PhD Qualifying Exam – Jan 2017

Manufacturing Processes-II

- Full marks 100
- No books and notes are allowed but two A4 sheets of handwritten formulae are allowed
- Please make any suitable assumption if required and state it clearly

1. For the orthogonal cutting of a particular work material, it has been found that the tool-chip contact length is always equal to the uncut chip thickness (t_0) and the mean shear stress (τ_f) at the tool-chip interface is a proportion k of the mean shear stress on the shear plane (τ), i.e., $\tau_f = k\,\tau$. Assume that the stresses uniformly distributed on the shear plane and at the tool-chip interface.

   a. Prove that the cutting force, F_c, is given by:

$$F_c = \frac{\tau w t_0 (1 + k\sin(\phi - \alpha)\sin\phi)}{\cos(\phi - \alpha)\sin\phi}$$

   where ϕ is the shear angle, α is the rake angle, w is the width of the tool. [10]

   b. In the absence of any experimental data, explain a methodology to estimate the orientation of the shear plane. If the rake angle is zero ($\alpha = 0$), estimate the numerical value of shear angle if k=1. [15]

2.

   a. In an assembly, holes of diameter $25.0^{+0.035}_{+0.005}$ mm are assembled with shafts of diameter $25.0^{\pm 0.015}$ mm. Draw the tolerance zones and identify the type of fit. Considering gage tolerance to be 10 % of the part tolerance, calculate the sizes of Go / NoGo gages to inspect the shaft and hole. Neglect wear tolerance on gages. [6]

   b. Two machines (A, B) are being evaluated in a shop to produce cylindrical pins of diameter $20.0^{\pm 0.020}$ mm. Following data was obtained by conducting Statistical Quality Control (SQC) tests on the machines.

	A	B
Mean diameter (mm)	20.005	20.012
Standard deviation (mm)	0.008	0.002

   Which machine would you recommend? Justify your answer [4]

3.

   a. A typical surface roughness profile produced on a component can be represented by the equation $y = 5 + 2\sin(\theta)$, where y is the profile height in microns and $0 \le \theta \le 2\pi$. Calculate the surface roughness parameters R_a, R_q and R_t (average, RMS, peak-to-valley) for the profile. (You can use numerical procedure, if you wish.) [6]

   b. Explain in brief, with examples, the differences between Circularity, Cylindricity and Runout geometric tolerances. [4]

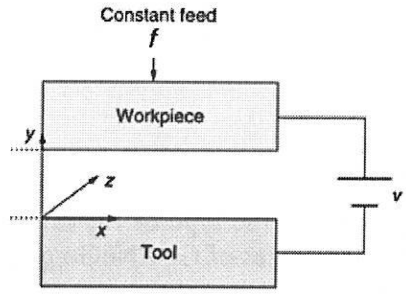
4.

a. A hole of diameter 15 mm is to be machined to a depth of 25 mm in Titanium workpiece using electrochemical machining (ECM). The electrode diameter used is 14.8 mm. The electrolyte with conductivity value of $k = 0.48\ \Omega^{-1}\text{cm}^{-1}$ is used. Calculate the tool feed rate for steady state machining using 5 V as a constant gap voltage. How much time is required to complete the machining operation?
For Titanium: $A = 22$; $Z = 2$; $\rho = 4.5\ g/cm^3$ [6]

b. Assuming a bare tool is used for ECM operation, calculate the approximate hole profile cross-section. Hint: Use side cut estimates at different depths 0, 5, 10, 15 and 20 mm from the workpiece surface. Assume initial gaps at the salient points are equal to the steady state or the equilibrium gap as observed in the part (a). [9]

c. Which geometric tolerance is affected due to such side cuts? Suggest any remedy to overcome or to minimize such errors. [5]

Useful equations:



Volume of material removed (cm^3) = CIt

Electrochemical constant, $C = \frac{A}{ZF\rho}$

At the feed rate f (cm/s), in the direction of decreasing y, the workpiece rate of change of position dy/dt can be written as:

$$\frac{dy}{dt} = \frac{CVk}{y} - f$$

where,

I = current (A); t =time (s); A = gram atomic mass; Z = number of valence electrons;

F = Faraday's constant (96500 $A.s$); ρ = density of work material (g/cm^3);

k = electrolyte conductivity ($\Omega^{-1}\text{cm}^{-1}$)

5. We need to create a hexagonal-shape 'blind cavity' having a depth of 8 mm in a 20 mm thick Tungsten plate by die-sinking EDM (see the top view of cavity). Discharge current varies with respect to time in a triangular wave fashion. Resistance and capacitance used in EDM relaxation circuit are 75 kΩ and 80 μF, respectively. Sideways overcut during EDM machining is 0.05 mm. Volumetric removal rate (R) in EDM is given as:

$$Volumetric\ removal\ rate\ R\left(\frac{mm^3}{s}\right) = \left(\frac{664 * I_{avg}\ (A)}{\theta^{1.23}\ (°C)}\right)$$

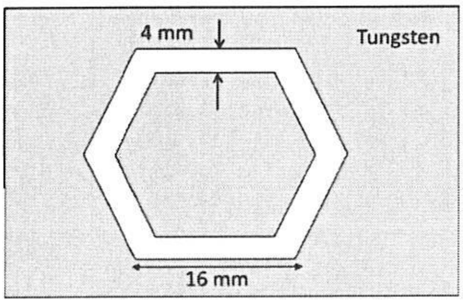
where θ is the melting point in °C and I_{avg} is the average current in A.

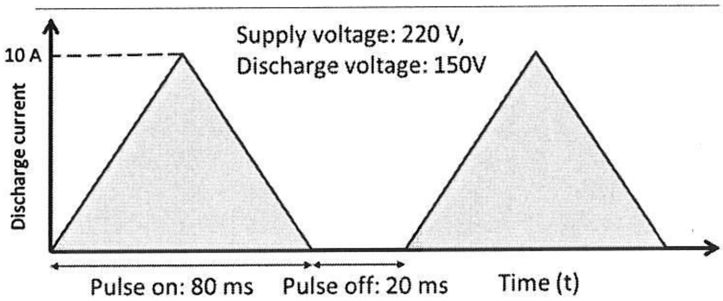
Important material properties of tungsten are: density: 19290 kg/m^3, specific heat 138 J/Kg.K, melting point 3410°C, thermal conductivity 166 W/m.K.

a. Calculate the time taken in machining this blind cavity? [8]

b. How many sparks will be generated per second during EDM? [6]

c. Given a choice between copper, glass, tungsten and graphite as tool material for EDM, which one will you choose and why? Give your order of preference for tool material with proper reasoning. [6]





6. A tool manufacturer specifies a tool life of 15 minutes for a turning insert for machining low-alloy steel at cutting speed of 267 m/min, feed of 0.15 mm/rev and cutting depth of 0.8 mm. It is further indicated that tool life of 60 minutes can be achieved at cutting speeds of 0.7 times the cutting speed for the tool life of 15 minutes (other parameters held constant).

   a. Using manufacturer supplied information, calculate the two constants in Taylor's tool life equation. [3]

   b. Assume that cost of each insert is Rs. 500/- and it has two cutting edges. Machine and labor rate is Rs. 2500/- per hr. It takes 10 seconds to index and 30 seconds to replace the insert. For turning a 500mm long bar at 0.15 mm/min, what is the tool life for minimum cost per piece? [10]

   c. What cutting speed will give the tool life estimated in (b)? [2]

Paper Ends

PhD Qualifying Exam – 2016-17 - (Manufacturing Processes-I)
(Attempt all questions, All parts of a question must be answered continuously)

[1] The *flow stress* of a material in uniaxial tension at unit strain varies with temperature as $A - B_1 T$ upto a temperature T_1 and thereafter to $Y/5$ as observed at a temperature T_2 at a rate B_2, where Y is the *flow stress* at unit strain at room temperature. With strain, the variation of *flow stress* follows the Hollomon Equation, $\sigma = K\varepsilon^n$. The value of n diminishes with temperature as: $n = n_0 - CT$, where n_0 is the value of n at the room temperature, T_0.

For hot forging of a mass of material, the total of thermal and mechanical energy needs to be minimized. Neglect heat losses and assume frictionless conditions at the tool-work interface. Assume thermal properties of the material to remain unchanged.

Set up an equation in terms of Y, n_0, A, B_1, B_2, C and T to determine the optimal temperature at which the forging should be performed to minimize the total energy input. **[10]**

[2] Show with neat sketches that equibiaxial tension amounts to uniaxial compression superimposed by hydrostatic tensile stress. **[05]**

[3] A cylindrical sample of initial diameter D_0 and initial height H_0 is placed concentric inside a cylindrical container of internal diameter D ($D > D_0$) and compressed to a height H ($H < H_0$). Determine the initial clearance ($D - D_0$) such that the compressive flow stress when the sample just touches the container wall is 30% higher than the yield strength of the material under uniaxial tension, Y. Determine the principal stresses at this stage.

If the stresses were increased further, determine the values of the principal stresses.

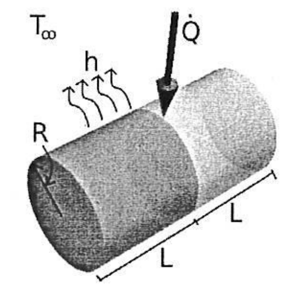
Assume that there is no clearance between the container and the tools used in compression, and that the material is isotropic. Neglect friction at the tool-workpiece interface. **[10]**

[4] Draw schematic of electric arc characteristics with respect to current and voltage, and explain the behaviour. How will it change with respect to electrode diameter and arc length for GMAW? **[05]**

[5] Explain the *Partially Melted Zone* during solidification and its mechanism with an example. **[05]**

[6] For a friction stir welding process, the rotating tool has a rectangular cross-section with sides a and b. If the applied normal load on the tool is P, the angular velocity of the tool is ω and the co-efficient of friction between the tool and work-piece is μ, derive an expression for the heat generated during the process during N rotations of the tool. **[10]**

[7] Two cylinders of infinite thermal conductivity, heat capacity, Cp, and density ρ are joined sidewise along the periphery with laser welding of constant heat input rate ($\dot{Q}$) (in J/s) and source velocity v (m/s), as shown in the figure. Assuming that the heat is lost from all the exposed surfaces just by convection, derive an expression for the sample temperature just after the completion of the process. If instead of one laser source, there are two laser sources always diametrically opposite during the welding, how the derived expression will change? (Assume that the initial temperature of the samples is equal to the temperature far away, that is T_∞). **[10]**

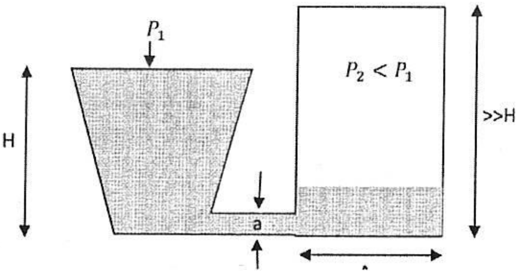


[8] Consider the schematic of a casting process as shown. Derive expressions of the following:

(a) The height by which the liquid will rise?

(b) Pouring time.

Assume that the pouring cup is always full. Expressions must be in terms of the given parameters, H, A, a, g, P_1 and P_2, where g is the gravitational acceleration, A and are the area (see figure) **[5 + 5]**



[9] Calculate the temperature rise in a high-strength steel that is adiabatically deformed to a strain of 1.0. Pertinent data are: $\rho = 7.87 \times 10^3$ kg/m³, $\sigma_a = 800$ MPa, $C = 0.46 \times 103$ J/kg°C. **[5]**

[10] The strain hardening behavior of an annealed low-carbon steel is approximated by $\sigma = 700\varepsilon^{0.20}$ MPa. Suppose a bar of this steel was cold-worked and then cold-worked 15% more and found to have a yield strength of 525 MPa. What was the unknown amount of cold work? **[10]**

[11] If the stress-strain relation is given by $\sigma = 700\varepsilon^{0.20}$, at what strain would the maximum load be observed in a tension test? **[5]**

[12] Two solid cylinders of equal diameter have different heights. They are compressed plastically by a pair of rigid dies to create the same percentage reduction in their respective heights. Consider that the die-workpiece interface friction is negligible. What would be the ratio of the final diameter of the shorter cylinder to that of the longer cylinder. **[5]**

[13] Two flat steel sheets, each of 1.0 mm thickness, are being resistance spot welded using a current of 5000 A and weld time of 0.1 s. The contact resistance at the interface between the two sheets is 200 μΩ and the specific energy to melt steel is 10×10^9 J/m³. A spherical melt pool of diameter 4 mm is formed at the interface due to current flow. Consider that electrical energy is completely converted to thermal energy. What would be the ratio of the heat used for melting to the total resistive heat generated? **[10]**

<u>Department of Mechanical Engineering, IIT Bombay</u>

Ph.D. Comprehensive Examination – Jan 2017

Heat Transfer

Maximum Marks: 100 **Passing Marks: 40**

Instructions:

- This is an open-book examination. You are allowed 3 books. No photocopied books.
- Please start each new question on a new page and keep all subparts of a question together.
- Maximum marks per question are given in the table below.
- Use of calculator is permitted.
- Make suitable assumptions where necessary and state them clearly.
- Make sure to cancel any unwanted work clearly, so that it is not graded.

Question	Marks
1	7.5
2	10
3	10
4	15
5	17.5
6	20
7	20

Problem 1. Answer the following questions in a succinct manner.

(a) What is a gray surface? Describe its spectral and directional characteristics? **[1.5 mark]**

(b) Define Biot number? If a surface interacts convectively and radiatively with its surroundings then define its Biot number. **[1.5 mark]**

(c) Define Peclet number and its significance for internal flows? **[1.5 mark]**

(d) What is the reason for existence of a critical radius of insulation? **[1.5 mark]**

(e) State the necessary conditions required for separation of variables method to work. **[1.5 mark]**

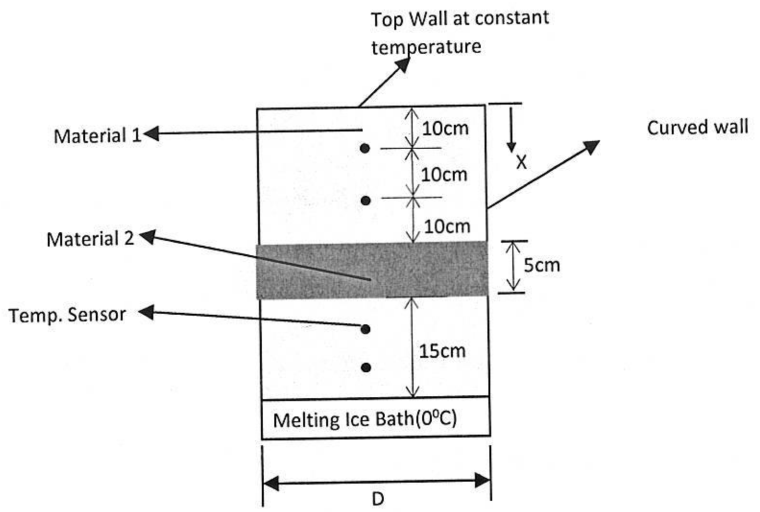
Problem 2. Packed bed of spheres is often employed in number of practical applications. When studying convective heat transfer through packed bed of spheres Nusselt number correlations as a function of particle Reynolds number can be measured by careful experiments. Situation exists where Nusselt number at no flow condition is needed, i.e. Nusselt number at conduction limit. Consider a heated (isolated) sphere placed in a quiescent/stagnant fluid medium. Assume that the heat conduction in the fluid surrounding the sphere has negligible convection effects. The thermal conductivity of the fluid may be considered constant.

(a) Determine the temperature distribution in the sphere? **[5 marks]**

(b) Derive the conduction limit Nusselt number? **[5 marks]**

Assume that the ambient temperature is T_{amb}, sphere diameter is D and the Nusselt number is defined as $Nu = hD/k_f$ where h is heat transfer coefficient and k_f is fluid thermal conductivity.

Problem 3. An industrial process is modelled as shown in the figure below where two cylinders of diameter 'D' of 'material 1' are separated by a disk of another 'material 2' of the same diameter. The process requires a steady temperature at the top wall of the cylinder of 'material 1'. The bottom wall is to be maintained at 0^0C which is done be keeping it in contact with melting ice where ice is added at 0^0C and water is removed at 0^0C. The thermal conductivity of the 'material 1' is known to be twice that of 'material 2'. Assuming the contact resistance between the sample and disk on either side to be 2.5 K/W determine the rate(kg/s) at which ice must be added to the constant temperature bath at steady state when the temperatures measured at the two locations shown are 125^0C(x=10cm), 100^0C(x=20cm). The rods and the disk are perfectly insulated on the curved surfaces and therefore heat transfer can be assumed to be one dimensional. Use latent heat of water=330J/g. **[10 marks]**



Problem 4. Processed plastic parts are dropped onto a continuously moving conveyor. The parts are square shaped with thickness of 2 mm and edge-length of 20 mm. The plastic part has thermal conductivity, specific heat and density given as 0.35 W/mK, 1900 J/kgK and 1100 kg/m^3, respectively. Conveyor belt rubber thermal properties are 0.14 W/mK, 700 J/kgK and 1300 kg/m^3. In order to ensure longer life of the conveyor belt, the contact temperature of the hot molded plastic part is not to exceed 140 °C.

(a) Estimate the contact temperature (between plastic part and conveyor belt) at the instant the plastic part drops on the belt if the plastic part is at 180 °C and conveyor belt at 20 °C? At the very instant when the bodies make contact, the bodies can be treated as semi-infinite medium. **[5 marks]**

Now, assume that the side of the part that faces the conveyor is adiabatic for simplicity. The top surface of the part is exposed to 20 °C with a heat transfer coefficient of 15 W/m^{2}K. Due to packaging considerations, the part must be cooled to 80 °C (maximum) before it can be stacked.

(b) Estimate the maximum available conveyor belt speed if the packaging station is located 22 meters away from the molding station? **[10 marks]**

Problem 5. The spectral, hemispherical emissivity of tungsten may be approximated as

$$\varepsilon_\lambda = 0.45 \text{ for } 0 < \lambda < 2\ \mu\text{m}$$

$$= 0.10 \text{ otherwise}$$

Consider a cylindrical tungsten filament (density: 19300 kg/m^3; specific heat: 132 J/kg-K) that is of diameter 0.8 mm and length 20 mm. The filament is enclosed in an evacuated bulb and is heated by an electric current to a steady-state temperature of 2900 K. Assume that the surroundings are at a temperature of 300 K. Recall: $\varepsilon = \dfrac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{E_b}$

(a) How much power should be supplied to the bulb to maintain it under steady state? **[10 marks]**

(b) What is the initial rate of cooling of the filament when the current is switched off? **[7.5 marks]**

Problem 6. Consider 2D constant property laminar flow with negligible viscous dissipation and axial conduction between two parallel plates which are 2h apart. One wall is insulated and the other wall has a uniform flux q into the fluid. The coordinate system is at the center in between the plates and at the beginning of the channel. The velocity profile is fully developed with mean velocity equal to $\bar{u}$, temperature is uniform and equal to $\bar{T}$ at x=0.

(a) Write the energy equation (use 'T' for temperature), momentum equation (use 'u' for velocity) with the proper boundary conditions which when solved will give the fully developed velocity and temperature profiles. Do not attempt to solve since the solution is available in the text books and is the following:

$$u = \frac{3}{2}\bar{u}\left(1 - Y^2\right)$$

$$T - T_w = \frac{qh}{2k}\left(\frac{3}{4}Y^2 - \frac{1}{8}Y^4 + Y + \frac{13}{8}\right)$$

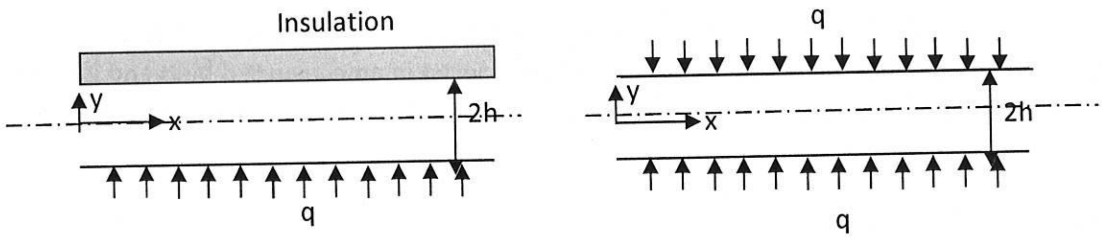
Where T_w is the wall temperature and k is the thermal conductivity of the fluid and Y=y/h. In addition, using the definition of bulk temperature the following is obtained:

$$T_b - T_w = \frac{3qh}{4k}(-.9905)$$

[2.5 marks]

(b) It is required to obtain the temperature distribution when the top wall also has a uniform heat flux q without actually solving the equations of motion and energy. The equation of motion remains the same and only the energy equation is different.
Write the energy equation (use 'θ' for temperature) for the case with two walls having uniform heat flux. Assume that the solution for this equation can be obtained by using $\theta = T_1 + T_2$ where T_1 and T_2 can be obtained using part (a) above. Note that you need to split

the equations (governing and other conditions) into two equations which are such that part (a) can be used. Note that the initial condition (i.e. that at x=0) will also be required even though it does not explicitly enter the solution given above.



Part (a) Part (b)

[17.5 marks]

Problem 7. For [fully developed] laminar flow of air in a tube of radius r_o = 1 cm [with constant heat flux boundary condition], the velocity and temperature profiles were measured to be:

$u(r) = 0.1\,[1 - (r/r_o)^2]$

$T(r) = 344.8 + 75.0\,(r/r_o)^2 - 18.8\,(r/r_o)^4$

with units of m/s and K, respectively.

(a) Calculate the values of mean velocity and mean (or bulk) temperature at the measurement station. **[6 marks]**

(b) Calculate the values of mean velocity and mean temperature 1 cm downstream of the measurement station. **[6 marks]**

(c) Find (to the extent possible), the velocity and temperature profiles 1 cm downstream of the measurement station. **[6 marks]**

State in words, how your answer of parts (b) and (c) above could change if the considered location was 10 cm upstream (rather than downstream) of the measurement station. **[2 marks]**

PhD Qualifying Examination – Solid Mechanics DES 1

Maximum marks 50, Passing marks 20

January 11, 2017 Duration: 3 hours

Instructions:

1. Exam is closed books, closed notes.

2. Each question carries 10 marks. Solve any 5 of 6 questions.

3. Solve each section (A, B, C) on a separate answer sheet.

4. If any information in the question paper is missing/incorrect/ambiguous, please make and clearly state appropriate assumptions and solve the problem.

SECTION A

A1(a) State any two major assumptions of Euler-Bernoulli beam bending theory and explain the need for those assumptions and how exactly they are deployed in developing the theory. [2M]

A1(b) Is it always necessary for the "neutral surface" in beam bending to be normal to the direction of application of load? Discuss with neat sketches and appropriate examples and counter-examples using square, rectangular and circular cross-sections. [3M]

A1(c) Consider a beam undergoing bending due to a transverse load. Let the shear force in a cross-section be "V". The shear stress distribution in the cross-section is given by the following formula: [5M]

$$\tau = \frac{VQ}{It}$$

where "I" is the moment of inertia of the cross-section about its neutral axis; "t" is the width or thickness at the point under consideration; "Q" is the moment of the area of the portion of the cross-section to one side ie from the point under consideration to the outer most fibre. Find the percentage of shear force "V" carried by the web and the flange of an I-section.

A2(a) Consider a thin cylinder under internal pressure "p". Let its inner radius be r_i and wall thickness "t" ($r_i >> t$). Draw a neat free body diagram and estimate the hoop stress. What were your assumption(s) in finding the hoop stress? [2M]

A2(b) Now consider that it is a thick cylinder with inner radius r_i and outer radius r_o. Would the same assumptions hold good? Explain. [2M]

A2(c) The formulae for radial and hoop stress in a thick cylinder are given below: [4M]

$$\sigma_r = a - \frac{b}{r^2} \quad \text{and} \quad \sigma_\theta = a + \frac{b}{r^2}$$

where the constants "a" and "b" have to be found using appropriate boundary conditions. Draw a neat free body diagram by taking a longitudinal section dividing the thick cylinder into two equal halves. Verify that the hoop stresses maintain the equilibrium against the applied internal pressure "p".

A2(d) Can the peak hoop stress be ever lower than the applied pressure "p"? Discuss. [2M]

SECTION B

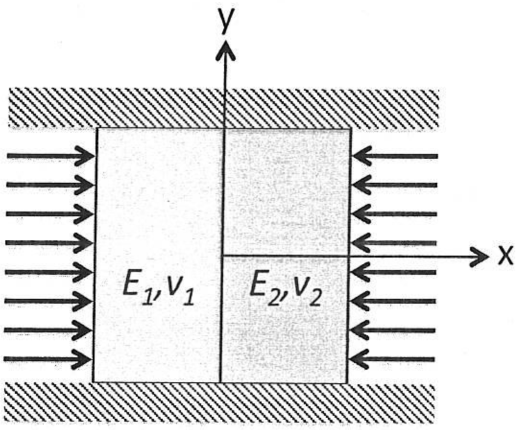
B1:

In the figure given below, two blocks of same dimensions (a, b, c) m in the (x, y, z) directions, respectively, are initially in contact and constrained between two rigid but frictionless walls in the y-direction, at $y = \pm b/2$. A uniformly distributed compressive force P is applied on one side of each block at $x = -a$ and $x = +a$. The materials of both the blocks are isotropic and homogeneous, with the respective elastic modulus and Poisson's ratio as indicated in the figure. Assuming that the interface between the blocks is frictionless:

(a) Write the all the boundary conditions required to solve this problem (3 marks)

(b) Find the stress and strain distribution in each block of materials (4 marks)

(c) Show that the solution obtained in (b) is unique (3 marks)



B2. For any given stress state ($\sigma_1, \sigma_2, \sigma_3$), where σ_i are the principle stresses, Find in the principle stress space:

(a) The stress vector along the vector $\mathbf{n} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ (2 marks)

(b) The stress vector perpendicular to the one calculated in (a) (3 marks)

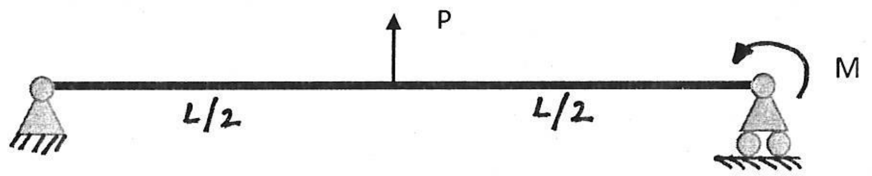
(c) The locus of the tip of the stress vector in (b) for any given stress state (3 marks)

(d) If the locus of the stress vector in (b) is cut by a surface parallel to the σ_1-σ_2 axes and passing through the origin, what is the resulting shape of the curve with dimensions in terms of the principle stress components? (2 marks)

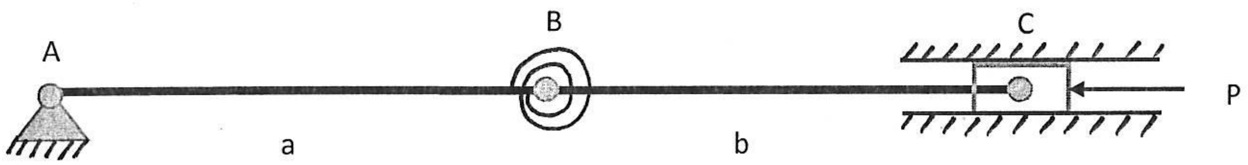
You have to show the calculations in detail for every part of this question.

SECTION C

C1 (10 marks). Consider a simply supported beam of length L and flexural rigidity EI. A transverse vertical force P is applied at the midpoint ($L/2$) of the beam and an anticlockwise moment M is applied at the end $x=L$. Choose a single degree of freedom function that approximates the transverse deflection curve of the beam. Write down the total potential energy of the system using the chosen deflection approximation. Using the principle of minimum potential energy, calculate the relationship between M and P which will ensure that the slope at the end $x=L$ is zero.



C2 (10 marks). Consider the mechanical system consisting of rigid bars AB (length a) and BC (length b) connected together at B through a torsional spring of stiffness k. The bar AB is connected to a pinned joint at A and the bar BC to a frictionless sliding pinned joint at C as shown. Before the application of the axial load, A,B and C are aligned along a horizontal line. Calculate the critical axial load P that will cause the system to buckle (become elastically unstable).



January 2017

DES2: Kinematics and Dynamics of Machines
Ph.D. Qualifying Examination

General instructions:

- Write clearly and legibly. Answers should be 'to-the-point', but should clearly show the important steps in the solution, including all assumptions and approximations.
- No queries will be entertained. If any information appears to be missing, make suitable assumptions and state them clearly. Clearly mark/highlight such assumptions, if any, by enclosing them in a box.
- The exam is 'closed book, closed notes.'
- Total = **points**. Time = **3 hours.**

Q1 [15 marks] Consider a point mass m **sliding down** (under gravity effect) a frictionless track along a conical spiral with cone angle α starting at point P_o with initial radius R_o and initial height H_o measured as shown in the figure below. Vertical pitch of spiral is p and Initial velocity of particle at point P is zero.

Given $\alpha = 60^{o}$, Radius R_o at point P_o = 5 m, Vertical pitch is p = 0.2 m, m = 1 kg.

a. (4) Obtain geometric relation expressing radius r and height z at ANY point P in terms of angle ϕ, (measured from starting position P_o as shown) and geometrical parameters. Express velocity of particle in polar coordinates in terms of $\dot{\phi}$.
b. (6) Determine tangential velocity and $\dot{\phi}$ for particle after 3 revolutions. What would be tangential velocity if instead of point mass a disc of same mass and radius of 5 cm is rolling (without sliding) down the spiral under gravity.
c. (5) Determine total acceleration of particle after 3 revolutions. Assume $\ddot{\phi} = 0.001\phi$.

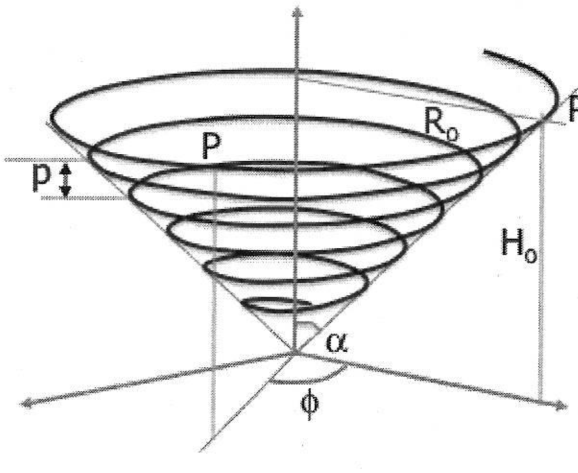


Figure 1

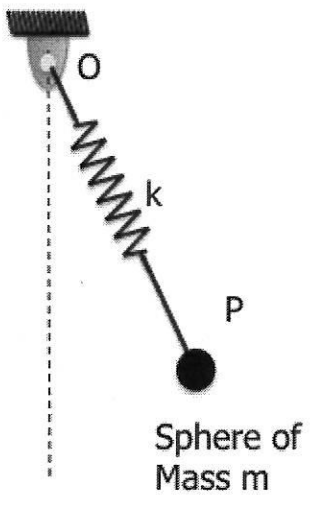


Figure 2

Q2 [15 marks] A solid sphere of mass m and radius r_m is attached to a rubberband and made to oscillate as a flexible string pendulum in a vertical plane. Motion could be in both vertical and tangential directions as shown.

The string stiffness is nonlinear with respect to displacement x measured from equilibrium position and is given by $k = k_o + k_1 r^2$.(k_o, $k_1 > 0$) Equilibrium length of pendulum string is l. (Formula for inertia of solid spehere = $2/5mr^2$)

a. (6) Identify degrees of freedom for the system and determine kinetic and potential energy of the system

b. (5) Using Lagrange formulation obtain equations governing the dynamics of the system

c. (4) Simplify equations with stiffness tending to ∞ and show that the equations reduce to compound pendulum equations.

Q3 [20 marks] Suppose you have an unknown dynamical system and you need to find the

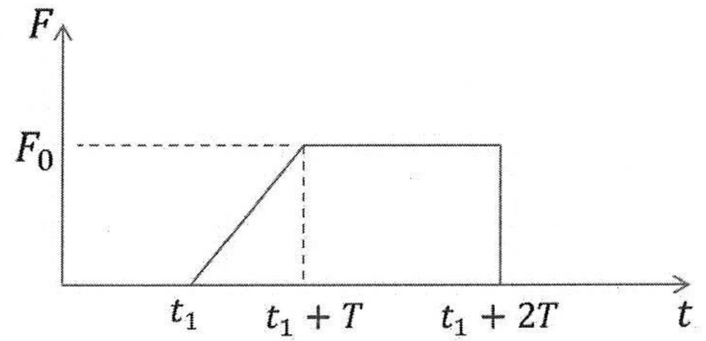
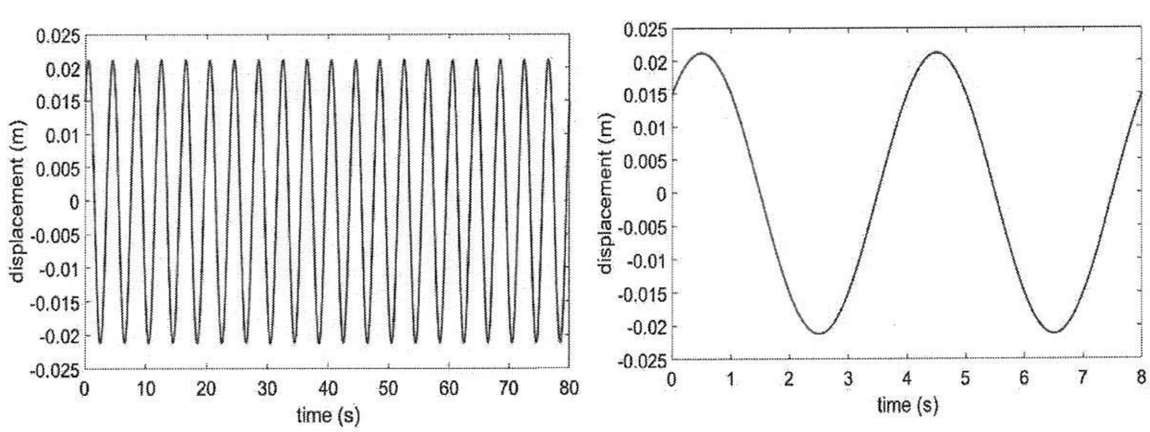


Figure 3

response of that system to a force in the form of trapezoidal pulse as shown in Fig. 3.

Even though you do not know the system, you weigh it on a scale and find the net mass to be 10 kg. While you are wondering how to determine the system parameters, your colleague gives it some initial conditions (without noting down what it is) and happens to luckily record the displacement of the system at one location. That data is shown in Figure 4. Both (a) and (b) show the displacement vs. time. Whereas (a) shows the response up to 80 seconds, (b) shows the same response zoomed-in up to 8 seconds.



(a) Response up to 80 seconds

(b) Same data as (a), response up to 8 seconds

Figure 4

Given the data in Fig.4:
a. Is it okay to assume a single degree of freedom model for this unknown system? Why or why not?
b. Is it possible to determine the unknown system parameters? If yes, determine the system parameters and the initial conditions corresponding to the response shown.
c. Is it possible to determine the response of the system to the force shown in Fig. 3? If yes, determine the response.

Q4[5 marks] Derive the equivalent spring constant for the system in the Figure 5. The system consists of a massless rigid bar, a lumped mass, and two springs as shown.

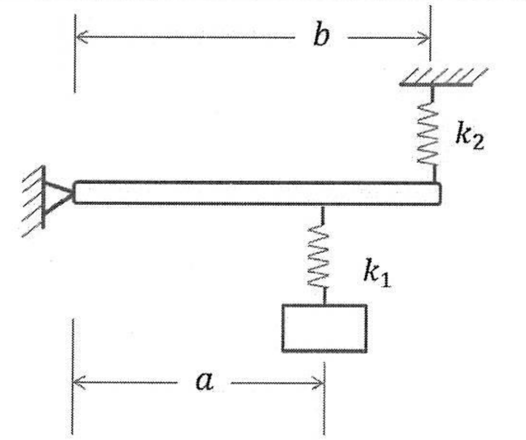


Figure 5

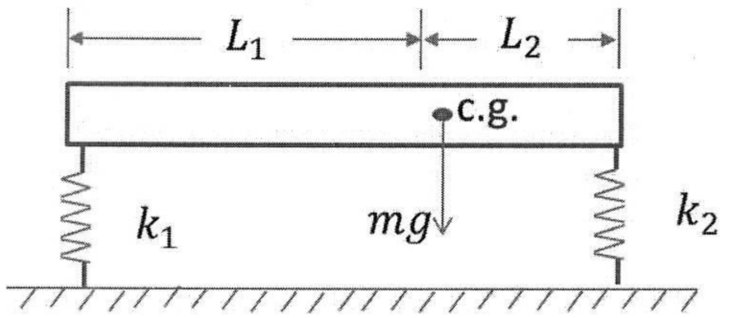


Figure 6

Q5 [15 Marks] Consider a crude model of an automobile shown in Fig.6. In the figure, 'c.g.' refers to center of gravity. Assume small oscillations only. Use k_1= 35025 N/m, k_2= 37944 N/m, L_1= 1.37 m, L_2= 1.68 m, m = 1460 kg, $I_{c.g.}$ = 40341 kg.m^2. For this system,

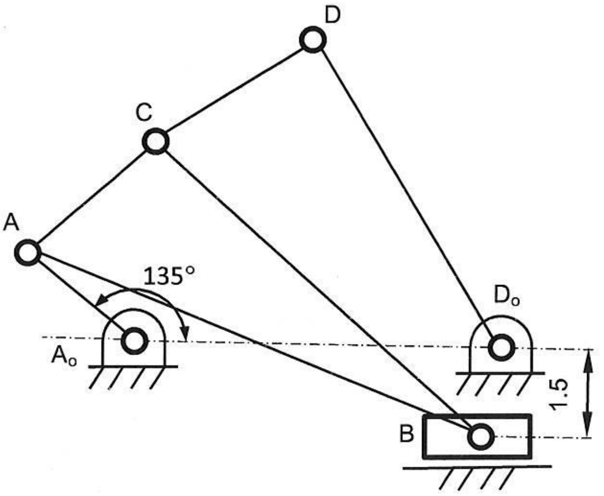
(a) [5 Marks] determine the natural frequencies of vibration

(b) [10 Marks] find the mode shapes

Q6:[10 + 10 marks] Details of the mechanism given below are as follows:

AoA = 2.5   AB = 10   BC = 8 AC = CD = 4   AoDo = 8   DDo = 6

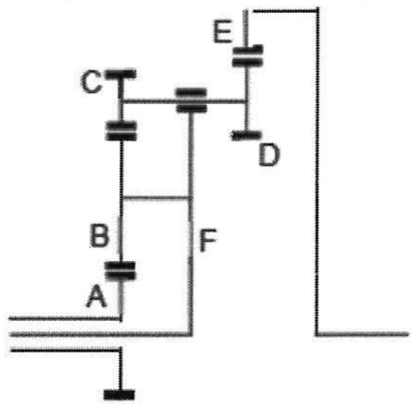
Assume any unit of length.
The angular velocity of AAo is 10 rad/s counterclockwise.
Obtain the angular velocity and angular acceleration of DDo.
(If you solve by a graphical method, then you will be judged primarily by the method of graphical construction. Accuracy of the drawing will carry low weightage.)



Q7[10 marks] An epicyclic gear train depicted below has the following gear teeth:

A : 16   B : 20   C : 18   D : 22   E : 101

Gear A rotates at 100 rpm clockwise and the internal gear E is not rotating. What is the speed and direction of rotation of the arm F ?



--------PAPER ENDS---------

Department of Mechanical Engineering, IIT Bombay

Ph.D. Comprehensive Examination – March 2017

Heat Transfer

Maximum Marks: 100 **Passing Marks: 40**

Instructions:

- This is an open-book examination. You are allowed 3 books. No photocopied books.
- Please start each new question on a new page and keep all subparts of a question together.
- Maximum marks per question are given in the table below.
- Use of calculator is permitted.
- Make suitable assumptions where necessary and state them clearly.
- Make sure to cancel any unwanted work clearly, so that it is not graded.

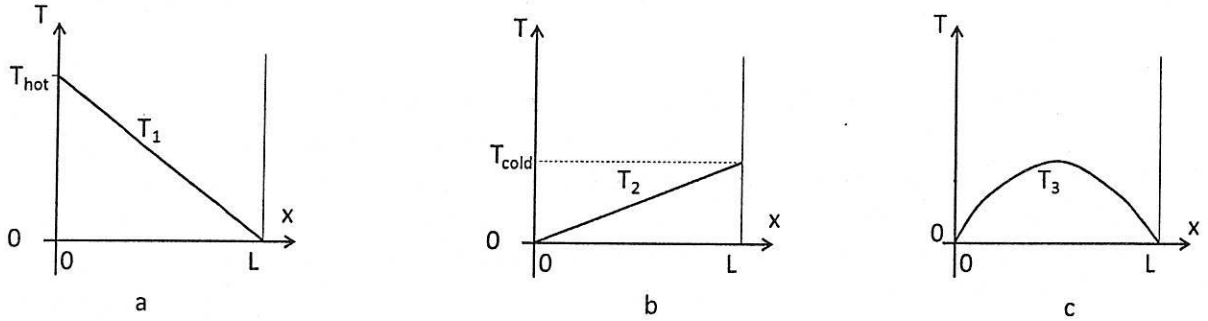
---

1. Figure (a, b, c) below shows 3 separate one-dimensional temperature profiles for solid conduction. Temperature profiles of figures a & b are linear with respect to x-direction. Figure c temperature profile is parabolic in nature and maximum temperature is observed at the center. (7.5 marks)

(a) Write the governing equation for the 3 scenarios along with its boundary condition?

(b) Using superposition principle, write the governing equation and associated boundary conditions after combining Figures a, b and c.

(c) Draw the temperature profile for the combined case and show where approximately peak temperature location will fall. Comment on the outcome.



2. Consider a large steel ingot in the form of a sphere of radius r_0. It is heated in a furnace for a long time till it reaches a uniform temperature T_0 throughout. The ingot is suddenly submerged into the tank of water to quench it so that some desired specific properties are obtained. At the end of a specified quenching time the ingot is removed from the tank, and the temperature in the ingot is of the form $T=ar^2+b$ where 'a' and 'b' are known constants. Determine the rise in temperature of the water bath if the mass of the water in the bath is M_0 and the water is assumed to be well mixed during the quenching process and there is no boiling taking place and all the properties of the water and ingot e.g. specific heat 'C_w and C_i', density 'ρ_w and ρ_i' respectively, etc. remain constant. (7.5 marks)

3. Consider the heating of a fluid flowing in a tube going through a furnace. The wall temperature of the tube can be assumed to have a constant value, T_W, and the pressure drop available for fluid flow through the tube, Δp, is fixed. The flow is flow is assumed laminar and fully developed both thermally and hydrodynamically throughout the length of the tube. The length of the tube is 'L' but the diameter is not yet fixed and it is expected to remove the maximum possible heat from the furnace when the inlet temperature is T_i. You are therefore required to obtain an expression for the heat removed by the tube as a function of the diameter, length, pressure drop Δp, T_w, T_i and fluid properties so that the expression can be used by someone to calculate the appropriate diameter given numerical values for the parameters that form part of your expression. Use k,ρ,μ,C_p for thermal conductivity, density, viscosity and specific heat of the fluid and define any other symbols that you may have used. (15 marks)

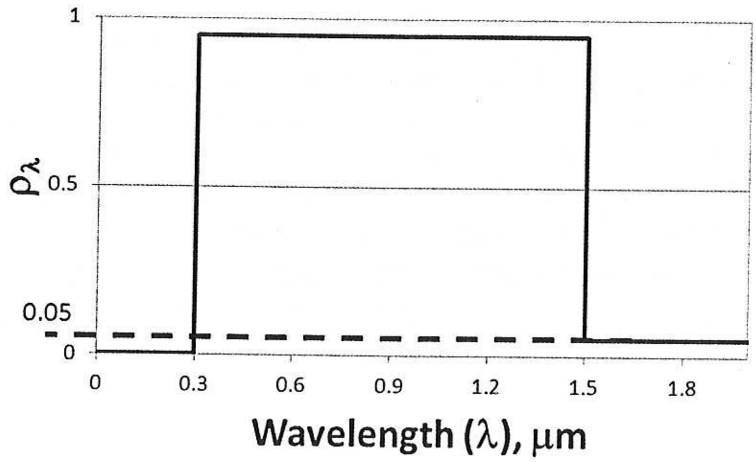
4. Hot combustion gases are flowing past a boiler tube of diameter 0.05 m. The boiler tube surface temperature is approximately at 600 K while the flowing gases are at 1800 K. The convective heat transfer coefficient is estimated to be 100 W/m^{2}K. Over a time period, ash deposits on the outside-tube surface and thickness of the ash deposit can be approximated as 5 mm at a given instant. Make necessary assumptions and state them clearly. At steady state and with surroundings at 1500 K

a. Calculate the rate of heat transfer per unit tube length when there are no ash deposits

b. Assuming that convective heat transfer per unit length reduces by 46% with ash deposits then calculate rate of heat transfer/length with ash deposits.

Emissivity of boiler tube and ash deposit are 0.8 and 0.9, respectively. Thermal conductivity of ash deposit is ~ 1 W/mK. (15 marks)

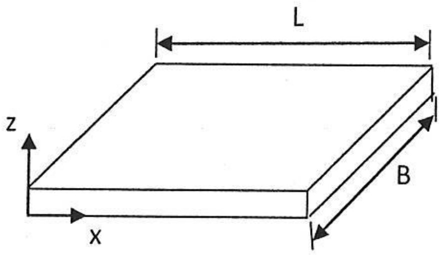
5. A spacecraft panel maintained at 300 K is coated with opaque, diffuse white paint having the spectral reflectivity distribution shown below. The spacecraft is in a near-earth orbit and is exposed to solar irradiation of 1353 W/m^2 as well as to deep space at 0 K. What is the net radiative heat flux leaving the panel surface? Comment on the obtained solution. Is this a radiator (radiating heat to space) or solar collector? Use blackbody radiation band emission fractions wherever necessary. Sun's surface temperature can be approximated to be ~ 5800 K. (15 marks)



6. A thin plate with length, L, breadth, B, and thickness, t, in the x,y,z directions respectively is exposed to a fluid with temperature T_1 and heat transfer coefficient h_1 on the top surface(i.e. z=t) and to a fluid with h_2 and T_2 on the bottom surface(i.e. z=0). The initial temperature of the plate is T_i throughout and the temperatures at x=0(and all y and z) and at y=0(and all x and z) are suddenly brought to T_0, T1 respectively at time θ=0 and then maintained at these values for higher time values. The other two side surfaces are insulated. Formulate the governing equation and boundary conditions for determining the temperature at any location in the plate at a given time, if the thickness, t, is unknown, but can be assumed to be negligible with respect to the other dimensions, 'L' and 'B', which are known. The thermal conductivity of the material varies with temperature and the functional dependence, k(T), is known. Assume density and specific heat of the plate and surrounding fluid to be constant.

a. Now assume that you know how to formulate the problem correctly and you formulate a very similar problem where the boundaries which were at constant temperature are also made insulated with all other conditions remaining same. Determine the temperature at steady state at x=L/2,y=B/2 and z=t/2.

(20 marks)



7. Laser heat treatment processes are used to modify the mechanical properties in the near-surface region of solid materials. For instance, an intense laser beam may be traversed over the surface of a ceramic material to form a consolidated glassy surface layer by melting and resolidification. A similar operation may be conducted to anneal the surface layer of a doped semi-conductor material. It is also possible to harden the surface layer of a steel component.

Consider a thick slab subjected to a laser source. The slab can be assumed to be semi-infinte in extent. Assuming that the laser heating can be treated as a surface flux, formulate the unsteady problem by writing the governing equation and boundary conditions. The material may be assumed to be at an initial temperature, $T_{Initial}$. Obtain the unsteady temperature distribution in the solid material.

(20 marks)

PhD Qualifying Re-exam – March 2017

Manufacturing Processes-II

- Full marks 100
- No books and notes are allowed but two A4 sheets of handwritten formulae are allowed
- Please make any suitable assumption if required

1. Following observations refer to an orthogonal machining experiment:

Tool rake angle	10 degrees
Cutting speed	2 m/s
Depth of cut (uncut chip thickness)	0.25 mm
Chip thickness	0.80 mm
Width of work piece	2.5 mm
Cutting force	850 N
Thrust force	300 N
Density of the work material ρ	7200 Kg/m^3
Specific heat of the work material C_p	502 J/Kg.K

Calculate:

a. Shear angle and coefficient of friction in machining.

b. Total power and the power expended in shear zone and chip-tool interface friction zone.

c. Assuming that 85 % of the total power (heat) is carried away by the chip, estimate the average rise in temperature of the chip. [20]

2. Explain in brief

a. Difference between *Roughness, Waviness* and *Lay* for a surface profile. Provide representative sketches and list the causes. [5]

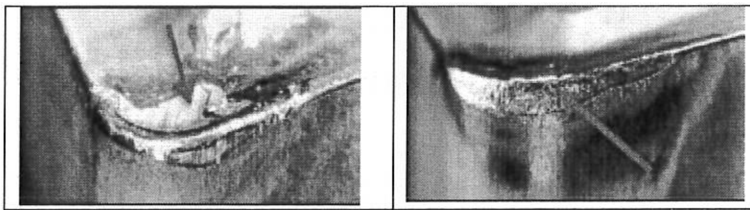
b. Difference between Straightness and Cylindricity tolerances for a cylindrical part (draw tolerance symbols). [5]

c. In a rough turning operation, the tool-life follows the following equation,

$$t = \frac{C}{v^3 f^{0.8} a^{0.1}}$$

where v, f and a are the cutting speed in m/sec, feed in m/rev and depth of cut in m. If only one parameter can be changed, which parameter should be increased to double the MRR so that the reduction in the tool-life is minimum? [5]

d. Identify the dominant wear mechanism present in each image of a cutting tool. How can the amount of wear or the wear rate be reduced in each case? [5]



(i)	(ii)

3. It is desired to machine a triangle shaped through cavity in a solid steel block of 85 x 75 x 20 mm (Fig. 1). You have to choose between wire EDM and conventional machining process. **The objective is to minimize the cutting power required.** A performance parameter α has been defined which is the ratio of the machining time in EDM to that in conventional machining ($\alpha = t_{EDM}/t_{MAC}$). Find the breakeven value of α below which conventional machining will be economical. Assume that there is no starting hole for EDM and during machining the entire volume of the cavity is to be removed.

Specific cutting energies for the two processes are $U_{EDM} = 690$ J/mm^3 and $U_{MAC} = 4.08$ J/mm^3. For EDM process assume: Wire diameter: d_w=0.2 mm and Gap: s=0.1 mm. [20]

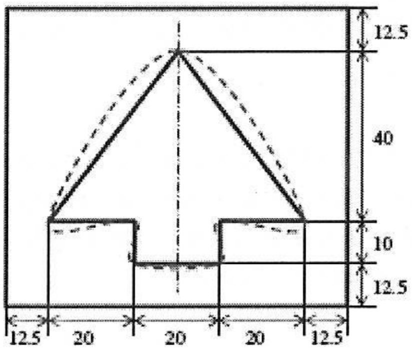


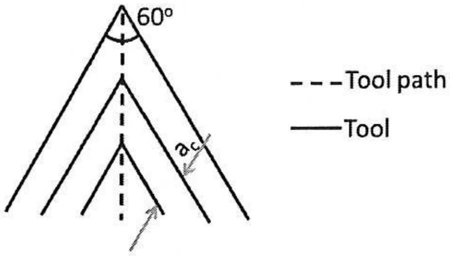
Fig. 1. Part geometry

4. We need to design plug and ring gauges to maintain quality control during the production of 50 mm shaft and hole pair of H7d8 fit as per the specification. Note that 50 mm lies in the diameter steps of 30 and 50 mm. Upper deviation for 'd' shaft is given by $16D^{0.44}$ and low deviation for hole H is zero. Tolerance factor i (μm) = $0.45D^{1/3} + 0.001D$. The relative magnitude of each grade is given in the following table.

a. Find the tolerance of hole and shaft.
b. Identify the fit.
c. Find the suitable dimensions of plug and ring gauges. Ignore the wear allowance. [15]

Relative magnitude of each grade (for basic size upto 500mm)							
Grade	IT5	IT6	IT7	IT8	IT9	IT10	IT11
Values	7i	10i	16i	25i	40i	64i	100i

5. A 50 mm diameter bar is to be screw-threaded, for 250 mm of its length on a lathe. The included angle of the thread is 60 degrees, the pitch is 2.5 mm, and the outside diameter of the threaded portion is to be 50 mm.



a. If the undeformed chip thickness, a_c is limited to 0.13 mm and tool is fed radially during each pass, how many passes of the tool will be required to complete the operation? [5]

b. If the rotational frequency of the workpiece is 0.8 rev/sec and it takes 20 sec to return the tool and engage the carriage with the lead screw after each pass, what will be the total production time? [5]

6. An abrasive water jet machining process is used to machine a steel workpiece with a kerf width of 2 mm at a transverse speed 200 mm/min. The contribution of water phase in material removal can be neglected and only kinetic energy of abrasive phase of the jet is responsible for material removal.

a. If mass flow rates of abrasives ($\dot{m}_{abr}$) and water ($\dot{m}_w$) are 1 kg/min and 4 kg/min respectively, find the penetration depth of the jet into the material. [7]

b. Suggest at least 2 ways to maximize the depth of penetration into the steel using above process. The abrasive water jet velocity (V_{awj}) is related with the water jet velocity (V_w) as:

$$V_{awj} = \frac{1}{1 + L_r} V_w$$

where L_r is the loading ratio, i.e., ratio of mass flow rate of abrasives ($\dot{m}_{abr}$) to the mass flow rate of water ($\dot{m}_w$). Specific cutting energy of steel = 13.6 J/mm^3. [8]

Paper Ends

Department of Mechanical Engineering, IIT Bombay
Ph. D. Qualifying Examination: Applied Mathematics
March 2017
Total points: 60, Time: 3 Hours
Minimum passing score: 24 points (40%)
Closed Book, Closed Notes Examination

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1. **(5 points)** Solve the initial value problem:

$$u\frac{du}{dt} = \frac{u^2}{t} + 2t^3 \cos t^2, \qquad u(\sqrt{\pi}) = 0$$

2. **(2 + 4 points)** A vector field $\mathbf{u}$ is conservative if curl $\mathbf{u} = 0$. It can be then expressed as the gradient of a scalar function ϕ, called the velocity potential function. Thus, $\mathbf{u} = \nabla\phi$ if $\mathbf{u}$ is conservative.

   (a) Show that for a conservative vector field $\mathbf{u}$, the integral $\int_A^B \mathbf{u} \cdot d\mathbf{r} = \phi_B - \phi_A$, i.e., the integral is independent of path between A and B.

   (b) The velocity potential ϕ can be determined the following way: Since the integral $\int_A^B \mathbf{u} \cdot d\mathbf{r} = \phi_B - \phi_A$ (independent of path between A and B), choose $\phi_B = \phi$ and $\phi_A = 0$, with $B(x, y, z)$ and $A(0, 0, 0)$. Choose the path from $A(0, 0, 0)$ to $B(x, y, z)$ as succession of three line segments from $(0, 0, 0)$ to $(x, 0, 0)$ to $(x, y, 0)$ to (x, y, z). Note that along each path some variable(s) is/are constant, which provides some simplification to the expression for $\mathbf{u} \cdot d\mathbf{r}$. Using this procedure, determine the velocity potential ϕ for the vector field $\mathbf{u} = (2xy - z^3)\mathbf{i} + x^2\mathbf{j} - (3xz^2 + 1)\mathbf{k}$, first verifying that it is conservative.

3. **(2 + 6 points)** Consider the system

$$\mathbf{x}' = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 3e^t \\ e^t \end{bmatrix}$$

   (a) Determine whether this system can be solved by diagonalizing the coefficient matrix.

   (b) Solve the system. If the system can be solved using the diagonalization of the coefficient matrix, **you MUST solve it in that manner to get any credit.** If it cannot be solved in that manner, use a different technique.

4. **(5 points)** Verify that $[x, x \ln x, x(\ln x)^2]$ is a solution of the ODE $x^3y''' + xy' - y = 0$ on $x > 0$, and check for linear dependence of the solution.

5. **(3 + 3 points)** The Cayley-Hamilton theorem states that if the characteristic equation for a square matrix $\mathbf{A}_{n \times n}$ is written as $\lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0 = 0$, then we have $\mathbf{A}^n + c_{n-1}\mathbf{A}^{n-1} + \cdots + c_1\mathbf{A} + c_0\mathbf{I} = 0$, where $\mathbf{I}$ is the identity matrix of order n.

   (a) Pre-multiplying each term in this equation (written for the matrix) with the inverse of the matrix can lead to the determination of the inverse. Using this approach, determine the inverse of

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

   (b) For the above matrix, evaluate the polynomial $\mathbf{A}^5 - 4\mathbf{A}^4 - 7\mathbf{A}^3 + 11\mathbf{A}^2 - \mathbf{A} - 10\mathbf{I}$. **(Hint:** Factorize the given polynomial using the characteristic equation.)

6. **(10 points)** The displacement $x(t)$ for a mechanical oscillator, with mass m, spring constant k, and a constant forcing function, F_0, satisfies the following equation

$$m\frac{d^2x}{dt^2} + kx = F_0,$$

subject to the initial conditions $x(0) = 1$ and $\frac{dx}{dt}(0) = 0$. Using Laplace transform find the desired solution for $x(t)$. The natural frequency of the system is $\omega = \sqrt{k/m}$.

7. **(10 points)** For a resistance-inductance-capacitance circuit, Kirchhoff's law leads to the following equation:

$$LI'' + RI' + \frac{I}{C} = V_0\omega\cos(\omega t)$$

Find the current $I(t)$ in the circuit with $R = 100$ ohms, $L = 0.1$ henry, and $C = 10^{-3}$ farad, which is connected to a voltage source $V(t) = 155\,\sin(377t)$. Assume zero charge and current when $t = 0$.

8. **(10 points)** A vibrating string, attached at two ends, with a potential term can be considered as an initial-boundary value problem. The equation governing this system is given below with appropriate initial and boundary conditions. Solve the equation to find the motion governing the displacement of the string. Show the physical importance of the solution using graphical representation. The governing equation is

$$y_{tt} - y_{xx} + y = 0 \tag{1}$$

for $0 < x < \pi$, $0 < t < \infty$, with the boundary conditions given as $u(0,t) = u(\pi,t) = 0$ for $0 < t < \infty$. The initial conditions are given as $y(x,0) = f(x)$ and $y_t(x,0) = 0$ for $0 < x < \pi$, where the function $f(x) = x$ for $0 < x < \pi/2$ and $f(x) = \pi - x$ for $\pi/2 < x < \pi$.

Useful information:

1. $\int x\sin(nx)\,dx = \frac{\sin(nx)}{n^2} - \frac{x\cos(nx)}{n}$.

2. Fourier series representation for a function $f(x)$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

3. Properties of Laplace transforms:

$$L[f(t)] = \overline{f}(s) = \int_0^{\infty} f(t)e^{-st}\,dt$$

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$L[f''(t)] = sL[f'(t)] - f'(0)$$

$$L^{-1}[F(s)G(s)] = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

$$L^{-1}\left(\frac{1}{s}\right) = 1, \quad L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}, \quad L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$$

20th March 2017

DES2: Kinematics and Dynamics of Machines
Ph.D. Qualifying Examination

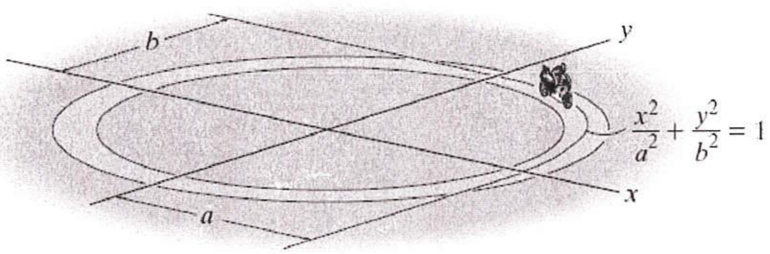
General instructions:

- Write clearly and legibly. Answers should be 'to-the-point', but should clearly show the important steps in the solution, including all assumptions and approximations.
- No queries will be entertained. If any information appears to be missing, make suitable assumptions and state them clearly. Clearly mark/highlight such assumptions, if any, by enclosing them in a box.
- The exam is 'closed book, closed notes.'
- Total = 100 **points**. Time = **3 hours**.

Q1 [10 marks] The motorcycle travels along the elliptical track (major axis of ellips x) at a constant speed V as shown below. Determine the greatest magnitude of the acceleration if a > b. Consider motorcycle as a point mass. (Formula of radius of curvature of a curve is given)

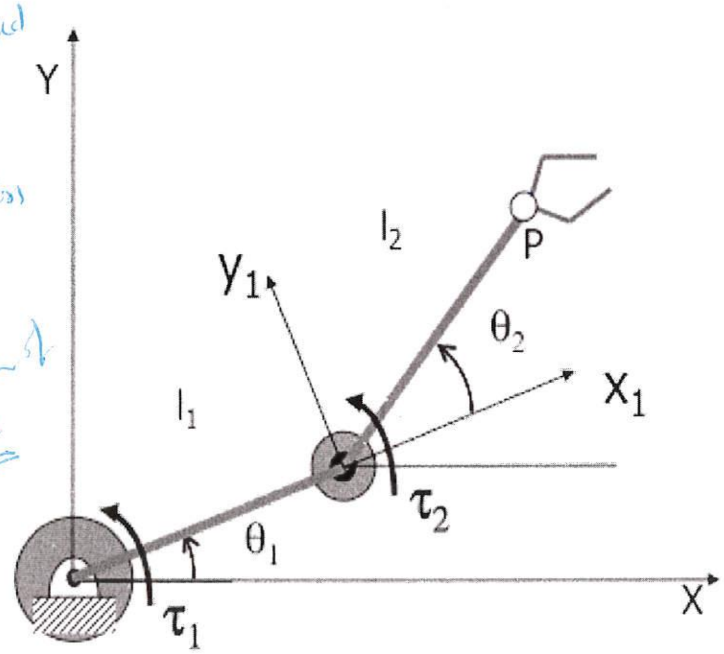
6 concept
4 derive

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

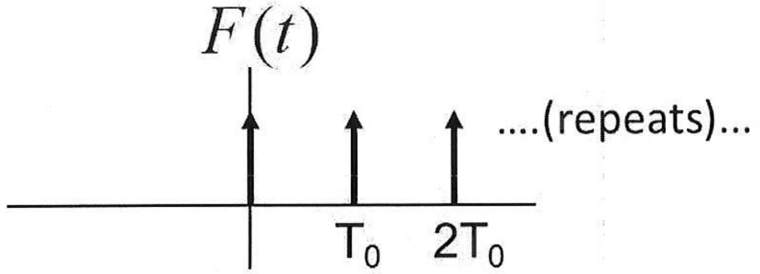


Q3 [20 marks] Consider two link robot manipulator with parameters as shown in the figure. Additionally masses of links 1 and 2 are given as m_1 and m_2 and center of gravities of links are located at distance l_{c1} and l_{c2} from respective drive axes. Using Lagrange formulation derive equations of dynamics.

6 - Geometry based kinematics
4 - KE express
4 - PE exp
6 - derivation of eqns.

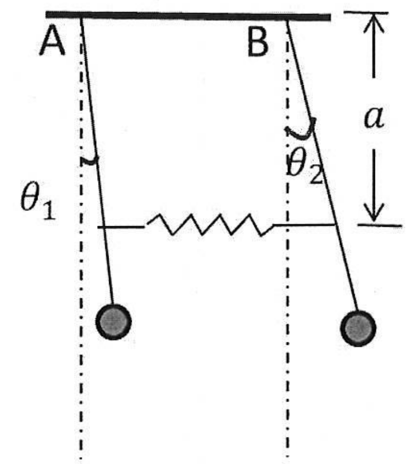


Q4 [20 Marks] Find the response of an undamped simple spring mass system system, initially at rest, to unit impulse train shown in the figure.



Q5 [20 Marks] Consider two identical pendulums connected by a spring k at a distance a from the ceiling (Fig. 2). The lengths of the two pendulums are L and their mass is m. Assuming θ_1 and $\theta_2$are small, determine the following:

(a) Governing equations of motion
(b) Natural frequencies of vibration
(c) Normal modes of vibration (or eigen vectors)



Q6 [20 marks] A cam rotates by 70 degrees for the displacement of the follower to change from 8 cm to 0 cm. The first 3 cm of this fall can be described by half of a cycloidal motion. The rest 5 cm of the fall is half of a harmonic motion. The equation for a cycloidal return is $y = L\left[1 - \frac{\theta}{\beta} + \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right]$ and the equation for a harmonic return is $y = \frac{L}{2}\left[1 + \cos\frac{\pi\theta}{\beta}\right]$

What should be the rotation of the cam (in degrees) at which the motion changes from cycloidal to harmonic so that the velocity profile of the complete motion is continuous?

Q7 [10 marks] A rack and pinion system has a pressure angle of 25°. The pinion has 22 teeth and a module of 8 mm/tooth. The addendum is equal to the module and the dedendum is 1.25 times the module. Determine the contact ratio.

--------PAPER ENDS---------

Question Paper for Ph.D. Qualifiers in Fluid Mechanics, March 2017

Time Duration: 3 hours; Total points: 60; *Exam is Closed Notes and Closed Books. All information you need has been provided in the QP*. Please keep your bags and mobile away. Only calculator is allowed. In case of any doubt, you may make appropriate assumptions and proceed to solve the problem. Please mention your assumptions carefully, and justify each step in your solution.

1. (10=5+5 points) Write down the expressions from the following tensor operations:

a) $\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}$, where $\hat{\boldsymbol{n}} = a\,\hat{\imath} + b\,\hat{\jmath}$. Here $\hat{\boldsymbol{n}}$ is a normal unit vector on an inclined surface and $\boldsymbol{\sigma}$ is a 2D Cartesian (x-y) stress tensor. Also $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the x- and y-direction, respectively. Thereafter, perform the operation: $(\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}) \cdot \hat{\imath}$ and $(\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}) \cdot \hat{\jmath}$.

b) Repeat (a) above considering momentum-flux tensor $\boldsymbol{mu}$ instead of stress $\boldsymbol{\sigma}$ as the 2D Cartesian (x-y) tensor, where $\boldsymbol{m} = m_x\,\hat{\imath} + m_y\hat{\jmath}$ is the mass-flux vector and $\boldsymbol{u} = u\,\hat{\imath} + v\hat{\jmath}$ is the velocity vector.

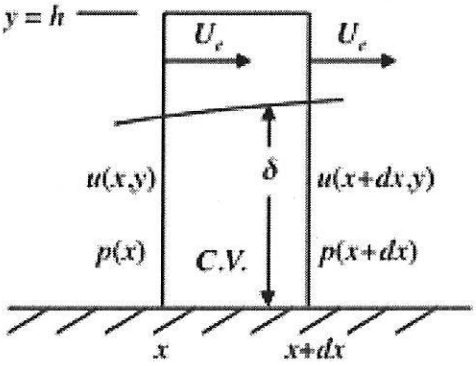
2. (10=5+5 points) Solve the following questions on the governing fluid equations:

a) Write down the 3D differential form of conservation equations, for continuity and momentum conservation in Cartesian (x-y-z) coordinate system; consider the flow as incompressible flow and gravitational force acting along y-direction. Also discuss the physical interpretation of the various terms. Thereafter, simplify the momentum equations for a steady and inviscid flow. Finally, discuss the difference in the results obtained from the differential as compared to the integral form of the conservation equations.

b) Considering the simplified form (steady & inviscid flow) of *y-momentum* equation obtained in part (a) above, present a non-dimensional form of the equation using a length-scale L_c and velocity-scale U_c. Thereafter, discuss the physical interpretation of the non-dimensional governing parameter encountered in the non-dimensional y-momentum equation. Finally, write down the expressions for non-dimensional form of the dimensional engineering parameters: lift as well as drag forces and pressure-gradient encountered in an external flow across a cylinder and internal flow in a pipe, respectively. For the cylinder as well as pipe, consider the diameter as D and free-stream/inlet velocity as U_∞.

3. (10 points) Derive the von Karman boundary layer integral equation by conserving mass and momentum in a control volume (CV) of width dx and height h that moves at the exterior flow speed $U_e(x)$ as shown in the figure below. Here h is a constant distance that is comfortably

greater than the overall boundary layer thickness. The equation should be expressed in terms of the wall shear stress τ_0, the displacement thickness δ^*, and the momentum thickness θ. Redraw your CV with the coordinate system clearly and list all assumptions. What does the equation devolve to, for a boundary layer over a flat plate?



4. (10=2+3+3+2 points) Refer to the schematic below, in which a parent pipe with flow rate Q, rotating at an angular velocity $\Omega(t)$, splits into two daughter pipes, each with flow rate $Q/2$. Here t is the time, and all pipes are rigidly attached to each other. The flow exits the daughter pipes a distance R_0 from the x axis tangential to the respective pipe axes, in the rotating frame of reference, as shown in the figure. Assume that fluid is inviscid, and that fluid velocity profile is uniform in the cross section of all the pipes. Also assume that the inner radii of the daughter pipes, R_d, is much smaller than R_0. Neglect the pipe mass for all the calculations below.

a) Starting with the equation for conservation of angular momentum of the fluid in a *material* volume $V(t)$ (i.e. in a volume following the fluid particles):

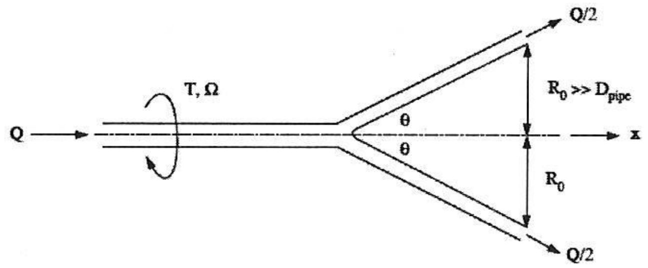
$$\frac{d}{dt}\int_{V(t)} (r \times \rho \boldsymbol{u})dV = \boldsymbol{T}^{Ext}$$

derive the equation for conservation of angular momentum of the fluid in a stationary control volume V_0 coinciding with $V(t)$ at a given time t. Here $\boldsymbol{T}^{Ext}$ is the external Torque acting on the material volume. You may denote A_0 as the boundary of volume V_0 and $\hat{\boldsymbol{n}}$ as the unit normal vector pointing *out* of V_0 at the boundary. Why is it convenient to choose such a stationary control volume ?

b) Using a 3D Cartesian coordinate system with the origin located at the bifurcation, calculate the net angular momentum of the fluid in the daughter pipes for a certain flow rate Q and angular velocity Ω. (Hint: Use the orientation where both the daughter pipes lie in the x-y plane)

c) Using conservation law for angular momentum of fluid derived in part (a), find the relation between torque T, angular velocity $\Omega(t)$ and angular acceleration $d\Omega(t)/dt$.

d) Given the initial condition $\Omega(0) = 0$, find $\Omega(t)$ for $t>0$ if a constant (in time) torque T is applied along axis of the parent pipe.



5. (10 points) Consider steady laminar flow through the annular space formed by two coaxial tubes aligned with the z-axis, where a is the radius of the inner tube and b is the radius of the outer tube. The flow is along the axis of the tubes and is maintained by a pressure gradient dp/dz.

   a) Draw a clear schematic of the flow and the coordinate system.

   b) List your assumptions clearly.

   c) Derive the expression for the axial velocity at any radius R.

   d) Find the expression for the radius at which the maximum velocity is reached.

   e) Find the expression for the volume flow rate.

   f) Determine the radial profile of shear stress and demonstrate its magnitude and direction on a schematic of the flow.

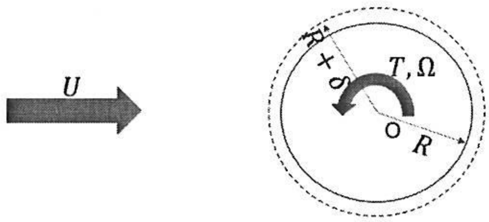
6. (10=4+3+3 points) An incompressible fluid of density ρ is flowing past a cylinder of radius R, centered at $x=y=0$, rotating around its axis at a steady angular velocity Ω (see schematic below). The velocity field far from the cylinder is steady and uniform, and is given as $\mathbf{U}(\mathbf{r})=U\mathbf{i}$, where U is a constant and $\mathbf{i}$ is the unit vector in the x direction. The fluid forms a thin viscous boundary layer around the cylinder with thickness $\delta = \left(\frac{\nu}{\Omega}\right)^{\frac{1}{2}}$, where ν is the kinematic

viscosity. The flow can be assumed to be inviscid and irrotational outside this boundary layer. The viscous traction at the surface of the cylinder is given by $\boldsymbol{\tau} = -\rho\nu \frac{[U(R+\delta,\theta)\cdot\widehat{\boldsymbol{\theta}}]}{\delta}\widehat{\boldsymbol{\theta}}$, where $\widehat{\boldsymbol{\theta}}$ is the unit vector in the azimuthal (θ) direction.

a) If we apply a counter-clockwise torque T on the cylinder, what is the resulting angular velocity Ω in terms of the other relevant parameters in the problem, at steady state ? Assume $\delta \ll R$

b) What are the location of stagnation points on the cylinder surface, for a given Torque T ? Assume that the torque is low enough such that stagnation points stay on the surface of the cylinder.

c) What is the net force acting on the cylinder, and in exactly which direction ? State your assumptions clearly.



Extra Information:

- For any field $F(\boldsymbol{x},t)$ and any control volume (CV) region $V^*(t)$, the Reynolds Transport Theorem states

$$\frac{d}{dt}\int_{V^*(t)} F(\boldsymbol{x},t)dV = \int_{V^*(t)} \frac{\partial F(\boldsymbol{x},t)}{\partial t}dV + \int_{A^*(t)} F(\boldsymbol{x},t)\boldsymbol{b}\cdot\widehat{\boldsymbol{n}}dA$$

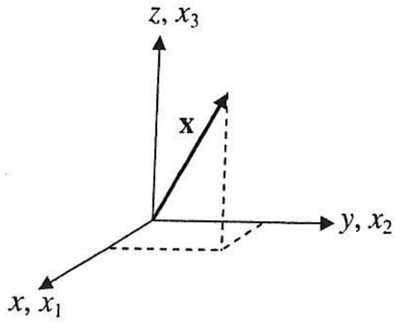
where $\boldsymbol{b}(x,t)$ is the velocity of the surface of the CV at any **x** location and time t, and $A^*(t)$ is the surface of the CV.

- The integral conservation equation for fluid momentum $\rho\boldsymbol{u}$, in the presence of a body force $\boldsymbol{g}(x,t)$ and surface traction $\boldsymbol{f}(\widehat{\boldsymbol{n}},\boldsymbol{x},t)$ is:

$$\frac{d}{dt}\int_{V^*(t)} \rho\boldsymbol{u}dV + \int_{A^*(t)} \rho\boldsymbol{u}(\boldsymbol{u}-\boldsymbol{b})\cdot\widehat{\boldsymbol{n}}dA = \int_{V^*(t)} \rho\boldsymbol{g}dV + \int_{A^*(t)} \boldsymbol{f}(\widehat{\boldsymbol{n}},\boldsymbol{x},t)\,dA$$

- Velocity potential ϕ for doublet of strength d oriented along x direction is $\phi(r,\theta) = \frac{d\cos\theta}{2\pi r}$
- Stream function ψ for point vortex with circulation Γ is $\psi(r,\theta) = \frac{\Gamma}{2\pi}\ln(r)$
- In polar coordinates, $u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r}$, $u_\theta = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$

Cartesian Coordinates (Figure B.1)



Position: $\mathbf{x} = (x, y, z) = (x_1, x_2, x_3) = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3$
Unit vectors: $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, or $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$
Unit vector dependencies: $\partial\mathbf{e}_i/\partial x_j = 0$ for i and $j = 1, 2$, or 3; that is, Cartesian unit vectors are independent of the coordinate values
Gradient operator: $\nabla = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z} = \mathbf{e}_1 \frac{\partial}{\partial x_1} + \mathbf{e}_2 \frac{\partial}{\partial x_2} + \mathbf{e}_3 \frac{\partial}{\partial x_3}$

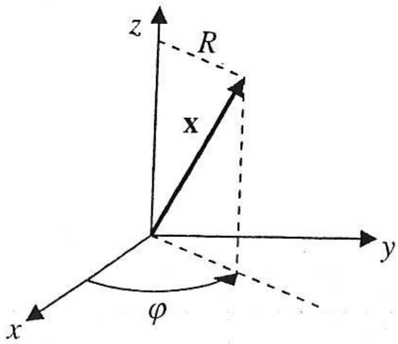
Surface integral, S, of $f(x,y,z)$ over the plane defined by $x = \xi$: $S = \int_{y=-\infty}^{+\infty} \int_{z=-\infty}^{+\infty} f(\xi, y, z)\,dz\,dy$

Surface integral, S, of $f(x,y,z)$ over the plane defined by $y = \psi$: $S = \int_{x=-\infty}^{+\infty} \int_{z=-\infty}^{+\infty} f(x, \psi, z)\,dz\,dx$

Surface integral, S, of $f(x,y,z)$ over the plane defined by $z = \zeta$: $S = \int_{x=-\infty}^{+\infty} \int_{y=-\infty}^{+\infty} f(x, y, \zeta)\,dy\,dx$

Volume integral, V, of $f(x,y,z)$ over all space: $V = \int_{x=-\infty}^{+\infty} \int_{y=-\infty}^{+\infty} \int_{z=-\infty}^{+\infty} f(x, y, z)\,dz\,dy\,dx$

Cylindrical Coordinates (Figure B.2)



Position: $\mathbf{x} = (R, \varphi, z) = R\mathbf{e}_R + z\mathbf{e}_z$; $x = R\cos\varphi$, $y = R\sin\varphi$; $z = z$, or $R = \sqrt{x^2 + y^2}$, $\varphi = \tan^{-1}(y/x)$

Unit vectors: $\mathbf{e}_R = \mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi$, $\mathbf{e}_\varphi = -\mathbf{e}_x \sin\varphi + \mathbf{e}_y \cos\varphi$, $\mathbf{e}_z$ = same as Cartesian

Unit vector dependencies: $\partial\mathbf{e}_R/\partial R = 0, \partial\mathbf{e}_R/\partial\varphi = \mathbf{e}_\varphi, \partial\mathbf{e}_R/\partial z = 0$

$$\partial\mathbf{e}_\varphi/\partial R = 0, \partial\mathbf{e}_\varphi/\partial\varphi = -\mathbf{e}_R, \partial\mathbf{e}_\varphi/\partial z = 0$$
$$\partial\mathbf{e}_z/\partial R = 0, \partial\mathbf{e}_z/\partial\varphi = 0, \partial\mathbf{e}_z/\partial z = 0$$

Gradient operator: $\nabla = \mathbf{e}_R\frac{\partial}{\partial R} + \mathbf{e}_\varphi\frac{1}{R}\frac{\partial}{\partial\varphi} + \mathbf{e}_z\frac{\partial}{\partial z}$

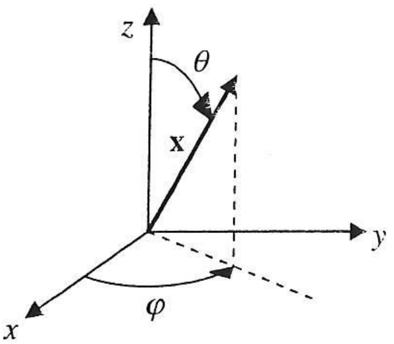
Surface integral, S, of $f(R,\theta,z)$ over the cylinder defined by $R = \xi$: $S = \int\limits_{\varphi=0}^{2\pi}\int\limits_{z=-\infty}^{+\infty} f(\xi, \varphi, z)\xi\, dz\, d\varphi$

Surface integral, S, of $f(R,\theta,z)$ over the half plane defined by $\varphi = \psi$: $S = \int\limits_{R=0}^{+\infty}\int\limits_{z=-\infty}^{+\infty} f(R, \psi, z)\, dz\, dR$

Surface integral, S, of $f(R,\theta,z)$ over the plane defined by $z = \zeta$: $S = \int\limits_{R=0}^{+\infty}\int\limits_{\varphi=0}^{2\pi} f(R, \varphi, \zeta) R\, d\varphi\, dR$

Volume integral, V, of $f(R,\theta,z)$ over all space: $V = \int\limits_{z=-\infty}^{+\infty}\int\limits_{R=0}^{+\infty}\int\limits_{\varphi=0}^{2\pi} f(R, \varphi, z) R\, d\varphi\, dR\, dz$

Spherical Coordinates (Figure B.3)



Position: $\mathbf{x} = (r, \theta, \varphi) = r\mathbf{e}_r$; $x = r\cos\varphi\sin\theta$, $y = r\sin\varphi\sin\theta$, $z = r\cos\theta$; or $r = \sqrt{x^2 + y^2 + z^2}$, $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$, and $\varphi = \tan^{-1}(y/x)$

Unit vectors: $\mathbf{e}_r = \mathbf{e}_x \sin\theta\cos\varphi + \mathbf{e}_y \sin\theta\sin\varphi + \mathbf{e}_z \cos\theta$,
$\mathbf{e}_\theta = \mathbf{e}_x \cos\theta\cos\varphi + \mathbf{e}_y \cos\theta\sin\varphi - \mathbf{e}_z \sin\theta$, $\mathbf{e}_\varphi = -\mathbf{e}_x \sin\varphi + \mathbf{e}_y \cos\varphi$

Unit vector dependencies: $\partial\mathbf{e}_r/\partial r = 0, \partial\mathbf{e}_r/\partial\theta = \mathbf{e}_\theta, \partial\mathbf{e}_r/\partial\varphi = \mathbf{e}_\varphi \sin\theta$

$$\partial\mathbf{e}_\theta/\partial r = 0, \partial\mathbf{e}_\theta/\partial\theta = -\mathbf{e}_r, \partial\mathbf{e}_\theta/\partial\varphi = \mathbf{e}_\varphi\cos\theta$$
$$\partial\mathbf{e}_\varphi/\partial r = 0, \partial\mathbf{e}_\varphi/\partial\theta = 0, \partial\mathbf{e}_\varphi/\partial\varphi = -\mathbf{e}_r \sin\theta - \mathbf{e}_\theta \cos\theta$$

Gradient operator: $\nabla = \mathbf{e}_r\frac{\partial}{\partial r} + \mathbf{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \mathbf{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}$

Surface integral, S, of $f(r,\theta,\varphi)$ over the sphere defined by $r = \xi$: $S = \int\limits_{\theta=0}^{\pi}\int\limits_{\varphi=0}^{2\pi} f(\xi, \theta, \varphi)\xi^2 \sin\theta\, d\varphi\, d\theta$

Surface integral, S, of $f(r,\theta,\varphi)$ over the cone defined by $\theta = \psi$: $S = \int_{r=0}^{+\infty} \int_{\varphi=0}^{2\pi} f(r,\psi,\varphi) r \sin\psi \, d\varphi \, dr$

Surface integral, S, of $f(r,\theta,\varphi)$ over the half plane defined by $\varphi = \zeta$: $S = \int_{r=0}^{+\infty} \int_{\theta=0}^{\pi} f(r,\theta,\zeta) r \, d\theta \, dr$

Volume integral, V, of $f(r,\theta,\varphi)$ over all space: $V = \int_{r=0}^{+\infty} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} f(r,\theta,\varphi) r^2 \sin\theta \, d\varphi \, d\theta \, dr$

B.6. EQUATIONS IN CURVILINEAR COORDINATE SYSTEMS

Plane Polar Coordinates (Figure 3.3a)

Position and velocity vectors $\mathbf{x} = (r, \theta) = r\mathbf{e}_r$; $\mathbf{u} = (u_r, u_\theta) = u_r\mathbf{e}_r + u_\theta\mathbf{e}_\theta$

Gradient of a scalar ψ: $\nabla\psi = \mathbf{e}_r \dfrac{\partial\psi}{\partial r} + \mathbf{e}_\theta \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$

Laplacian of a scalar ψ: $\nabla^2\psi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\psi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2\psi}{\partial\theta^2}$

Divergence of a vector: $\nabla\cdot\mathbf{u} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(ru_r) + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial\theta}$

Curl of a vector, vorticity: $\boldsymbol{\omega} = \nabla\times\mathbf{u} = \mathbf{e}_z\left(\dfrac{1}{r}\dfrac{\partial(ru_\theta)}{\partial r} - \dfrac{1}{r}\dfrac{\partial u_r}{\partial\theta}\right)$

Laplacian of a vector: $\nabla^2\mathbf{u} = \mathbf{e}_r\left(\nabla^2 u_r - \dfrac{u_r}{r^2} - \dfrac{2}{r^2}\dfrac{\partial u_\theta}{\partial\theta}\right) + \mathbf{e}_\theta\left(\nabla^2 u_\theta + \dfrac{2}{r^2}\dfrac{\partial u_r}{\partial\theta} - \dfrac{u_\theta}{r^2}\right)$

Strain rate S_{ij} and viscous stress σ_{ij} for an incompressible fluid where $\sigma_{ij} = 2\mu S_{ij}$:

$$S_{rr} = \frac{\partial u_r}{\partial r} = \frac{1}{2\mu}\sigma_{rr},\ S_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r} = \frac{1}{2\mu}\sigma_{\theta\theta},\ S_{r\theta} = \frac{r}{2}\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{2r}\frac{\partial u_r}{\partial\theta} = \frac{1}{2\mu}\sigma_{r\theta}$$

Equation of continuity: $\dfrac{\partial\rho}{\partial t} + \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\rho u_r) + \dfrac{1}{r}\dfrac{\partial}{\partial\theta}(\rho u_\theta) = 0$

Navier-Stokes equations with constant ρ, constant ν, and no body force:

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}\right),$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + \nu\left(\nabla^2 u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2}\right),$$

where $\nabla^2 = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$.

Cylindrical Coordinates (Figure B.2)

Position and velocity vectors: $\mathbf{x} = (R, \varphi, z) = R\mathbf{e}_R + z\mathbf{e}_z$; $\mathbf{u} = (u_R, u_\varphi, u_z) = u_R\mathbf{e}_R + u_\varphi\mathbf{e}_\varphi + u_z\mathbf{e}_z$

Gradient of a scalar ψ: $\nabla\psi = \mathbf{e}_R\dfrac{\partial\psi}{\partial R} + \mathbf{e}_\varphi\dfrac{1}{R}\dfrac{\partial\psi}{\partial\varphi} + \mathbf{e}_z\dfrac{\partial\psi}{\partial z}$

Laplacian of a scalar ψ: $\nabla^2\psi = \dfrac{1}{R}\dfrac{\partial}{\partial R}\left(R\dfrac{\partial\psi}{\partial R}\right) + \dfrac{1}{R^2}\dfrac{\partial^2\psi}{\partial\varphi^2} + \dfrac{\partial^2\psi}{\partial z^2}$

Divergence of a vector: $\nabla \cdot \mathbf{u} = \frac{1}{R}\frac{\partial}{\partial R}(Ru_R) + \frac{1}{R}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_z}{\partial z}$

Curl of a vector, vorticity: $\boldsymbol{\omega} = \nabla \times \mathbf{u} = \mathbf{e}_R\left(\frac{1}{R}\frac{\partial u_z}{\partial \varphi} - \frac{\partial u_\varphi}{\partial z}\right) + \mathbf{e}_\varphi\left(\frac{\partial u_R}{\partial z} - \frac{\partial u_z}{\partial R}\right)$

$$+ \mathbf{e}_z\left(\frac{1}{R}\frac{\partial (Ru_\varphi)}{\partial R} - \frac{1}{R}\frac{\partial u_R}{\partial \varphi}\right)$$

Laplacian of a vector: $\nabla^2\mathbf{u} = \mathbf{e}_R\left(\nabla^2 u_R - \frac{u_R}{R^2} - \frac{2}{R^2}\frac{\partial u_\varphi}{\partial \varphi}\right) + \mathbf{e}_\varphi\left(\nabla^2 u_\varphi + \frac{2}{R^2}\frac{\partial u_R}{\partial \varphi} - \frac{u_\varphi}{R^2}\right)$
$+\mathbf{e}_z\nabla^2 u_z$

Strain rate S_{ij} and viscous stress σ_{ij} for an incompressible fluid where $\sigma_{ij} = 2\mu S_{ij}$:

$$S_{RR} = \frac{\partial u_R}{\partial R} = \frac{1}{2\mu}\sigma_{RR},\ S_{\varphi\varphi} = \frac{1}{R}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_R}{R} = \frac{1}{2\mu}\sigma_{\varphi\varphi},\ S_{zz} = \frac{\partial u_z}{\partial z} = \frac{1}{2\mu}\sigma_{zz}$$

$$S_{R\varphi} = \frac{R}{2}\frac{\partial}{\partial R}\left(\frac{u_\varphi}{R}\right) + \frac{1}{2R}\frac{\partial u_R}{\partial \varphi} = \frac{1}{2\mu}\sigma_{R\varphi},\ S_{\varphi z} = \frac{1}{2R}\frac{\partial u_z}{\partial \varphi} + \frac{1}{2}\frac{\partial u_\varphi}{\partial z} = \frac{1}{2\mu}\sigma_{\varphi z},$$

$$S_{zR} = \frac{1}{2}\left(\frac{\partial u_R}{\partial z} + \frac{\partial u_z}{\partial R}\right) = \frac{1}{2\mu}\sigma_{zR}$$

Equation of continuity: $\frac{\partial \rho}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\rho u_R) + \frac{1}{R}\frac{\partial}{\partial \varphi}(\rho u_\varphi) + \frac{\partial}{\partial z}(\rho u_z) = 0$

Navier-Stokes equations with constant ρ, constant ν, and no body force:

$$\frac{\partial u_R}{\partial t} + (\mathbf{u}\cdot\nabla)u_R - \frac{u_\varphi^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial R} + \nu\left(\nabla^2 u_R - \frac{u_R}{R^2} - \frac{2}{R^2}\frac{\partial u_\varphi}{\partial \varphi}\right),$$

$$\frac{\partial u_\varphi}{\partial t} + (\mathbf{u}\cdot\nabla)u_\varphi + \frac{u_R u_\varphi}{R} = -\frac{1}{\rho R}\frac{\partial p}{\partial \varphi} + \nu\left(\nabla^2 u_\varphi + \frac{2}{R^2}\frac{\partial u_R}{\partial \varphi} - \frac{u_\varphi}{R^2}\right),$$

$$\frac{\partial u_z}{\partial t} + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 u_z,$$

where: $\mathbf{u}\cdot\nabla = u_R\frac{\partial}{\partial R} + \frac{u_\varphi}{R}\frac{\partial}{\partial \varphi} + u_z\frac{\partial}{\partial z}$ and $\nabla^2 = \frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial}{\partial R}) + \frac{1}{R^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$.

Spherical Coordinates (Figure B.3)

Position and velocity vectors: $\mathbf{x} = (r, \theta, \varphi) = r\mathbf{e}_r$; $\mathbf{u} = (u_r, u_\theta, u_\varphi) = u_r\mathbf{e}_r + u_\theta\mathbf{e}_\theta + u_\varphi\mathbf{e}_\varphi$

Gradient of a scalar ψ: $\nabla\psi = \mathbf{e}_r\frac{\partial \psi}{\partial r} + \mathbf{e}_\varphi\frac{1}{r}\frac{\partial \psi}{\partial \theta} + \mathbf{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial \psi}{\partial \varphi}$

Laplacian of a scalar ψ: $\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \psi}{\partial \varphi^2}$

Divergence of a vector: $\nabla\cdot\mathbf{u} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r\sin\theta}\frac{\partial (u_\theta\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial u_\varphi}{\partial \varphi}$

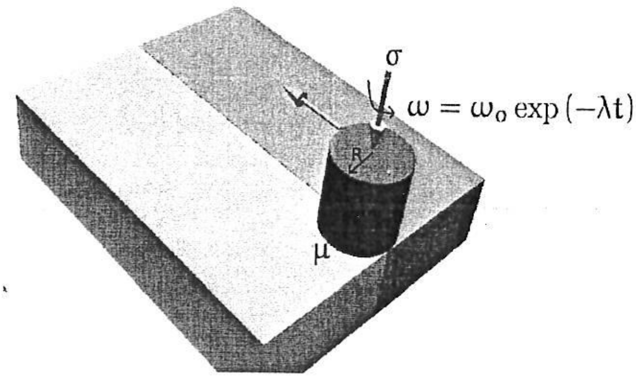
PhD Qualifying Exam -Repeat – 2016-17 - (Manufacturing Processes-I)

(Attempt all questions, All parts of a question must be answered continuously, Write down clearly the assumptions made during answering each question)

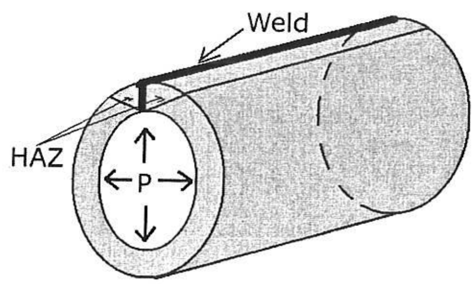
[1] In a tensile test, two pairs of values of stress and strain were measured for a specimen alloy after it had yielded: (a) true stress = 217 MPa and true strain = 0.35, and (b) true stress = 259 MPa and true strain = 0.68. Based on these two data points, determine the strength coefficient and the strain-hardening exponent of the specimen alloy. **[10]**

[2] (a) Explain the significance of *carbon equivalent* and *chromium equivalent* with regard to welding processes. (b) How to control sensitization in welding of stainless steel by controlling carbon and chromium equivalents? **[05 + 05]**

[3] The figure on the side shows a schematic of friction stir welding process. Following information are given for a friction stir welding process: R: tool radius, μ : coefficient of friction between the tool and workpiece, σ : stress applied on the tool, and ω : angular speed of the tool. It is stated further that the angular speed of the tool decays with time as: $\omega = \omega_0 \exp(-\lambda t)$. Derive an expression for the total heat generated due to friction during the process. Neglect the translational speed of the tool. **[10]**



[4] A schematic of a welded cylindrical pipe is shown below. It has inner radius and outer radius of 10 cm and 10.10 cm, respectively. The base metal, welded region and HAZ have yield strengths of 100 MPa, 110 MPa and 90 MPa, respectively. Assuming the material follows Von-Mises Yield Criteria, estimate the maximum internal pressure that the pipe can sustain without yielding. **[10]**



[5] (a) Show schematically the forces & stresses experienced by strip during rolling operation. (b) A strip with a cross-section of 150 mm x 6 mm is being rolled with 20% reduction of area using a pair of steel rolls of diameter 400 mm. What is the final thickness of the strip? (c) What would be the angle subtended by the deformation zone at the roll center? (d) How roll radius, coefficient of friction, front and back tension affect the roll separating force? **[10]**

[6] A cylindrical riser must be designed for sand-casting of a steel rectangular plate with dimensions 7.5 cm x 12.5 cm x 2.0 cm. Experiments have shown that the total solidification time of this casting is around 1.6 min. The cylinder for the riser should have a diameter-to-height ratio of 1.0. Determine the dimensions of the rise so that its solidification time is at least 2.0 min. **[10]**

[7] Propose a simple methodology based on scientific principles to estimate the minimum required rotational speed of the mold during a horizontal centrifugal casting of an alloy to ensure that the molten alloy will always remain pressurized against the mold wall inside the mold cavity. **[10]**

[8] A cylindrical workpiece of 75 mm height and 50 mm diameter is subjected to a cold upset forging and reduced to a height of 36 mm. The workpiece material follows a flow stress curve defined by the strength coefficient as 350 MPa and strain hardening coefficient as 0.17. Assume a coefficient of friction of 0.1. Determine the force as the process begins, at an intermediate height of 62 mm and at the final height of 36 mm. **[10]**

[9] With a clear schematic diagram of ram pressure as function of the ram stroke length in direct and indirect extrusions, explain working principle, merits and demerits of each of these processes. **[10]**

[10] (a) If you pull and break a tension-test specimen rapidly, where would the temperature be the highest and why? (b) The values of strength coefficient (K) and strain-hardening coefficient (n) of two different alloys are given. Will these values be sufficient to evaluate which of the two alloys is more tough and why? **[4 + 6]**