DEPARTMENT OF MECHANICAL ENGINEERING, IIT Bombay

Ph.D. Qualifying Examination – 11th July 2017

Heat Transfer

Maximum marks: 90 Passing Marks: 36

Instructions:

1. All questions are compulsory. This is an **open book/closed notes** examination.
2. **Please start each new question on a new page and keep all subparts of a question together.**
3. Maximum marks per question are given in parenthesis.
4. Use of calculator is permitted.
5. Make suitable assumptions where necessary and state them clearly.
6. Make sure to cancel any unwanted work clearly, so that it is not graded.

[14 marks] Question 1: The temperature distribution *T(x,t)* in a 2-m long brass rod is governed by the following equation:

$$\alpha^2 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, \qquad (0 < x < 2, t > 0)$$

where $\alpha^2 = 2.9 \times 10^{-5}$ m^2/s. The following boundary and initial conditions apply:

$$T(0,t) = T(2,t) = 0, \qquad (t > 0)$$

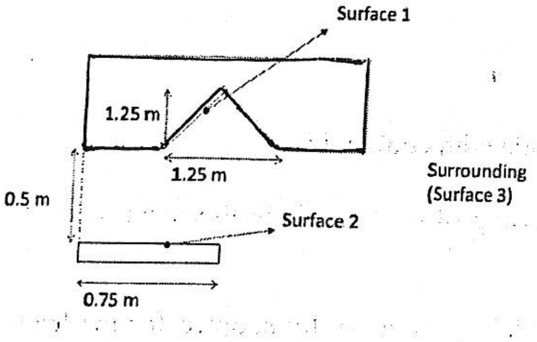
$$T(x,0) = \begin{cases} 50x, & (0 < x < 1) \\ 100 - 5x, & (1 < x < 2) \end{cases}$$

(a) Determine the solution *T(x,t)*

(b) Compute the temperature of the midpoint of the rod at the end of 1 hour

(c) Compute the time required for the temperature of the midpoint to reduce to 5°C.

[20 marks] Question 2: A long beam has a triangular cavity trenched along one edge (as shown below) that is about 1.25 m deep and about 1.25 m wide at its base. A parallel plate that is about 0.75 m wide is positioned below the trenched beam at a distance 0.5 m, as shown in the Figure. The right edge of the plate is directly under the center of the triangular cavity and left edge is aligned with left edge of trenched beam. Determine the view factor from the triangular enclosure to the surroundings. Note that the entire triangular trench is treated as surface 1.



[6 marks] Question No. 3: The roof of a car in a parking lot absorbs a solar radiant flux of 800 W/m^2, while the underside is perfectly insulated. The convection coefficient between roof and ambient air is 12 W/m^2-K and the ambient air temperature is 20°C.

(a) Neglecting radiation exchange with the surroundings, calculate the steady-state temperature of the roof.

(b) Calculate the temperature of the roof if its surface emissivity is 0.8.

[20 marks] Question No. 4: The following question relates to flow over a flat plate.
Consider the case of boundary layer flow over a flat plate. The free stream velocity and temperature are U_∞ *and* T_∞ respectively. The *x* coordinate is along the plate and *y* is normal to the plate. The plate temperature at any location can be taken as $T_w(x) > T_\infty$. Note that $T_w(x)$ may be constant or varying. As is the accepted convention, $u \sim U_\infty$ *and* $x \sim L$. The hydrodynamic and thermal boundary layer thicknesses are given by δ *and* δ_t respectively.

For a case of a fluid with very **high** Prandtl number, proceed systematically in performing the following analysis.

a. List the assumptions made in the analysis of boundary layers. From the continuity equation, estimate how the *y* component of the velocity, *v* scales. First, sketch the boundary layer profiles and label the velocities and temperatures. **[5 marks]**
b. Simplify the x-momentum equation using boundary layer approximations, then use the scales for u, v, x *and* y to show that $\frac{\delta}{L} = Re_L^{-0.5}$ **[5 marks]**
c. Simplify the energy equation using boundary layer approximations, then use the scales, as well as results from part (b) to show that $Nu_L \sim Re_L^{1/2} Pr^{1/3}$ **[10 marks]**

[15 marks] Question 5: Consider the case of steady, fully developed laminar flow of a viscous fluid with constant thermophysical properties between two parallel plates. The width into the plane of the paper is *b* and the plates are separated by a distance *'2d'* between them. The coordinate system is such that *y* = 0 represents the mid plane between the two plates. Appropriate conventional symbols may be used for thermophysical properties etc.

The solution of the momentum equation gives the velocity distribution to be

$$u(y) = -\frac{d^2}{2\mu}\left(\frac{dp}{dx}\right)\left[1 - \left(\frac{y}{d}\right)^2\right]$$

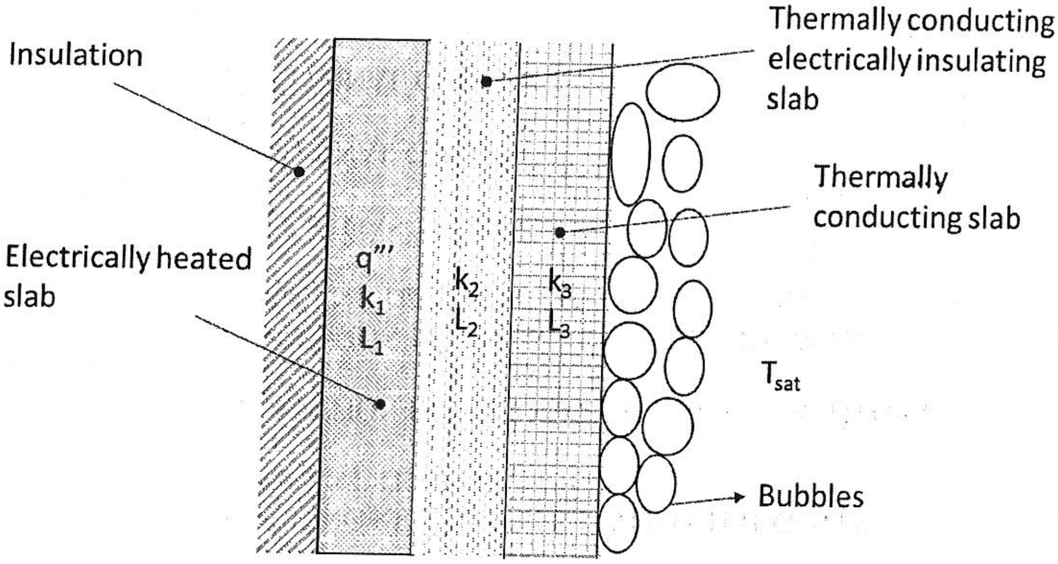
With this as the starting point, consider the case where the top and bottom plates are subjected to uniform heat fluxes $q''_{w,1}$ and $q''_{w,2}$ respectively. Proceed systematically as given below.

a. List all assumptions you will make to obtain the temperature distribution for the fluid. **[2 marks]**
b. Starting with a suitable control volume, perform an energy balance to obtain the expression for the bulk mean temperature of the fluid $T_m(x)$ in terms the heat fluxes and the temperature at the inlet $T_{m,i}$ **[5 marks]**
c. Start with the full form of the energy equation, simplify the same giving reasons and obtain the differential equation for the temperature distribution of the fluid $T(y)$. State the boundary conditions needed to solve the same. Do not attempt to solve the same. Can the flow be treated as thermally fully developed? Explain very briefly **[6 marks]**
d. If the wall heat fluxes are both equal to q''_w, sketch the variation of the fluid temperature at a given axial location. Can the flow be thermally fully developed under this condition, explain briefly. **[2 marks]**

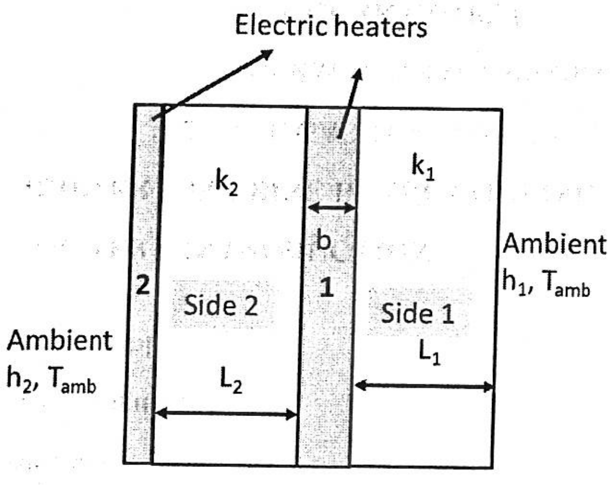
[15 marks] Question 6: Draw the temperature profiles for the following situations. Clearly state your assumptions.

a. An infinitely long slab is suddenly dipped in a large tank and the slab is generating heat volumetrically. Assuming steady state condition prevails draw the one-dimensional temperature profile if (i) heat transfer coefficient is very large and (ii) heat transfer coefficient is small. **[4.5 marks]**

b. Assuming steady-state conditions to prevail draw the temperature profile for the schematic shown. Assume heat flow to be one-dimensional and nucleate boiling occurs on the external surface. **[4.5 marks]**



c. Aim here is to design an electric heater (1) that transfers heat from one surface through a solid plate of thickness L_1 and thermal conductivity k_1, see Figure below, for a specific purpose. Any heat transfer from the other surface of the heater is considered to be a heat loss, and is not desired. Draw the ideal profile that would allow zero heat loss to ambient from side 2 when a guard heater (2) was present as shown in Figure (b)? (Assume negligible ambient convective loss through backside of heater 2) **[6 marks]**



(b)

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PhD Qualifying Exam – July 2017 - (Manufacturing Processes-I)
Department of Mechanical Engineering,
IIT Bombay

Marks 80

1. Feeders supply liquid metal to compensate volumetric shrinkage during casting solidification. For this, a feeder has to remain in liquid state longer than the casting, and the amount of liquid metal in feeder (about 15% for uninsulated cylindrical feeders) has to be greater than casting shrinkage volume. The relative solidification time of simple shapes can be compared using the square of their geometric modulus, which is given by the ratio of their volume to cooling surface area. The yield is given by the ratio of only part weight to full casting weight (including feeders).
   (a) Design a cylindrical feeder for the above casting, with 20% higher modulus (than the casting), and height/diameter ratio of 1.5. Assume feeder bottom is in contact with casting (hence negligible heat transfer). **[10]**
   (b) Check the shrinkage volume criteria, given the volumetric contraction of the cast metal is 5%. Can this feeder be used to feed another casting? If no, why? If yes, what will be the yield improvement. **[10]**

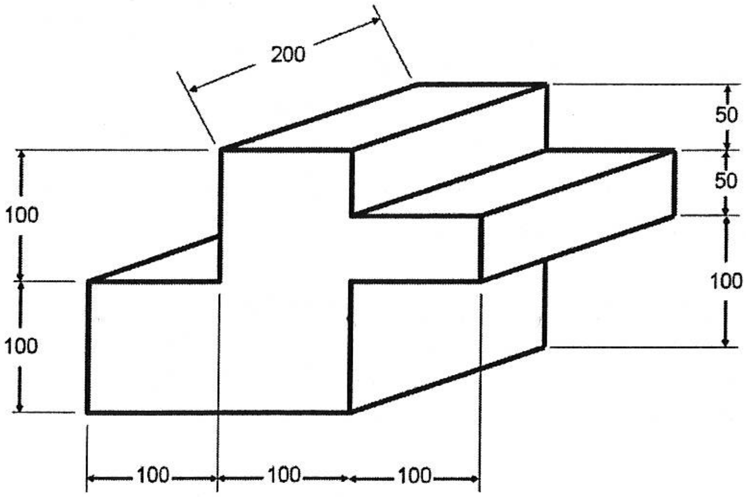

Figure 1

2. In a sintering process, particle growth occurs due to motion of atoms across grain boundaries via a process known as Ostwald ripening in which larger particles grow on the expense of smaller particles. This process leads to overall increment in mean particle radius as follows:

$$\langle R\rangle^3 - \langle R_o\rangle^3 = kt$$

where $\langle R \rangle$ is the mean particle radius at time t, $\langle R_o \rangle$ is the initial mean particle radius and k is a constant associated with the sintering process. Sintering is done for a system of particles in which 30% particles are of size 5 μm and 70% particles are of size 10 μm, and size distribution of particles at time $t = 10s$ is given by $P(R) = \frac{3}{1720} R^2, R \in (2\mu m, 12\mu m)$. Find out mean radius of particles at $t = 50s$. **[10]**

3. The following expression gives temperature as a function of time (t) and the distance (r) from the weld centerline during welding of a thick plate:

$$T(r,t) = T_o + \frac{q}{2\pi\lambda vt} \exp\left(-\frac{r^2}{\alpha t}\right)$$

   (a) Based on the equation above, derive an expression for the time it takes to reach maximum temperature at a given distance r from the heat source during arc welding of a thick plate. **[4]**

   (b) Estimate maximum temperature at a point 5 mm away from the heat source during welding of carbon steel (Melting Point $\approx$ $1500K$). Material and process parameters are as follows: thermal conductivity (λ) = 50W/m/K, density(ρ) = 7850kg/m3, specific heat (C_p) =0.45kJ/kg-K, initial temperature (T_o) of the plate = 300K, weld current (I) = 150 A, voltage (V) = 20 volts, welding speed (v) = 2.5mm/s, weld efficiency= 0.6. **[3]**

   (c) Estimate the distance at which temperature would reach the melting point **[3]**

4. In the course of deep drawing of a part, fracture occurs

   (a) at the early stage of the draw, and

   (b) towards the end of the draw

   With clear pictures, provide explanation of likely source(s) of problem(s) and possible remedies in each case. **[10]**

5. It is suggested that the punch force during deep drawing operation is influenced by (a) blank diameter, (b) clearance between punch and die, (c) workpiece alloy properties, (d) blank thickness, and (e) blank-holder force. Explain each of the above by a clear picture and maximum three points. **[10]**

6. The schematic in Figure 2 illustrates sheet drawing process in plane strain condition through a pair of wedges. *P* is the pressure acting on metal sheet via die. Wedge angle is α.

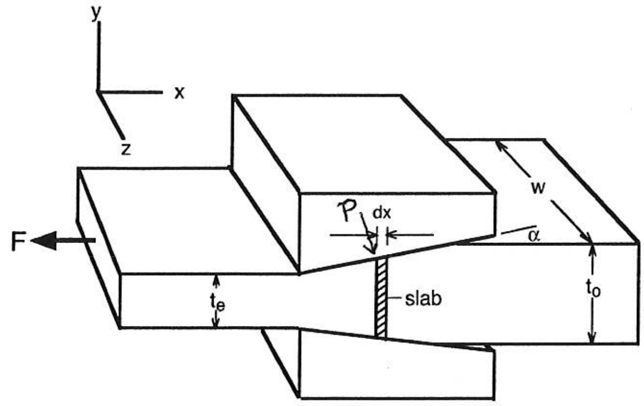

Figure 2

(a) Draw the differential slab element with all the forces / stresses acting on it. **[5]**
(b) Write equation for force balance along the X direction. **[5]**
(c) Under equilibrium condition, determine σ_x as a function of *P* by using Tresca criterion of yield. **[5]**
(d) Establish relation between stress with which sheet is drawn and *P*. State your assumptions clearly. **[5]**

PhD Candidacy Qualification Exam – Solid Mechanics

6^{th} July 2017 (Thursday)

Time Allowed = 180 minutes; Maximum Marks = 50; Pass Marks = 20

READ ALL THE INSTRUCTIONS BELOW, BEFORE ANSWERING THE QUESTIONS.

- This question paper has 2 pages
- This is a close book and closed notes exam.
- Answer questions (A1, A2), (B1, B2) and (C1, C2) in separate booklets, by clearly writing your roll number on all answer sheets you submit for grading.
- The number of the question and sub-part, if any, need to be clearly mentioned against your answer. Cross-out sections of the answers you do not want to be graded.
- Only regular scientific calculators are allowed.
- No queries on the question paper are entertained during the exam. If necessary, made and state appropriate assumptions, with proper justification for use, to solve the problem.

There are 6 (six) questions in this paper. You should attempt a maximum of 5 (five) questions of your choice for grading. Make sure to cross out the answer(s) you do not want to be evaluated.

A1. The right-angle bar of Figure 1 is built in at A and rests on a frictionless support at C. Both segments of the bar have flexural rigidity *EI*. Use Castigliano's second theorem to find the horizontal displacement at C, when the bar is loaded by a moment M_0 at B.
[10 Marks]

A2. A cantilever beam of length, *L* and cross-section area, *A*, and second area moment of inertia, *I*, is subjected to a compressive load on one end, as shown in figure 2. The other end of the beam is fixed into the wall. The elastic modulus of the beam is *E*. At the free end of the beam, two point loads, *P* and *F*, act at the area centroid, such that they are parallel and perpendicular to the beam, respectively. Using the Euler buckling equation given below, find the critical buckling load, P_{cr} and the maximum bending moment M_{max} at P_{cr}. The Euler buckling equation is given as $EI\frac{d^2w(x)}{dx^2} = M(x)$, where *w(x)* is the transverse displacement and *M(x)* is the moment at any cross-section, along the length of the beam.
[10 Marks]

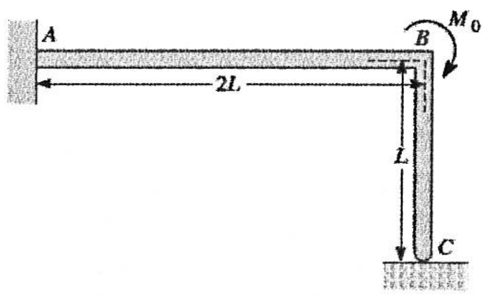

Figure 1

Figure 2

B1. The 2D displacement fields in a linear elastic (Young's modulus E and Poisson's ratio v) body are given by $u(x,y) = ax^2+cy^2$, $v(x,y) = bxy$, $w(x,y)=0$; where, *u,v,w* are displacements in the *x,y,z* directions, respectively, and a,b,c are constants with appropriate units. Determine the interrelationship(s) between the constants *a,b,c* in order for the given displacement fields to be valid.
(10 points)

B2. Consider the following bent elastic beam ABCD with the dimensions shown in Figure 3. The beam has uniform flexural rigidity EI. Two moments each of magnitude M are applied to the beam at points A and D. Note that the initial distance between points A and D is a. Determine the final distance between A and D after the application of moments. Ignore axial deformations. Do not use energy methods to solve this problem. Clearly show the free body diagrams of all relevant members. Assume small deformations/angles.
(10 points)

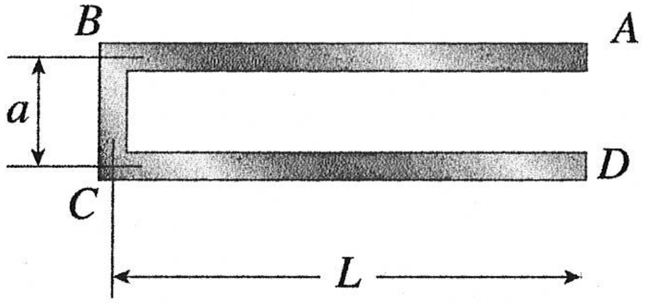

Figure 3

C1. Consider a hollow circular disk of inner radius a = 20 mm, outer radius b = 100 mm and thickness t = 1 mm made of a homogeneous, isotropic, linear elastic material with mass density ρ = 4000 kg/m^3, Young's modulus E = 100 GPa and Poisson's ratio ν = 0.2. The disk is spinning about its axis with angular velocity ω = 10000 rad/s. The disk is constrained in the radial direction at both the inner and outer radii, respectively. State the boundary conditions of the disk. Starting with the equations of equilibrium in polar coordinates and assuming plane stress conditions, obtain the expressions for radial displacement u_r and stress components σ_θ and σ_r in the disk. Also, find the value of the stress ratio σ_θ/σ_r at the inner and outer radii, respectively.
[Useful data:
The equilibrium equation in polar coordinates for axisymmetric problems is

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}+b_r=0.$$

The stress-strain relations in polar coordinates for plane stress conditions are

$$\sigma_r=\frac{E}{1-\nu^2}\left[\varepsilon_r+\nu\varepsilon_\theta\right];\sigma_\theta=\frac{E}{1-\nu^2}\left[\varepsilon_\theta+\nu\varepsilon_r\right].$$

The strain-displacement relations in polar coordinates for axisymmetric problems are

$$\varepsilon_r=\frac{du_r}{dr};\varepsilon_\theta=\frac{u_r}{r}.$$

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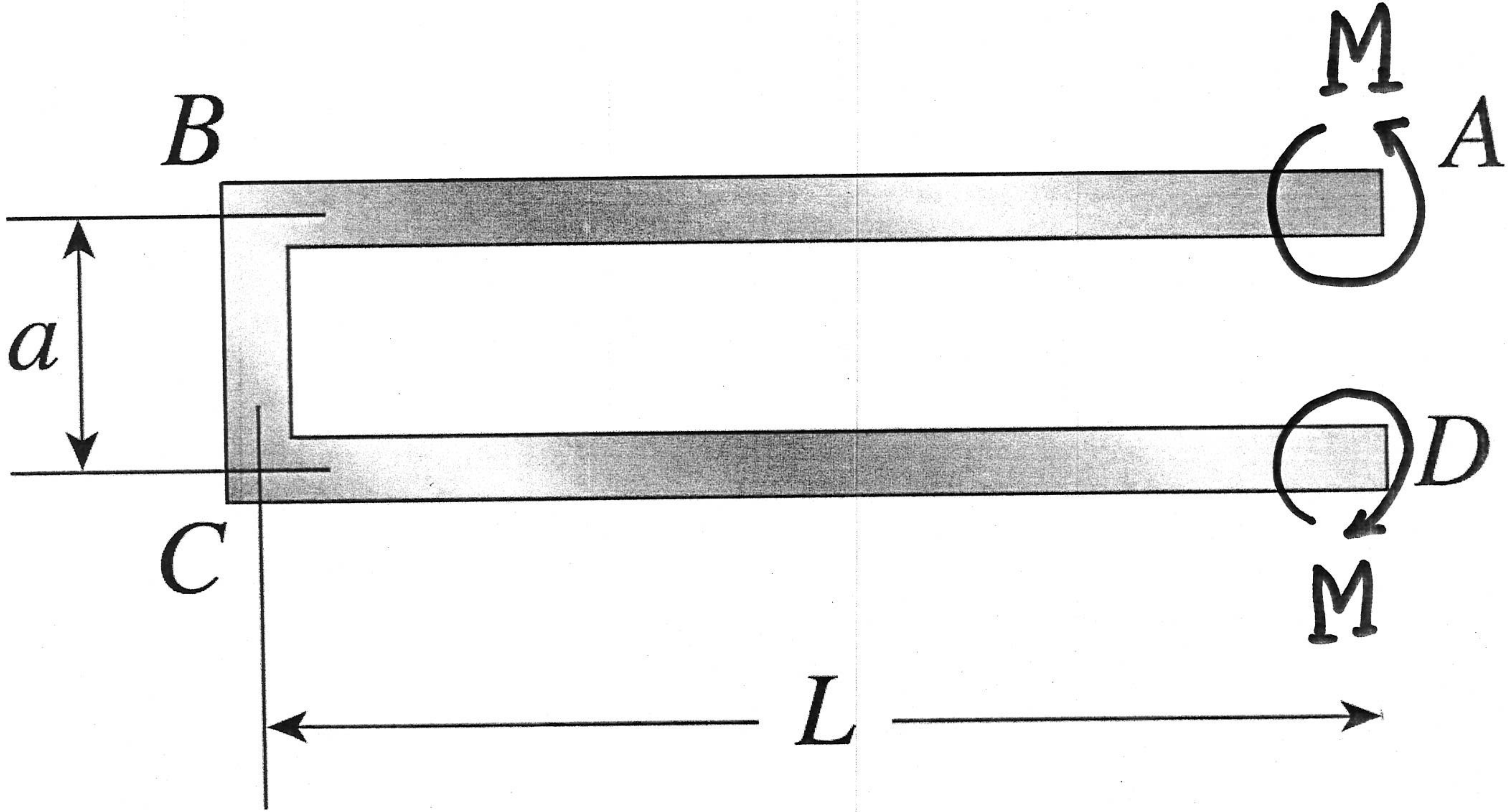
(10 Marks)

C2. Let $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ be the three ortho-normal base vectors of a Cartesian coordinate system in a three dimensional vector space. The traction vectors at a point in a solid on planes with unit outward normal vectors $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{n}$ are given as $\mathbf{t}(\mathbf{e}_1) = \mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3$, $\mathbf{t}(\mathbf{e}_2) = 2(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$ and $\mathbf{t}(\mathbf{n}) = 2\sqrt{3}\mathbf{e}_1 + 2\sqrt{3}\mathbf{e}_2 + 0\mathbf{e}_3$ where normal vector $\mathbf{n} = (1/\sqrt{3})(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$. Find the traction vector $\mathbf{t}(\mathbf{e}_3)$ on the plane with unit outward normal $\mathbf{e}_3$ and the Cauchy stress tensor $\mathbf{T}$ at the same point.
(10 Marks)

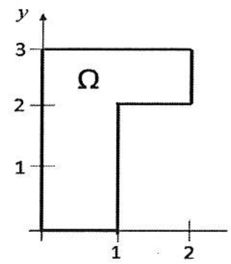

M
A
B
a
D
C
M
L

Department of Mechanical Engineering, IIT Bombay

Ph. D. Qualifying Examination: Applied Mathematics, 10 July 2017

Total marks: 60, Minimum passing score: 24 points (40%) Time: 3 Hours

Closed Book, Closed Notes Examination



1) Let $\rho(x, y)$ be the density of a distribution of mass over a region Ω in the x-y plane. The moments of inertia $\boldsymbol{I_x}$ and $\boldsymbol{I_y}$ about the x, y axes are defined as $I_x = \iint\limits_{\Omega} \rho(x,y)\, y^2\, dA,\quad I_y = \iint\limits_{\Omega} \rho(x,y)\, x^2\, dA$ respectively.

Evaluate $\boldsymbol{I_x}$ and $\boldsymbol{I_y}$, if Ω is defined by the adjacent figure. Assume the density to be constant, and equal to 6.0. *(10 marks)*

2) The Sturm-Liouville equation has the following form:

$$-\left[\frac{d}{dx}\left(p(x)\frac{dy_n}{dx}\right) + q(x)y_n\right] = \lambda_n \rho(x) y_n, \qquad (1)$$

where p, q, r and y_n are real functions of x which ranges in $[a, b]$.

a) If $\rho(x) = 1/x$, show that equation given below can be rewritten in the Sturm-Liouville form of equation (1),

$$x^2\frac{d^2y_n}{dx^2} + x\frac{dy_n}{dx} + \lambda_n y_n = 0 \qquad (2)$$

Thus identify the corresponding $p(x)$, $q(x)$.

b) For $y_n(a = 1) = y_n(b = e) = 0$, solve (2) to find λ_n and y_n i.e. eigen values and eigen vectors. Here e = 2.71828182... is the base of natural logarithm. *(1+5=6 marks)*

Hint: Assume $y_n = C_1\, x^{\alpha}$. Note that $x^q = e^{q \log_e x}$ where q is a constant.

3) The Chebyshev equation is given as,

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0 \qquad (3)$$

a) Show that $x = 0$ is an ordinary point.

b) Determine a series solution for the equation at the ordinary point $x = 0$. *(1+3 = 4 marks)*

4) A n x n matrix A has n eigenvalues A_i. If $B = e^A$, show that B has the same eigenvectors as A, with corresponding eigenvalues B_i given by $B_i = e^{A_i}$. *(5 marks)*

Hint: e^A is defined by the Maclaurin series expansion: $e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots\ldots$

5) Find a normal $\hat{\mathbf{n}}$ and the equation of the tangent plane for the surface $S: x = yz$ and a point P = (3, −1, −3). *(5 marks)*

6) A model is represented using two variables $x_1(t)$ and $x_2(t)$ through the following differential equations:

$$\begin{aligned} \frac{dx_1}{dt} &= x_1 + 3x_2 + 5e^{-3t} \\ \frac{dx_2}{dt} &= 2x_1 - 4x_2 - 6e^{-3t} \end{aligned} . \qquad (4)$$

a) Write the equation (4) in the form $\frac{dx}{dt} = \boldsymbol{Ax} + \boldsymbol{b}$,

where $\boldsymbol{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ is a 2×1 vector and identify matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$.

b) Justify that the matrix $\boldsymbol{A}$ is invertible. Calculate its eigen values and eigen vectors.

c) Use the answer to sub-question (b) to form a matrix which diagonalizes matrix $\boldsymbol{A}$. Carry out the diagonalization of $\boldsymbol{A}$.

d) Use the diagonalizing matrix to uncouple equation (4).

e) Solve the uncoupled equations. *(1+3+2+1+3 = 10 marks)*

7a) For a general second-order equation of the form $au_{xx} + bu_{xy} + cu_{yy} = d$, the characteristic equation is $\frac{dy}{dx} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$.

Using the above analogy, classify and find the characteristics of the partial differential equation given by *(5 marks)*

$$xu_{xx} + (x - y)u_{xy} - yu_{yy} = 0, \quad x > 0, \quad y > 0 \qquad (5)$$

7b) Using Fourier transform, solve the following initial value problem, *(7 marks)*

$$\begin{cases} u_t = u_{xx}, & t > 0, \; x \in (-\infty, \infty) \\ u(x, 0) = g(x), & x \in (-\infty, \infty) \end{cases} \qquad (6)$$

Useful properties of Fourier transform

$$F\{f(x)\} = \hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx, \qquad F^{-1}\{\hat{f}(\omega)\} = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}d\omega,$$

$$F^{-1}\{e^{\alpha\omega^2 t}\} = \frac{1}{\sqrt{4\pi\alpha t}} e^{-\frac{1}{4\alpha t}x^2}$$

8) For the partial differential equation $v_t = v_{xx} + 2vv_x$, it is possible to look for a similarity solution of the form $v(x,t) = t^\alpha w(y)$, $y = \frac{x}{t^\beta}$, and α, β are constants. *(4+4 = 8 marks)*

(a) Find the parameters α and β that would eliminate t from the similarity equation.

(b) Find an equation for *w(y)* and show that this *ODE* can be reduced to first order.