Ph.D. Comprehensive Exam in Mathematics, Dec 2014 (Total points : 50)

Note: This is NOT an open book exam. NO cheat sheet is allowed. Only calculator is allowed. Put your mobiles and bags away. Justify each step in your solution as far as possible.

1. (6 points) Solve the following equation:

$$\frac{dy}{dt} = (\Gamma \cos t + T)y - y^3$$

where Γ and T are constants. (Hint: Consider a transformation of the form $v = 1/y^n$, where n is an integer, for simplifying the above equation).

2. (6 points) Given:

$$A = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$$

Find A^n, where n is a positive integer.

3. (10 points) Consider the following problem:

$$\alpha^2 u_{xx} - u_t = a, \quad -\infty < x < \infty, \quad t > 0$$

where a is a constant, along with the initial condition:

$$u(x,0) = f(x), \quad -\infty < x < \infty$$

Obtain the solution $u(x,t)$ for $-\infty < x < \infty$ and $t > 0$.

4. (10 points) Consider the following boundary value problem ovar a semi-circular domain:

$$\begin{aligned} \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} &= 0, \quad 0 \le \theta \le \pi, \quad 0 \le r \le a \\ u(a,\theta) &= f(\theta), \quad 0 \le \theta \le \pi \\ u(r,0) &= g(r), \quad u(r,\pi) = 0, \quad 0 \le r \le a \end{aligned}$$

Obtain solution for $u(r,\theta)$.

5. (4+4 points) A closed curve in 2D is described by the equations $x(\theta) = r(\theta)\cos(\theta)$ and $y(\theta) = r(\theta)\sin(\theta)$, where $r(\theta) = R + a\sin(m\theta)$. Here $\theta \in [0, 2\pi]$ is the parametrization for the curve, and R, m, a are positive constants, with m being an integer. Denoting the region enclosed by the curve as D:

   (a) Find an expression for the components $n_x(\theta), n_y(\theta)$ of the unit normal vector to the curve, $\hat{\mathbf{n}} = n_x(\theta)\mathbf{i} + n_y(\theta)\mathbf{j}$, pointing *out* from the enclosed domain D. Note that $\hat{\mathbf{n}} \cdot \hat{\mathbf{n}} = 1$.

(b) Given the scalar field $\phi(x, y) = x^2 + y^2$, find the area integral of $\nabla^2\phi$ over D, or $\int_D \nabla^2\phi dA$, in terms of R, a, and m.

6. (10 points) A point mass (of mass unity) is attached at $x = 0$ to a semi-infinite string (with tension unity) extending from $0 \leq x \leq \infty$. The displacement of the string, $y(x)$ is then governed by the equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

with the boundary condition $\frac{\partial^2 y}{\partial t^2}|_{x=0} = \frac{\partial y}{\partial x}\Big|_{x=0}$. Here, c is a positive constant. Given that $y(x, 0) = f(x)$ and $\frac{\partial y}{\partial t}|_{t=0} = g(x)$ for $x > 0$, find an expression for the displacement of the mass, $y(0, t)$.

Department of Mechanical Engineering, IIT Bombay
PhD Qualifying Examination – Autumn 2014
Solid Mechanics

Duration: 3 hours | Total Points: **100**

Instructions:

- Attempt all questions.
- Do not write your Roll number or name on the answer book. Just write the special examinee id assigned to you on the answer book.
- Examination is open notes and closed book. No Xerox/photocopied material is allowed. Anything handwritten is permitted.
- *Please read the questions completely before attempting them.*
- Clearly mention any assumption you make.
- In case of lack of time, please mention the steps carefully. Appropriate credit will be given.
- Refrain from cheating. If caught cheating, disciplinary action will be taken.
- No electronic devices other than calculator are allowed.

Q.1. Euler column buckling is a classical example of structural instability whereby a structure subjected to same loading can produce two different deformation states based on the load value. Figure 1(a) shows a column pin-joined at both ends and subjected to compressive axial load P. The column material is assumed to be linear elastic isotropic solid with Young's modulus E. Column is homogeneous with length L, uniform cross-section with area A and moment of inertia I. Assume deformations are infinitesimally small and buckled state comprises of bending deformation only. Buckled state is a very slight deviation from the vertical straight line.

(a) Complete the partial free body diagram (F.B.D) shown in Figure 1(b) with forces and moments at the exposed section at a distance x from A. Apply equilibrium conditions to the F.B.D to obtain governing equation for transverse deflection $w(x)$ (See Figure 1(b)). Use the boundary conditions at ends A and B to solve for $w(x)$. Thus find the minimum critical load P_{cr} for buckled configuration shown in Figure 1(a).

(b) Similar result can be obtained using Energy method. Express the strain energy (U) of the buckled column in terms of the unknown $w(x)$. Similarly express the work done (W) by applied load P to produce the buckled state. In the buckled state shown through dash lines in Figure 1(a), assume the column length as L'. Note that owing to infinitesimal deformation assumption and buckled state not being too far from vertical column position, L' is very close to L. Apply principle of conservation of energy and express the applied load in terms of unknown $w(x)$. Use the solution for $w(x)$ from previous section to recover the result for P_{cr}. Suggest any other suitable choice of kinematically admissible solution for $w(x)$ and thus compute P_{cr}. Finally compare the magnitudes of P_{cr} vis-à-vis the choice of $w(x)$.

[15 points]

Q.2. Figure 2 shows an elastic plate ABCD lying in x-y plane and confined in an enclosure AEFG with rigid boundaries. The plate and enclosure is infinitely long in the z-direction. Initially the plate is stress-free and at a temperature of 0°C. Plate ABCD is made up of elastic material with Young's modulus E, Poisson's ratio v and coefficient of thermal expansion α. Plate is uniformly heated to temperature ΔT and not subjected to any form of mechanical loading.

(a) Use strain decomposition and stress-strain relations to express stress components σ_x, σ_y, σ_{xy} in terms of total strain components ε_x, ε_y, ε_{xy} and temperature change ΔT.

(b) If $\Delta b/b < \Delta a/a$, what is the critical temperature when the plate will touch either of the walls of the enclosure. What is the temperature at which the plate will touch both the walls? Thus, plot σ_x, σ_y, σ_{xy} as a function of increasing temperature ΔT in the plate.

[10 points]

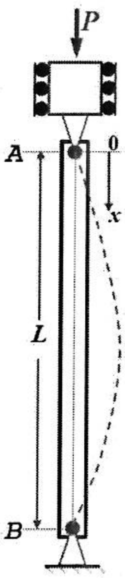


Figure 1(a)

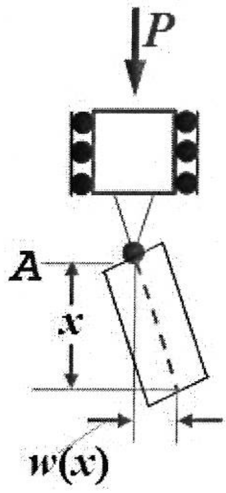


Figure 1(b)

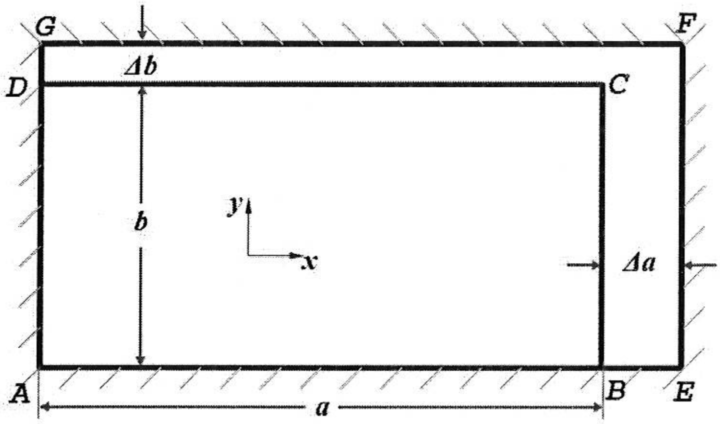


Figure 2

Q.3. A solid circular cylinder of length L and radius R with its axis aligned along z-direction is fixed at one end and the other end is subjected to torque T. Cylinder material is elastoplastic with shearing stress-strain diagram as shown in Figure 3.

(a) Within elastic range, how do the shear strain γ and stress τ vary as a function of r?

(b) What is the largest torque T^* for which the deformation remains fully elastic? Also, calculate the corresponding twist φ^* in the cylinder.

(c) If the applied torque $T > T^*$, the deformation in the cylinder will be elastic-plastic. Find out the radius r_Y which separates the elastic and plastic regions in terms of T. Thus, calculate the shear stress distribution along the radial direction in the cylinder.

(d) Will the relationship between shear strain, twist and radius affected due to plastic deformation?

(e) Express the twist φ for part (c), in terms of φ^*, R and r_Y. Note $\varphi > \varphi^*$.

(f) What is the torque T_P corresponding to complete plastic deformation?

(g) Use results of (b), (c) and (e) to plot the Torque-twist (T- φ) curve for elastic-plastic solid governed by stress-strain behavior of Figure 3.

[15 points]

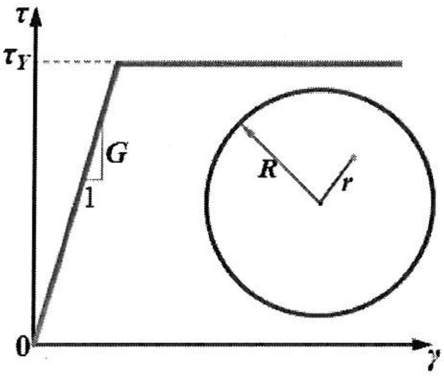


Figure 3

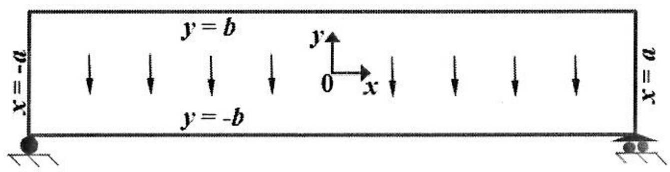


Figure 4

Q.4. A linear elastic isotropic homogeneous body is modeled using the plane stress assumption. The body is stressed due to body force per unit volume $\vec{b} = b_x e_x + b_y e_y$. Stress components are $\sigma_x, \sigma_y, \sigma_{xy}$.

(a) Write down the equilibrium equations.

(b) Show that the equations in (a) are trivially satisfied if body force vector can be expressed as $\vec{b} = -\nabla V$ where V is a scalar potential and the stresses are expressed as $\sigma_x = \partial^2\varphi/\partial y^2 + V$, $\sigma_y = \partial^2\varphi/\partial x^2 + V$, $\sigma_{xy} = -\,\partial^2\varphi/\partial x\partial y$.

(c) The compatibility condition is $\partial^2\varepsilon_x/\partial y^2 - 2\,\partial^2\varepsilon_{xy}/\partial y\partial x + \partial^2\varepsilon_y/\partial x^2 = 0$ where $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ are the strain components. Use Hooke's law to derive the governing equation for φ based on (b).

(d) Figure 4 shows a simply supported beam stressed due to self weight which corresponds to $V(x, y) = \rho g y$ where, ρ, g are mass density and acceleration due to gravity. Write down the stress boundary conditions along edges $x = \pm a$ and $y = \pm b$. [**Hint**: Lateral load on beam gives rise to shear forces].

(e) Rewrite the boundary conditions in terms of the Airy stress function φ.

(f) Show that $\varphi(x, y) = c_1 x^2 y - 5c_2 x^2 y^3 + c_3 y^3 + c_2 y^5$ is a valid solution based on the answer to (c).

(g) Use answer to (e), to find out the coefficients c_1, c_2, c_3. Coefficient c_3 can be found by enforcing moment balance along edge $x = a$.

[20 points]

Q5. Consider a slender bar of uniform cross-sectional area and length $2L$ rotating at constant angular velocity ω about an axis passing through the mid-span and perpendicular to the axis of the cross-section of the bar. Assume the material of the bar is homogeneous, isotropic and linear elastic. Let ρ be the mass density and E the Young's modulus of bar material. Place origin of the coordinate system at the center of rotation of the bar and obtain the expressions for the distribution of stress and displacement along the length of the bar in terms of the tip Mach number $M_t = V_{\text{tip}}/c$, where, V_{tip} is the tip velocity of the bar and c is the bar wave speed.

[15 points]

Q6. Consider a solid disk of radius b and thickness t made of a homogeneous, isotropic, linear elastic material with mass density ρ, Young's modulus E and Poisson's ratio v. The disk is spinning about its axis with angular velocity ω. Starting with the equations of equilibrium in polar coordinates and assuming plane stress conditions, obtain the expressions for distribution of stress components σ_θ and σ_r, and the radial displacement u, in the disk.

[25 points]

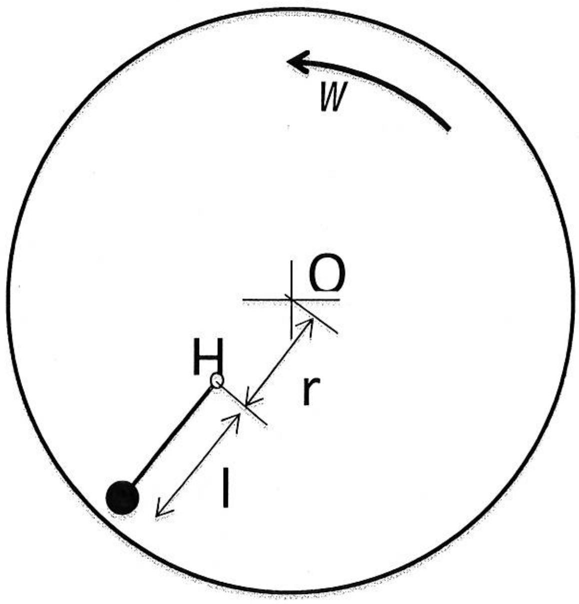
January 2015

DES2: Kinematics and Dynamics of Machines
Ph.D. Qualifying Examination

General instructions:

- Write clearly and legibly. Answers should be 'to-the-point', but should clearly show the important steps in the solution, including all assumptions and approximations.
- No queries will be entertained. If any information appears to be missing, make suitable assumptions and state them clearly. Clearly mark/highlight such assumptions, if any, by enclosing them in a box.
- The exam is 'closed book, closed notes.'
- All questions are compulsory.
- Total = **50 points**. Time = **3 hours**.

1. **[10 marks]** Consider a pendulum with a point mass m and length l, attached to a hinge point H on the disc rotating at uniform angular velocity ω in the **horizontal plane**. Position of pendulum is as shown in the figure. All joints are frictionless. A person standing on the surface of the disc at center point O observes the pendulum motion.
   a. The person disturbs pendulum by very small perturbation δ and pendulum starts oscillating. Derive expression for frequency of oscillation of pendulum.
   b. Now the perturbation given is really a large angle (but < 180 deg). Derive a nonlinear equation governing dynamics of oscillations of pendulum. Show that under special case of pendulum angle (wrt to the disc) θ tending to zero, you get the frequency of oscillation indeed as found in part a. above.
   c. Find out the reaction force at hinge H in case b.



c. Determine velocity $\dot{\psi}$ in terms of $\dot{\phi}, \phi$ and other parameters.
d. Determine expression for $\ddot{\psi}$ in terms of $\ddot{\phi}, \dot{\phi}, \phi, \dot{\psi}, \psi$ and other parameters
e. Draw free body diagram showing forces of interaction between pallet pin and the escape wheel teeth (assume coefficient of friction to be μ). Outline steps to derive equation of motion if applied torque on the escape wheel is T.

3. [15 points] Consider a train comprising N wagons, running with a constant speed v on a horizontal track. Each wagon has a mass M, and is connected to its adjacent wagon through a coupling of stiffness k. We want to model the horizontal motion of this train due to any excitations.

a. Construct a suitable model for the horizontal motion of the wagons. Clearly state your assumptions and approximations. How many degrees-of-freedom does this system have?

b. Select a suitable set of coordinates governing the motion of this system. Now derive the expressions for the kinetic energy T and potential energy V of the system in terms of these coordinates.

c. Derive the equations of motion of the above system. Rewrite them in matrix-vector form. [Hint: derive the equation for the i^{th} wagon, and note the pattern relative to the adjacent wagons.]

d. Now consider a subset of the above system comprising only two wagons. Obtain the equations of motion for this system by either reducing the result obtained in the last step or deriving a new. Write the eigenvalue problem for this system, and solve to obtain the eigen solutions. Physically interpret what the natural frequencies and modeshapes signify?

e. State the orthogonality condition of the eigenvectors relative to the mass and stiffness matrices. Do the eigen-vectors obtained in part (d) above satisfy this condition? Clearly justify your answer.

f. Consider the two-wagon train of part (d) above to be stationary. A third wagon that is to be attached to this train travels from the right at speed v towards the stationary train, and impacts the right-most wagon ('Wagon 2') through its coupler, such that the new wagon comes to a rest, and Wagon 2 is imparted an instantaneous speed ev (where e is a constant, $0<e<1$). [This practice is similar to what is known in the railways as 'loose shunting'.] Sensitive/fragile equipment is stored in Wagon 1 (i.e. the left-most wagon) that can only withstand a maximum shock, defined in terms of the acceleration amplitude A_0. Using modal analysis, determine the maximum speed at which Wagon 3 can be loose shunted onto the stationary train without risk to the equipment?

4. **[10 points]** An aircraft is moving horizontally on the runway for take-off. As it approaches its take-off speed, it 'rotates' such that its nose (or longitudinal axis) now points up at an angle θ to the horizontal. It is powered by a single gas turbine engine located within its body along its longitudinal axis, that has rotational inertia J and that rotates at an angular speed Ω. In this position, only the two rear wheels of the undercarriage are in contact with the ground. A vertical lift force and a horizontal drag force act through a point on the longitudinal axis known as the centre of pressure that coincides with the centre of mass. Assume a suitable geometry for this aircraft, with appropriate symbols.

a. Derive the direction and magnitude of the reaction forces between the wheels and the ground, as the plane rotates from horizontal to the angle θ?

PhD Qualifying Exam – 2014
Manufacturing Processes-I

- Full marks 100
- No books or notes allowed
- Please make any suitable assumption if required

1. Consider that an arc welding process with an arc power of 2.5 kVA has to be used for joining of two plates in a typical single V-groove butt joint configuration in a single-pass. The included angle of the V-groove is 60° and the groove extends up to the complete thickness of the plates. Data given for the welding conditions include: plate thickness = 3 mm; melting temperature, average thermal conductivity and diffusivity of workpiece material = 1530°C, 43.6 W/m°C and 1.2 x 10^{-5} m^2/s; and ambient temperature = 30°C. Estimate the maximum possible welding speed that can be used if the process efficiency is 80%. Given that the required rate of heat input (Q) for such case is

$$Q = 8 \cdot k \cdot (T - T_0) \cdot h \cdot \left(\frac{1}{5} + \frac{vw}{4\alpha}\right)$$

where k and α are respectively the average thermal conductivity in W/m°C and diffusivity in m^2/s, v is welding speed in m/s, T_0 is ambient temperature, T is the melting temperature, h is the plate thickness (in m) and w is the width (in m) of the weld.

[15 Marks]

2. Which will lead to a deeper weld pit and a cleaned weld surface (provide one line reason for each).
   i. Non-consumable tungsten arc welding (electrode -ve) with inert shielded gas
   ii. Non consumable Tungsten arc welding (electrode +ve) in vacuum

[5 Marks]

3. Which of the following welding processes cannot be used for (a) Overhead welding, (b) Long-continuous welding and (c) welding of materials requiring post-weld surface protection? Provide a one line explanation.
   i. Gas metal Arc welding (GMAW)
   ii. Shielded Metal Arc welding (SMAW)
   iii. Submerged Arc welding (SAW)

[5Marks]

4. A steel sheet of 10 mm initial thickness and feed velocity 2cm/s is being deformed to 5mm final thickness via cold rolling process with roll radius 5mm. It is known that the neutral point is located exactly at the midpoint of sheet/roll contact interface; calculate the rotational speed of rolls.

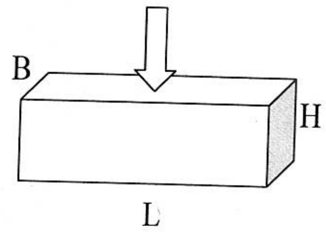
[10Marks]

5. An isotropic elastic cubic block under perfect hydrostatic compressive load does not deform at all, irrespective of the amount of load applied. Is this possible? If yes, explain when. Derive if required. **[10 Marks]**

6. Draw 2D yield surfaces for Rankine, Tresca and Von-Mises yield criteria all together in a single plot. For the 3D stress tensor $\sigma_{ij} = \sigma_o x^{3-j}\delta_{ij}$, plot effective (a) Rankine (b) Tresca and (c)Von-Mises yield stresses as a function of increasing x in $[0,1]$. Note that δ_{ij} is Kronecker delta function, defined as $\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$

[15 Marks]

7. Calculate total load required to initiate forging of a block (see figure, L=2m,B=50m,H=1m), for the following cases:
   a. No friction
   b. Friction coefficient=0.5
   c. Sticking friction



Given: (i) Uniaxial yield strength = 500 MPa

(ii) Forging load distribution for coulomb and sticking friction:

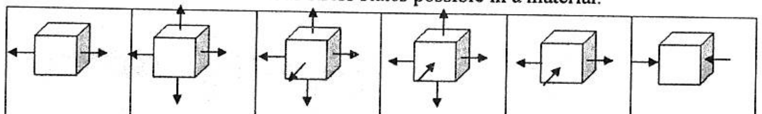
$$P_{coulomb}(x) = \frac{2\sigma_o}{\sqrt{3}} e^{\left(\frac{2\mu}{h}(a-x)\right)} \qquad P_{sticking}(x) = \frac{2\sigma_o}{\sqrt{3}}\left(1 + \frac{a-x}{h}\right)$$

Draw load distribution profiles for cases (a), (b) and (c).

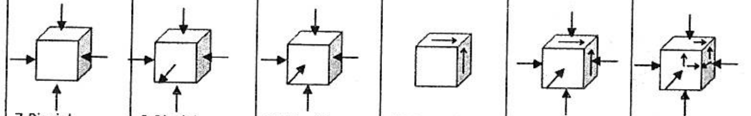
[15 Marks]

8. Which of the following stress state is seen in the metal forming operations given in Table 2? **[10 Marks]**

Table1: Various stress states possible in a material.

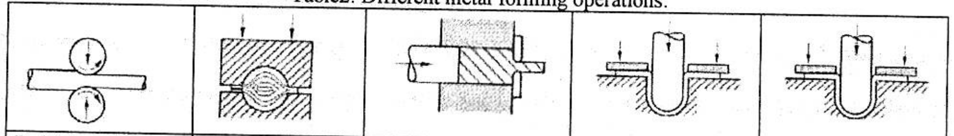


1. Simple uniaxial tension	2.Biaxial tension	3. Tiaxial tension	4. Biaxial tension, compression	5. Biaxial tension and compression	6.Uniaxial compression

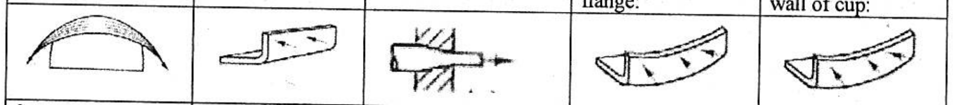


7.Biaxial compression	8.Biaxial compression, tension	9. Triaxial compression	10 Pure shear	11. Simple shear with Triaxial compression	12. Biaxial shear with Triaxial compression

Table2: Different metal forming operations:



Rolling:	Forging:	Extrusion:	Deep drawing at flange:	Deep drawing in wall of cup:



Stretching:	Straight bending:	Wire drawing:	Convex bending at the bend:	Convex bending at the flange:

9. Molten metal is being filled from a pouring basin into a rectangular mould of height h_m and base area A_m as shown in the figure below. h_t is the supply height in the pouring basin. The velocity at the entry of the gate may be assumed to be uniform. As the fluid fills the mould, the height h increases. Assuming that the free surface is flat and horizontal, determine the time required to fill the mould in terms of the cross-sectional area of the gate is A_g, A_m, h_m, h_t. Assume that the h_t, is constant throughout the filling process. Assume that h=0 at the beginning of the filling process. **[15 Marks]**

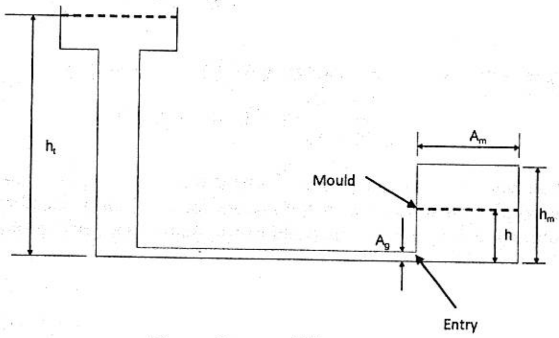


Figure: Bottom filling mould

PhD Qualifying Exam – 2014

Manufacturing Processes-II

- Full marks 100
- No books and notes are allowed but two A4 sheets of handwritten formulae are allowed
- Please make any suitable assumption if required

1) A cutting fluid company aims to develop a cutting fluid that could reduce power consumption by 20% with their fluid as compared with dry cutting for a typical cutting operation described below:
   A cylindrical workpiece made of carbon steel having 250 mm in diameter is turned on lathe with $+5^0$ rake cutter. This process is done in one pass to reduce the workpiece diameter by 5 mm. It takes 30 seconds to complete the entire axial length of 150 mm at a spindle speed of 1000 rpm. The power requirement of spindle motor for the above operation is 5 hp when no coolant is used. The coefficient of friction in dry contact is measured as 0.3. Find the following:
   a) The cutting force required with the fluid and the shear strength of the material used. [10]
   b) The coefficient of friction at tool-chip interface the new cutting fluid should yield. [10]

   Assume this process to be orthogonal and the material to be perfectly plastic. Lee-Shafer shear angle relationship is given by $\phi+\beta-\alpha=\pi/4$. 1 hp = 746 W.

2) Blind holes 10 mm diameter; 15 mm deep are drilled in a component. The drill has a point angle of 120 degrees. Axial feed during the operation is set to 0.2 mm/rev. An experiment was conducted in the shop to study the effect of spindle speed (N) on the number of holes (H) produced before the tool change.

Spindle Speed (N) rpm	No of holes produced (H)
318	802
1120	213

   a) Calculate the Taylor's tool life equation ($VT^n = C$) from the above data. (Choose V – m/min, T – min) [10]
   b) Shop engineers feel that it will be more useful to get an equation between **H** and **N**. Suggest the form of equation and compute its coefficients. Draw a graph to show the nature of variation of the curve (H-N) approximately [10]

Ph.D. Qualifying Examination (19^{th} Dec 2014)

Fluid Mechanics (TFE-1); Maximum marks: 100; Time: 9:30 AM-12:30 PM

This is a **closed book, open hand-written notes** examination. Answer all questions. Make suitable assumptions if required and state them clearly. Assume fluid as Newtonian and flow as incompressible for all problems given below.

Problem 1: Governing equation [15 Marks]:

a) Write down the 3D unsteady form of continuity and momentum equation in Cartesian coordinate system, for incompressible flow; and discuss the physical interpretation of the various terms.
b) Simplify the momentum equation for a steady and inviscid flow.
c) Write the momentum equation obtained in (b) for gravity acting along –y direction.
d) Using equation of a streamline and equations obtained in (c), derive Bernoulli equation.
e) List all assumptions for which Bernoulli equation is valid.

Problem 2: Plane Couette-Poiseuille flow[20 Marks]: *Plane Couette flow* is generated by placing a viscous fluid between two plates and moving one plate (say, the upper one) at a velocity U. *Plane Poiseuille flow* is generated by forced flow of a viscous fluid between two plates which are stationary. *Plane Couette-Poiseuille* flow corresponds to the flow generated by the combination of the upper plate motion and forced flow. For Plane Couette-Poiseuille flow, consider 2D, steady, incompressible *fully-developed* flow (density ρ, viscosity μ)in the channel of height h. Consider two dimensional coordinates in which x-axis is along the channel and y-axis is normal to x axis. Origin of the coordinate system lies on the bottom channel wall. The bottom plate is fixed with time while top plate is moving with velocity U. The velocities u and v are in the respective x and y directions:

a) Simplify Navier-Stokes equations for the present case. Neglect gravity.
b) Prove that pressure gradient, dp/dx, along x direction, is constant.
c) Derive expression for $u(y)$.
d) Draw velocity profiles for the following three cases:
   (i) $dp/dx<0$ (ii) $dp/dx = 0$ (iii) $dp/dx>0$
e) Find the critical value of dp/dx at which backflow will start near the bottom plate. Justify the criterion you use.

Problem 3: Non-Dimensional expression for Local Drag and Lift Coefficient[20 Marks]: Consider a 2D free-stream flow across a cylinder of square cross-section (of dimension $b \times b$), with free-stream velocity u_∞.

a) Start with the general expression for drag and lift force on a solid surface, given in dimensional form, as $D = \oiint (\hat{n}.\sigma).\hat{i}ds$ and $L = \oiint (\hat{n}.\sigma).\hat{j}ds$, where $\hat{n}$ is the unit vector normal to a surface and σ is second-order stress tensor. Consider square cylinder made up of left/right (vertical) and top/bottom (horizontal) surfaces, denoted by l, r, t and b, respectively, with x- and y-axis along top/bottom and left/right, respectively. Using the complete expression of stress-tensor σ as a function of velocity-gradients and pressure, draw a figure of square cylinder and show expanded form of $(\hat{n}.\sigma)$ for the various surfaces of the cylinder.

b) Derive expressions for the non-dimensional forces (coefficient of drag $C_D = 2D/(\rho u_\infty^2 b)$ and lift $C_L = 2L/(\rho u_\infty^2 b)$; where D is drag and L is lift force) acting on the cylinder in terms of local non-dimensional pressure ($P = 2p/(\rho u_\infty^2)$, velocity gradients ($\partial U/\partial Y$ and $\partial V/\partial X$ where $U = u/u_\infty$, $V = v/u_\infty$, $X = x/b$ and $Y = y/b$) and Reynolds number ($Re=\rho u_\infty b/\mu$). Show all the steps for the derivation of non-dimensional equation for C_D and C_L.

Problem 4: Drag and Thrust Generation[20 Marks]:

a) For free-stream flow across a body with flow separation, show the streamlines considering the body as square-cylinder, circular-cylinder and airfoil; of same frontal area. Discuss the reasons for flow-separation and role of wake-formation on the generation of drag force; and relative magnitude of drag force for the various bodies.

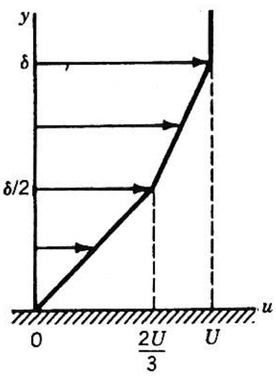
b) Under this condition, a human body also experiences a drag force; however, it changes to thrust (negative drag) during swimming. Draw the stream-wise velocity profile at a same stream-wise location behind the human body for the drag as compared to thrust generating case. Discuss the fluid-dynamic reasons for thrust generation during swimming and compare it with that of fish and motor-boat locomotion.

c) Discuss the effort (propulsive power) required for propulsion/swimming of a human body *behind* free-stream flow across (a) stationary object and (b) moving motor boat; as compared to isolated swimming.

Problem 5: Boundary Layer Flow[15 Marks]: A laminar boundary layer profile is approximated by the two straight-line segments indicated in the figure below. U is the free stream velocity.

a) Use the momentum integral equation to determine the boundary layer thickness $\delta(x)$.

b) Also determine the wall shear stress $\tau_w(x)$.



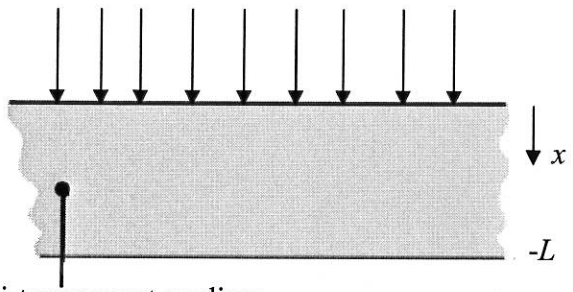
Problem 6: Vorticity Dynamics [10 Marks]: Take a plane polar control-volume of fluid of dimension dr and $rd\theta$. Evaluate the right-hand side of Stokes theorem $\int \vec{\omega}.d\vec{A} = \int \vec{u}.d\vec{s}$. Thereby, derive the expression for vorticity ω_Z in polar coordinates, with $A_Z = rd\theta dr$ and ds as the elemental surface areas of the CV. In the 2D Cartesian coordinate system, derive the vorticity transport equation from the X- and Y- momentum equation; and discuss the physical significance of the various terms in the equations.

Ph.D. Qualifying Examination 2014
Heat Transfer

1. The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form $T(x)$ given as

$$T(x) = -\frac{A}{ka^2} e^{-ax} + Bx + C$$

where A, a, B, and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $q(x)$. For your reference, the schematic of the problem statement is shown below:



For the above case, find: (a) The expressions for the heat flux at the front and the rear surfaces; (b) Heat generation rate, and (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L and K. Clearly mention the assumptions made by you. (10)

2. Figure below shows the triangular cross section through a long bar. One of the two mutually perpendicular sides is maintained at T_b and another one is kept at T_0. The hypotenuse is perfectly insulated. Determine analytically the temperature distribution for steady conduction in the rectangular area, $T(x,y)$. (15)

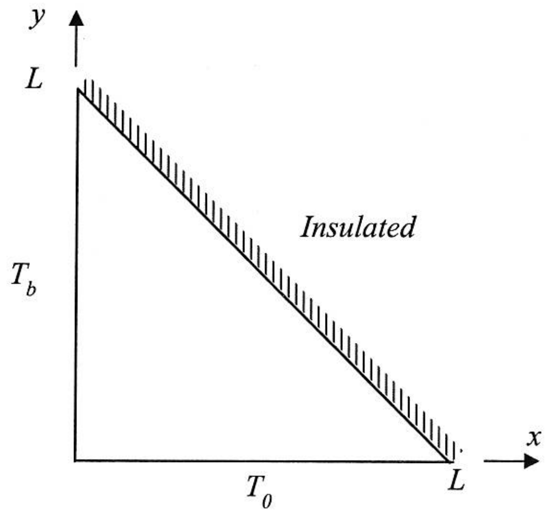


Figure: Schematic diagram of triangular cross section

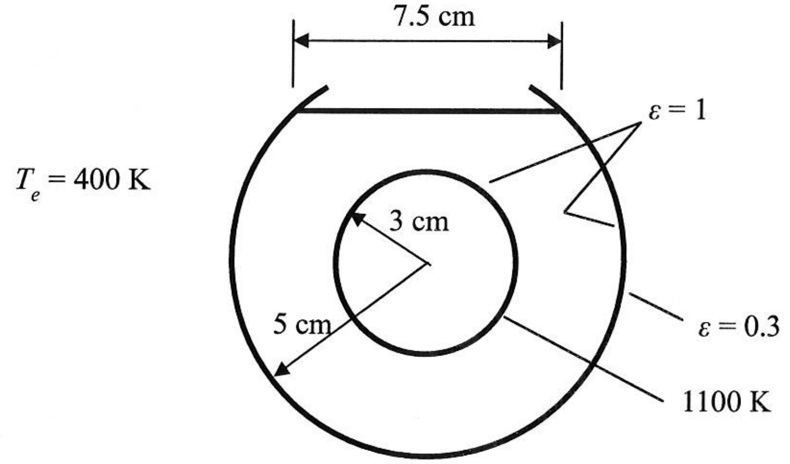
3. A thin wire is drawn by passing heated metal through a set of dies. The velocity of drawing is maintained constant at V_0 and the temperature of the wire at the dies is fixed at T_0. The wire then passes through air with temperature T_f and heat transfer coefficient h for some distance before it is rolled onto a spool at temperature T_L. The distance L between the spool and the dies must be enough to permit cooling of wire to temperature T_L.
   a) Obtain an expression governing the wire temperature as a function of the distance x from the dies.
   b) Now consider the radius R of the steel wire to be 1 mm and its length L to be equal to 10 m. This wire is being drawn with velocity $V_0 = 1$ m/s. If $T_0 = 500°C$, $h = 100$ W/m^{2}K, find the value of temperature at $x = L$? (15)

4. A slab of width L is initially maintained at temperature T_0 . At $t = 0$, the temperature of the right wall of the slab is raised to temperature T_L. Please note that the left wall of the slab is always maintained at the initial temperature T_0.
   a) Starting with non-dimensionalizing the governing equation, find out the non-dimensional transient temperature distribution of the wall. (You may restrict the solution up to the first term of the series.).
   b) It is proposed to determine the thermal diffusivity (α) of the material of the slab. For this, you are being provided with the experimentally measured temperatures (non-dimensionalized) at the mid-point of the slab at three time instants given as below:

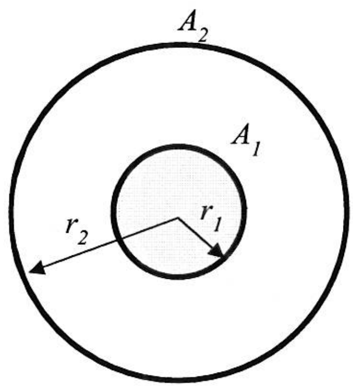
T (Non-dimensional)	0.06	0.19	0.240
t (seconds)	600	1200	1800

Assume the width of the slab L = 5 cm for this case. Suggest a methodology on how would you proceed ahead for the determination of thermal diffusivity. Go to the maximum possible extent and discuss in detail the methodology adapted. (15)

5. A black 6 cm diameter sphere at a temperature of 1100 K is suspended in the center of a thin 10-cm-diameter partial sphere having a black interior surface and an exterior surface with a hemispherical total emissivity of 0.3. The surroundings are at 400 K. A 7.5-cm-diameter hole is cut in the outer sphere. What is the temperature of the outer sphere? What is the Q being supplied to the inner sphere? *(For simplicity, do not subdivide the surface areas into smaller zones.)* (17+3)

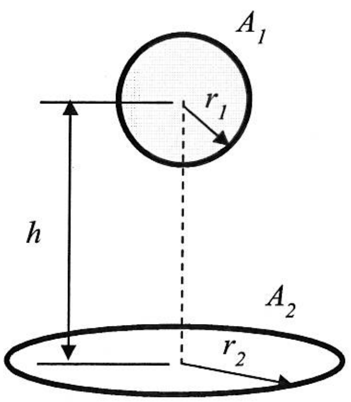


Given:



Concentric spheres,

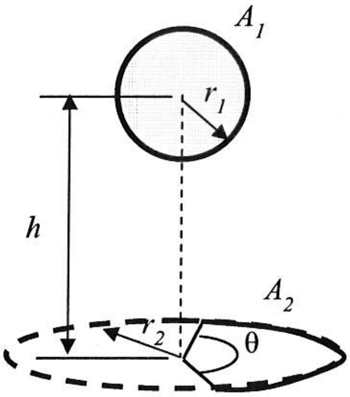
$$F_{2-1} = \left(\frac{r_1}{r_2}\right)^2$$



Sphere of radius r_1 to disk of radius r_2; normal to centre of disk passes through centre of sphere.

$$R_2 = \frac{r_2}{h}$$

$$F_{1-2} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + R_2^2}}\right)$$



Sphere to sector of disk; normal to centre of disk passes through centre of sphere.

$$R_2 = \frac{r_2}{h}$$

$$F_{1-2} = \frac{\theta}{4\pi}\left(1 - \frac{1}{\sqrt{1 + R_2^2}}\right)$$