INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
DEPARTMENT OF MECHANICAL ENGINEERING

Ph.D. Qualifying Examination **DES 2**
Kinematics and Dynamics of Machinery

8 questions, Total 100 marks **All questions compulsory**

Question 1 :Consider the robotic arm carrying a sphere of mass *m* as shown in the figure given below. At the instant shown, link 4 is telescoping out of link 3 at a constant speed v_{32}. Simultaneously, link 3 rotates relative to link 2 at a constant rate of$\dot{\gamma}$, while link 2 rotates relative to link 1 at a constant rate of $\dot{\beta}$, and link 1 rotates with a constant angular velocity $\dot{\alpha}$ with respect to the fixed base.Links 2, 3 and 4 are of length L, and at the instant shown, link 4 is halfway out of link 3.

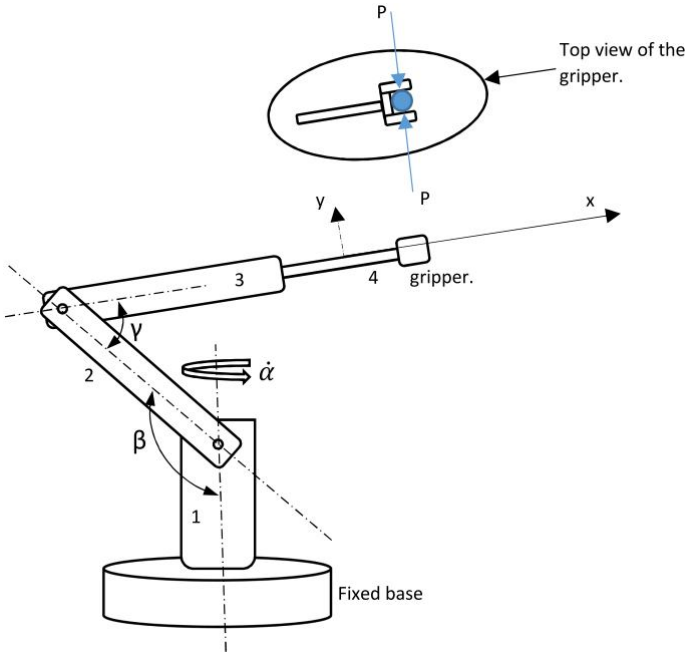
(a) Determine the velocity V_m of the sphere relative to the ground frame. Express it in terms of the unit vectors x, y and z, of the frame attached to link 4.

(b) Find the acceleration of the mass, expressed in terms of the unit vectors x, y and z of the frame attached to link 4.

(c) Draw the free body diagram of the mass, showing clearly all the forces acting on it.

(d) Determine the minimum gripping force P if the coefficient of static friction between the sphere and the gripping surfaces is μ.

[25 marks]



Question 2 : Consider a uniform thin disk of radius r, which is rolling without slipping on a horizontal surface such that the point of contact (A) moves in a circle of radius a, as shown in the figure given below. The disk makes an angle θ with the horizontal surface. The rate of spin of the disk is ω_s, whereas the rate of precession is ω_p. The disk rolls such that r_c, the horizontal distance of the center of mass of the disk from the axis of precession, is zero.

Determine

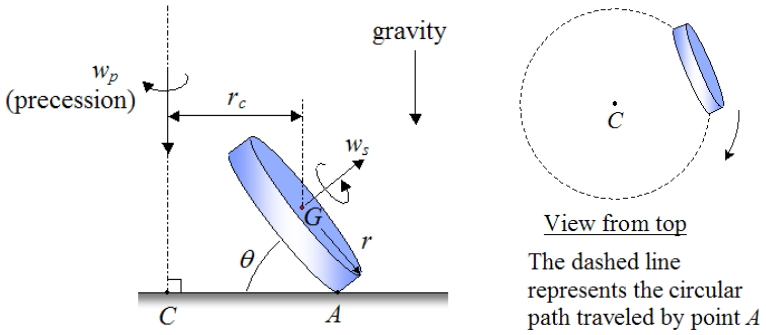
(a) The angular velocity and angular acceleration of the disk with respect to ground;

(b) The relationship between ω_s and ω_p;

(c) The force between the surface and the disk; and

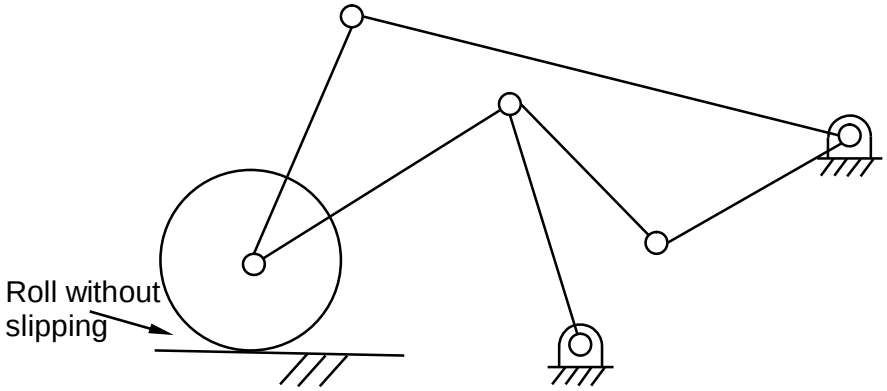
(d) The rate of precession of the disk.

[15 marks]



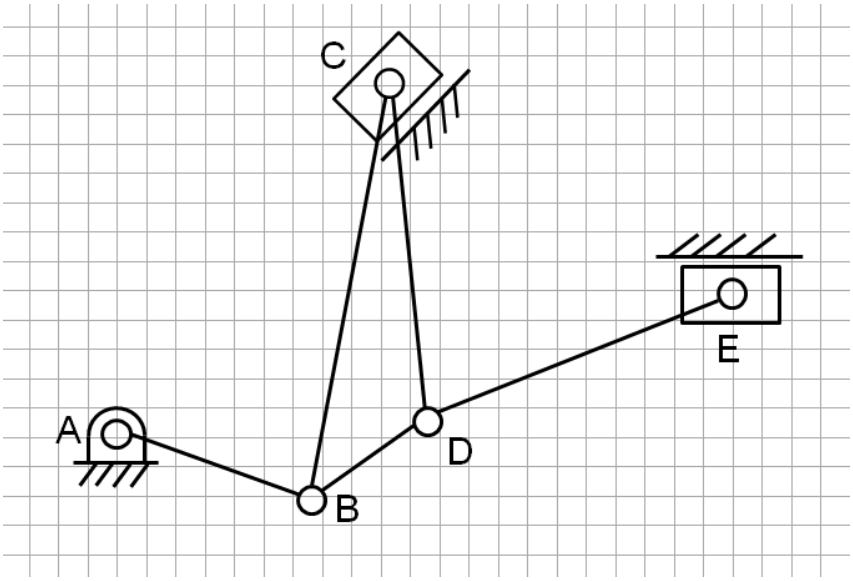
Question 3: Apply Kutzbach or Grubler's criteria (whichever is applicable here) to obtain the degrees of freedom of the mechanism shown below. Explain the manner of counting the number of links and joints.

[10 marks]



Question 4 : The mechanism shown below has been drawn on a 5mmX5mm grid. The link AB rotates about A at 20 rpm in the clockwise direction. Obtain the velocity of E by graphical technique. (You will be judged primarily by the method of graphical construction. Accuracy of drawing carries low weightage.)

[10 marks]

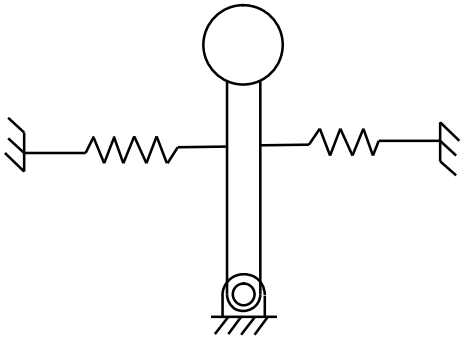


Question 5 : Obtain the acceleration of E of question 6 by graphical method.(You will be judged primarily by the method of graphical construction. Accuracy of drawing carries low weightage.)

[10 marks]

Question 6 : In the figure shown below, the inverted pendulum is kept in the vertical position by the action of the two springs of spring constants k_1 and k_2. The bob of mass m is connected to the pivot by a rigid rod of length *l*and both the springs are connected to the rod at a distance x from the pivot. The pendulum is in static equilibrium in this condition. What is the natural frequency of vibration (about the pivot) for small amplitudes?

[10 marks]



Question 7 : A 100 kg mass undergoes damped vibrations. The spring constant is 100 N/m. The ratio of damping coefficient to critical damping coefficient is 0.1. What is the damped natural frequency? If the initial amplitude of oscillation is 10 mm, what will the amplitude be after 3 oscillations?

[3 + 7 marks]

Question 8 : Determine the mode shape displacements for the following system. Show that the modes are orthogonal.

$$\begin{bmatrix} 2m & 0 \\ 0 & 3m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 3k & -k \\ -k & 5k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[7 + 3 marks]