e, \theta, \beta$ as depicted).



Q2 [13pts]: Consider a 2-link open-chain mechanism shown in Figure 1 below with link lengths l_1 and l_2. Angles θ_1 and θ_2 are the two degrees-of-freedom and (x_B, y_B) which are coordinates of point B are considered to be the output of this mechanism.

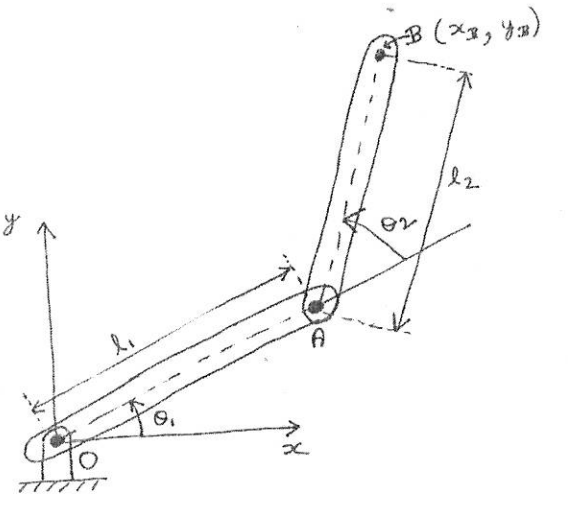


Figure 1

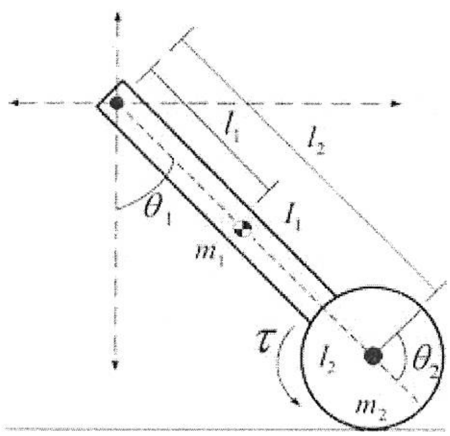


Figure 2

(i) [5pts] Suppose you are given specific values (x_B, y_B) and you are interested in finding values of angles θ_1 and θ_2 that would lead to those values at the output. Derive expressions for θ_1 and θ_2 in terms of (x_B,

y_B) and other linkage parameters.

(ii) [5pts] Consider the vector $\boldsymbol{X} = [v_x, v_y, \omega]^T$ where v_x, v_y are linear velocities of point B and ω is the angular velocity of the second link. Also, the two joint angle derivatives are contained in the vector $\boldsymbol{q} = [\dot{\theta}_1, \dot{\theta}_2]^T$. Derive analytical expressions for a matrix $\boldsymbol{J}$ such that $\boldsymbol{X} = \boldsymbol{Jq}$.

(iii)[3pts] Find angles θ_1 and θ_2 at which the matrix $\boldsymbol{J}$ is singular. Give an intuitive explanation about why those specific angles lead to a singular $\boldsymbol{J}$.

Q3 [17] Consider inertia wheel pendulum system in vertical plane, as shown in the Figure 2 above. Pendulum link 1 has mass m_1, length l_2 and distance of its CG at l_1, while inertia wheel has mass m_2 and inertia I_2 about its own CG. As inertia wheel is rotated using torque τ the reaction generated would start moving the pendulum link.

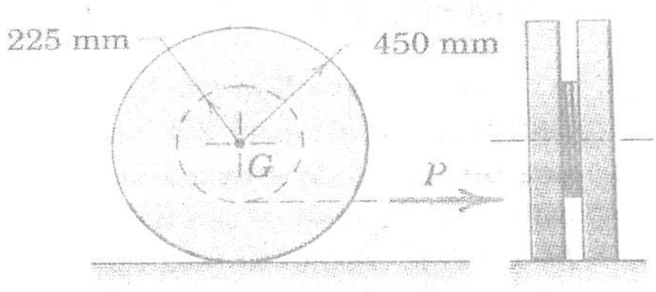
(i) [5] Detemine the kinetic and potential energies of system in terms of parameters given in the figure.

(ii) [8] Using Lagrange formulation derive all equations governing the dynamics of this system.

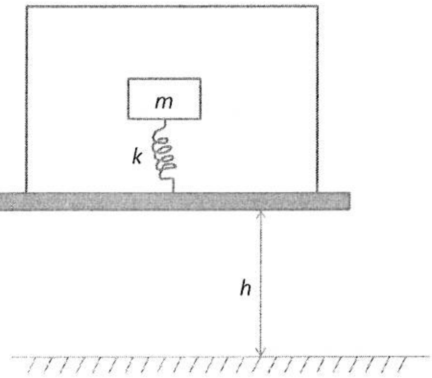
(iii) [2] How many initial conditions are required to solve these equations? List them.

(iv) [2] Based on equations write down expression for reaction torque due to rotation of inertia wheel on pendulum?

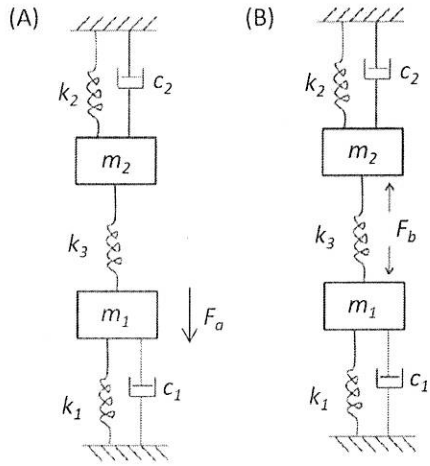
Q4 [8] Consider a wheel with central hub on which a rope is wrapped and force P is applied at the end of the rope. P is gradually increased as per the equation P=6.5t where P is in N and t is in sec. Determine using impulse-momentum equations, angular velocity of wheel 10 sec after P is applied if the wheel is initially at rest. Wheel having mass 60kg and radius of gyration about its center 250 mm is given to be rolling without slipping.



Q5 [12 Marks] To ensure survivability of portable devices, electronics industry uses drop test whereby the device is dropped to the ground from a distance h. Assuming the device can be approximated by a single degree of freedom system as shown in Fig. 1, predict the displacement response of the mass as well as its maximum acceleration.



Q6 [13 marks] Consider the two degree of freedom system shown in Fig. 2. The system is excited by two types of forces F_a and F_b. Determine the compression in the middle spring (k3) resulting from (A) force $F_a = F_0 \cos \omega t$ acting on mass m1 and (B) equal and opposite force $F_b = F_0 \cos \omega t$ acting on both the masses.



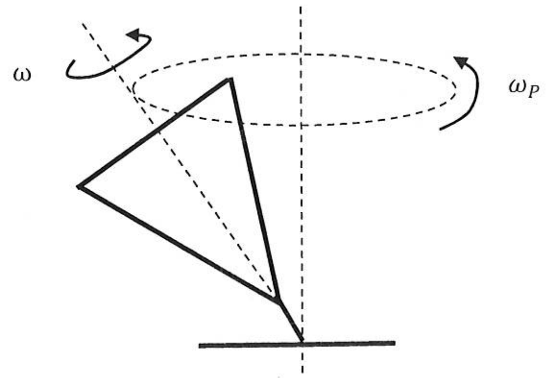
Q7 [13 marks] Consider the motion of the earth around the sun. Let (r, θ) be the polar coordinates representation of the position of the earth with respect to the sun. Let the unit radial vector be denoted as $\mathbf{e}_r$ and the unit tangential vector be denoted as $\mathbf{e}_\theta$

(i) [6 marks] Derive expressions for velocity and acceleration vectors of the earth (you may treat it as a point mass) in terms of $r, \theta, \dot{r}, \dot{\theta}, \ddot{r}, \ddot{\theta}$, $\mathbf{e}_r$, $\mathbf{e}_\theta$

(ii) [3 marks] Derive the equations of motion of the earth around the sun assuming the mass of the earth and sun are respectively M_e and M_s

(iii) [4 marks] Show that $r^2\dot{\theta}$ is a constant. Which physical quantity does $r^2\dot{\theta}$ relate to?

Q8 [12 marks] Consider the spinning of a top as shown in the figure below with gravity acting downward. Let ω represent the angular speed around the axis of spin and ω_P represent the angular speed around the axis of precession.



(i) [6 marks] Explain why the top precesses ie explain why there is any ω_P at all. Hint: Draw a free body diagram and consider the angular momentum of the top as well as the torque acting on it

(ii) [6 marks] Show that ω_P would increase as ω decreases.

--------PAPER ENDS---------

Ph.D. Qualifying Exam 2018
Manufacturing Process I
Max Marks 80, Duration 3 hrs.
Only hand written notes are allowed (no book or printed material).
Please make (and clearly state) any suitable assumption if required.

==

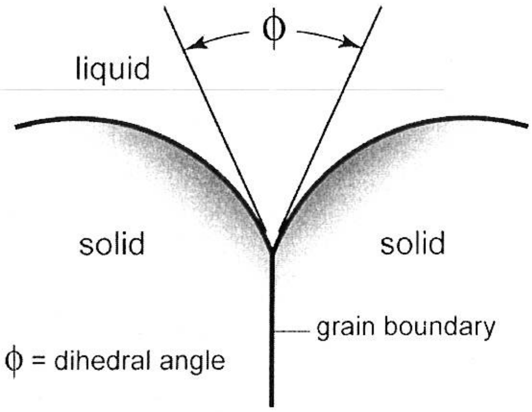
1. Derive lever rule for two phase materials (solid + liquid) for the following cases:
   i. Equilibrium solidification **[2]**
   ii. Non-equilibrium solidification assuming no diffusion in solid and perfect mixing in liquid **[4]**
   iii. Schematically explain the mushy zone and dendrite formation **[2]**

2. Derive expressions for heat generated/supplied per unit time during: **[4X3=12]**
   i. Arc welding
   ii. Resistance welding
   iii. Friction stir welding

   Clearly define the process parameters involved in the expressions.

3. i. What is green density? Explain the role of binder in sintering process. **[2]**

   ii. Derive the expression of dihedral angle between two solid particles during liquid phase sintering if the interfacial energy per unit area for soild-soild interface is γ_{ss} and soild-liquid interface is γ_{sl}. **[5]**

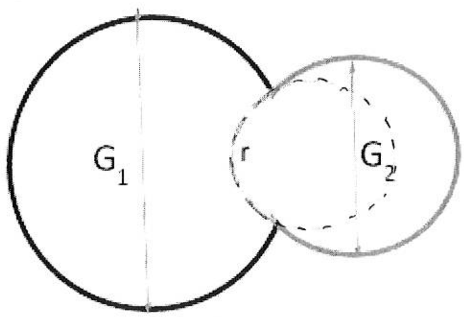


   iii. Derive the relation between γ_{ss} and γ_{sl} for the dihedral angle to be an obtuse angle. **[2]**

   iv. Coalescence of two particles with diameters G_1 and G_2 during sintering leads to the interface with radius (r) given by the following expression:

$$r = \cos\left(\frac{\phi}{2}\right)\frac{G_1 G_2}{|G_1 - G_2|}$$

Plot the variation of interface radius(r) with the grain size ratio (G_1/G_2) from 1 to ∞ assuming that dihedral angle remains constant. **[3]**

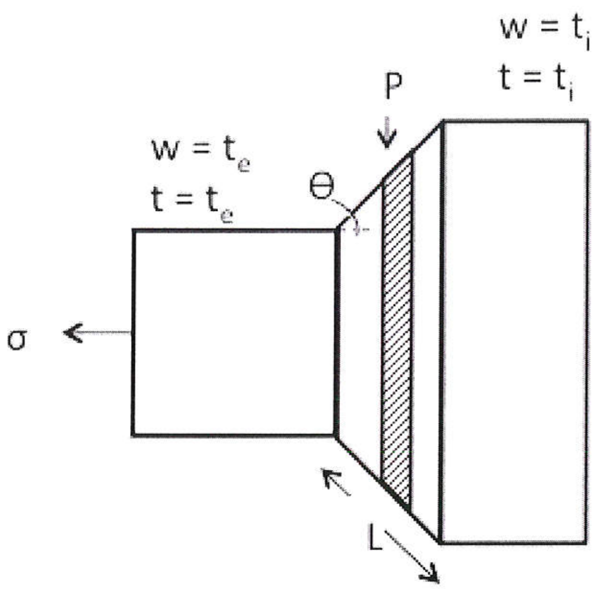


4. A single full penetration weld pass is made on steel using the following parameters: E=20V, I=200A, v = 5 mm/s, T_o=25°C, T_m=1510°C, $\rho C_P = 0.0044 \frac{J}{mm^3 - {}^oC}$, t = 5mm, $\eta = 0.9$, $H_{net} = 720 \frac{J}{mm}$, Recrystallization temperature=730°C. Calculate the peak temperatures at distances (i) 1.5mm and (ii) 3.0mm from the weld fusion boundary (iii) HAZ width (iv) HAZ width if the steel plates are preheated at 200°C (v) HAZ width if the steel plates are tempered at 430°C (vi) HAZ width if the steel plates are tempered at 430°C and preheated at 200°C

[3x6=18]

$$\frac{1}{T_p - T_o} = \frac{\sqrt{2\pi e}\rho C_p t y}{H_{net}} + \frac{1}{T_m - T_o}$$

5. Figure shows drawing of a square section bar.
   a. Write force balance equation for the drawing process of a square cross-section. **[8]**
   b. Establish relation between pressure P acting on the thin element and stress σ. **[6]**
   c. Determine average P over the contact length with die. **[6]**



6. A material yielding at 250 MPa was subjected to the stress state $\sigma_1 = 200$ MPa , $\sigma_2 = 150$ MPa and $\sigma_3 = -100$ MPa. Check if the material would yield under this state of stress. Thereafter, the magnitudes of the stresses were changed as follows : σ_1 increased to 250 MPa and σ_3 changed to -150 MPa. If the material got deformed by a true plastic strain increment of 0.2 during this step, determine the principal strain components. Assume material to have a plastic strain ratio (r) of 1.8. **[5]**

7. (a) Two sheets have the anisotropy values in the three directions as follows :

Material	R_0	R_{45}	R_{90}
A	2.38	3.45	2.27
B	0.79	0.64	0.42

$\bar{R} = 2.89$, $\Delta R = -1.125$
$\bar{R} = 0.62$ $\Delta R = -0.035$

Calculate the average anisotropy and the planar anisotropy in the two materials. Comment on the suitability of the two materials for forming sheet metal parts involving stretching and drawing. **[3]**

7 (b) How does one minimise / live with springback in products made from sheet metal ? *List four methods of doing so.* **[2]**

Ph.D. Qualifier – January 2018

Manufacturing Processes - II

1. During deformation of material in machining, the shear strain (γ) is a function shear angle, (ϕ) and rake angle, (α)
   a. Derive the expression for shear strain in orthogonal machining. [5]
   b. Find the expression for shear angle as a function of (α)□for minimum shear strain condition. [10]
   c. Prove that the expression for chip ratio, r, as a function of (α)□for minimum shear strain condition is: [5]

$$r = \frac{\text{Sin}\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}{\text{Cos}\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)}$$

   d. What is the expected value of the chip ratio at minimum shear strain and provide the physical explanation for this value. [5]

2. For the orthogonal cutting of a particular work material, μ is the coefficient of friction at tool-chip interface and r is the cutting ratio. Assume that rake angle is zero.

   Show that the ratio of material shear strength (τ_s) to the specific cutting energy (u) is given by: $\frac{\tau_s}{u} = \frac{(1-\mu\, r)r}{1+r^2}$ [15]

   Show that the rate of heat generation P_s in the shear zone is given by: $P_s = F_c V(1 - \mu\, r)$; where F_c is the cutting force. [10]

3. Bring out the difference between *Cylindricity* and *Perpendicularity* geometric tolerances. Draw neat part sketches with Tolerance symbols. [5]

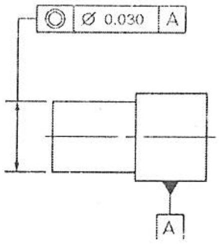
   Following data refers to the measurement of surface roughness of a component using stylus profilometer.

No.	1	2	3	4	5	6	7	8	9	10
Profile Height (μm)	3	4	6	2	1	4	5	3	4	2

   Calculate the R_a, R_q, and R_T (average, RMS and Peak to valley) parameters to characterize surface roughness. [10]

4.(a)For a number of interchangeable mating parts (holes and shafts), the average allowance is 0.04 mm and the allowance must not exceed +/-0.012 mm from the average value.The basic size is 100 mm. Tolerance of holes is twice the tolerance of the shaft. Determine the sizes of hole and shafts using hole basis system and unilateral tolerance system. [6]

(b)Which geometric tolerance is shown in the diagram below? With a neat sketch show the tolerance zone. [4]



5. In electrochemical machining process, the initial inter-electrode gap is k_0. Electrode area is A, density of the electrode material is ρ. During ECM, the applied voltage is V, while the total overvoltage at the electrodes is V_0 and supplied current is I. Current efficiency is α.

   Using fundamental Faraday's principles of electrolysis (not by using direct formula), prove that the variation in the inter-electrode gap k after time t will follow the following parabolic profile if no tool feed is given.

$$k^2 = k_0^{\ 2} + 2\lambda t$$

   where, λ is a constant. Express λ in terms of process parameters [5]

6. A single-pass turning operation is performed on workpieces 125 mm in diameter and 300 mm in length. A feed of 0.225 mm/rev is used for this operation. It is found that for cutting speeds of 3 m/sec and 2 m/sec, the tool must be replaced after every 5^{th} and 25^{th} workpiece, respectively. This is due to excessive tool wear. Determine the Taylor's tool life equation for this job. [5]

7. (a) On a single plot, show trends for machining-cost (equipment + labour)/piece, tool-changing-cost/piece, tooling-cost/piece, handling-cost/piece and overall-production-cost/piece with respect to cutting velocity (on x-axis). [3]

   (b) Based on the plot, explain the requirement for 'optimum cutting velocity for minimum production cost'. [2]

   (c) Derive the expression for optimum cutting velocity for minimum production cost. [5]

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