Indian Institute of Technology Bombay
Department of Mechanical Engineering
2019–2020 Semester 2: PhD Qualifying Examination Paper
TFE2 AdvancedHeat Transfer : 21st Jan 2020, 14.30 to 17.30 HRS

Please note:

- Closed book examination. However, you may use a self-prepared A4 sheet (both sides may be used) for the examination. It must be in your own handwriting and must be attached finally to the answer-book. **Do not** put your name on this A4 sheet
- **Begin each question on a new page.**
- The duration of this examination is three (3) hours.
- Total marks: 100. Passing marks: 40.
- List any assumptions you make.
- Be clear and neat. Marks also depend on the quality of your answers.

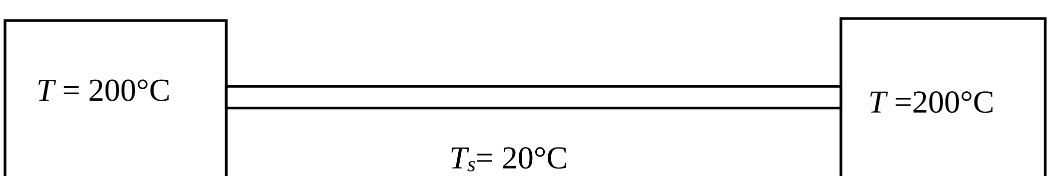
Question 1: Liquid metal at uniform temperature T_∞ and uniform velocity U_∞ flows over a heated flat plate of length L. The wall temperature T_w varies along the plate according to the equation: $T_w = T_\infty + Ax^n$. Here, x is the axial coordinate measured from the leading edge of the plate. 'A' is the proportionality constant and 'n' is a positive index. The flow can be assumed to be steady, two-dimensional, laminar and incompressible. The properties can be assumed constant.

a. Perform an order-of-magnitude analysis of the continuity, x-momentum and energy equation to show that $Nu_x \sim Re_x^{0.5} Pr^{0.5}$. **[12 marks]**

b. Using the wall temperature profile given above, the similarity variable, $\eta = y\sqrt{\frac{U_\infty}{\alpha x}}$, and $\theta = \frac{T - T_\infty}{T_w - T_\infty}$, transform the BL energy equation toobtaion the transformed governing equation and the transformed boundary conditions. **[8 marks]**

Question 2: A bar of square cross-section connects two metallic structures; both structures are maintained at a temperature 200 °C. The bar, 20 mm × 20 mm, is 100 mm long and is made of mild steel (k = 0.06 kW/mK). The surroundings are at 20 °C and the heat transfer coefficient between the bar and the surroundings is 0.01 kW/m^{2}K. Find the followings:

a. Starting with an infinitismal control volume, listing all assumptions, derive an equation for the temperature distribution along the bar.
b. Determine the temperature distribution at *0.25L, 0.5L, 0.75L and L* along the length of *2L* = 100 mm.
c. Calculate the total heat transfer rate from the bar to the surroundings.



T_s= 20°C
k = 0.06 kW/mK
$2L$ = 100 mm
h = 0.01 kW/m^{2}K

[12 marks]

Question 3: Consider two very large parallel plates (both plates with emissivity 0.5) separated by a distance of 0.1 m. The top plate 2, is held at a constant temperature of 330 K. The space between the plates is filled with air and the bottom plate, 1 supplies a uniform heat flux of 250 W/m^2.

Set up the energy balance from first principles.

For the first iteration, assume the bottom plate temperature to be 370 K. You will have to show calculations for 370 K and for another temperature which needs to be logically correct.

Roughly ***estimate*** the ratio of the convective to radiative heat transfer contributions for the case of T_1 = 370K.

Without calculations, what can you say about the convective and radiative components if the emissivities of the plate is increased to 0.75?

Assume that natural convection can be correlated using the formula $\overline{Nu_L} = 0.069 Ra_L^{\frac{1}{3}} Pr^{0.074}$

For air at 350K: $k = 0.03 \frac{W}{mK}, \nu = 2.092 \times 10^{-5} \frac{m^2}{s}, \alpha = 2.99 \times 10^{-5} \frac{m^2}{s}$ **[14 marks]**

Question 4: Consider natural convection over a vertical plate heated with **constant** $\boldsymbol{q''}$. Using the following velocity and temperature profiles,

$$v = V(y)\frac{x}{\delta}\left(1-\frac{x}{\delta}\right)^2 \quad \text{and} \quad T - T_\infty = (T_0(y) - T_\infty)\left(1 - \frac{x}{\delta_T}\right)^2$$

a. Express q'' in terms of $(T_0(y) - T_\infty)$.

b. Consider $\delta = My^{\frac{1}{5}}$ and $V(y) = Ny^{\frac{3}{5}}$, $T_0(y) - T_\infty = Py^{\frac{1}{5}}$ where M, N and P are constants. Performing **an integral analysis,** find $\frac{\delta}{y}$, $T_0(y) - T_\infty$ and the local Nusselt number $\left(Nu_y\right)$ in terms of ***Ra* (defined appropriately)** and *Pr*.

(For simplicity: Integrate the integral equations from the plate surface to the thickness of boundary layer. Assume $\delta = \delta_T$, however <u>*do not consider*</u> *Pr = 1 in the analysis)*

Momentum equation: $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$

Energy equation: $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$

[20 marks]

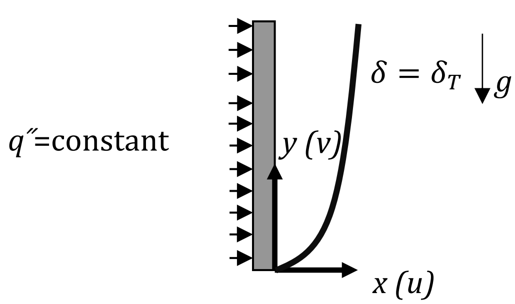


Figure for Question 4

Question 5: An aluminium plate of 1 m x 1 m face area and 1 mm thickness is coated with a selective paint on one face and is insulated on the other face and the edges. Surface coated with selective paint is exposed to sky, plate is located on a horizontal roof top of a building. Clear sky temperature may be assumed to be at ~3 K. Assume that the earth's atmosphere is transparent to the radiation between 8 to 13 μm wavelengths. Following properties are reported for the selective paint: reflectivity in the solar spectrum 90%, emissivity in the 8 to 13 μm wavelengths 86%.

a. Draw the plan and sectional elevation of the plate and mark the given information on it. [*3 marks*]
b. If the ambient temperature at night is 27^{o}C and convective heat transfer coefficient from the selective surface of the aluminium plate is 10 W/m^{2}K, show that the minimum temperature achieved by the plateat its centre is about -0.13^{o}C. [*5 marks*]
c. If the ambient temperature at noon is 34^{o}C, convective heat transfer coefficient from the selective surface of the aluminium plate is 10 W/m^{2}K and solar radiation incident on the sky facing surface is 1000 W/m^2, show that the minimum temperature achieved by the plate at its centre is about 11.81^{o}C. [7 *marks*]
d. If the plate is located along a vertical wall of a building with insulated surface touching the wall, the minimum temperature at night will **_____________** (***increase/decrease/remain same)***. Select one and justify your selection. [*3 marks*]
e. What challenges do you anticipate for using this type of radiating surface for cooling buildings, instead of using conventional air conditioners. [2 *marks*]

[Total: 20 marks]

Question 6: A large square aluminium tube, with 100 mm x 100 mm inner open cross section area, wall thickness of 2 mm and length of 500 mm is closed at the bottom. It is filled with 5 litres of water at 30^oC. The tube is well insulated from all four sides and the bottom. Top open edge of the Al tube is cooled with a cooling source whose cooling capacity varies with temperatures of the tube edge, t_c in ^oC, as follows: $q_c = (1 + t_c/36^oC)\ 30W$; average heat transfer coefficient on the water side may be assumed to be constant at 50 $W/(m^2.K)$.

a. Draw the sectional elevation of the Al tube with water and insulation and mark the given information on it [3 marks]

b. Derive the relationship for temperature of the top edge [5 marks]

c. Calculate the temperature of the top edge when water temperature is 30^oC, in °C [2 marks]

d. Calculate the temperature of the top edge when water temperature reached 4°C, in °C [2 marks]

e. Calculate the heat flux through the top edge when water temperature is 30°C, in W/m^2 [1 mark]

f. Calculate the heat flux through the top edge when water temperature reached 4°C, in W/m^2 [1 mark]

Note:

i. Al tube weight 1.2 kg, ρ_{al} = 2,760 kg/m^3, cp_{al} = 0.9 kJ/(kg.K) and k_{al} = 234 W/(m.K)

ii. Density of water is highest at 4^oC

[Total 14 marks]