Department of Mechanical Engineering, IIT Bombay

Ph.D. Qualifier Examination in Fluid Mechanics (January 21, 2023)

Max marks: 100 **Duration: 3 hours**

Note:

1. All questions are compulsory. This paper consists of 6 questions over 3 pages.
2. The maximum marks per question are given in parenthesis.
3. **Please begin each question on a new page and keep ALL subparts of a question together.** This is important as multiple faculty are involved in grading.
4. Strike out all unwanted work neatly, else, the work that appears first will be taken up for evaluation.
5. This is an open book and open notes examination. Tables/charts from Text books can be used. You are requested to write the Table no/figure number and name of the text book/author from which you choose values.
6. Make suitable assumptions when needed and clearly state them.

(10 marks) Problem 1: Consider the case of one-dimensional, steady flow of air through a subsonic diffuser of length L. The maximum velocity decreases from $u_1$at inlet $(x = 0)$ to u_2 at exit $(x = L)$. Measurements indicate that the variation of the maximum velocity is parabolic along the length. Obtain an expression for the maximum velocity distribution and the acceleration of a fluid particle along the centreline in terms of the known parameters.

If the length of the diffuser is 1.56 m, $u_1 = 24.3\ m/s$ and $u_2 = 16.8\ m/s$; evaluate the centreline acceleration at the inlet, $x = 0\ and\ x = 1\ m$.

(16 marks) Problem 2: In a certain medical application, water at room temperature (25°C, $\mu = 10^{-3}$ kg/m.s) and atmospheric pressure flows through a rectangular channel of length L = 50 cm, width s = 3 cm, and height b = 0.3 mm. The volume flow rate is Q = 0.5 m^3/s. Assume steady, laminar flow, and $\partial/\partial z = 0$. Of course, please assume any other facts you would need.

Starting with the momentum equations, answer the following.

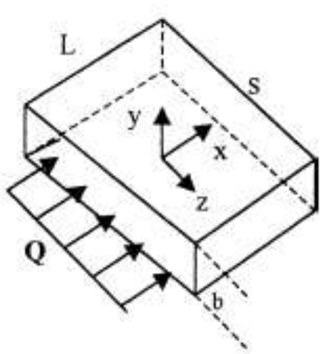

Figure for Problem 2, not to scale

(a) Logically argue that the flow is fully developed for most of the channel.

(b) Find an expression for streamwise velocity u as a function of (y, μ, dp/dx, and b), where dp/dx is the pressure gradient required to drive the flow through the channel.

(c) Find the pressure drop (Δp) for the above-mentioned values of L, s, b and Q.

(d) Determine the friction factor.

(17 marks) Problem 3: Consider a baffle plate on which a water jet impinges at a distance 15 cm from a hinge 'A' as shown in the figure below. A part of the jet turns towards the hinge and then turns again and exits the plate. The other part exits at the top. Jet diameter can be approximated to be very small compared to the size of the plate and thus force over the cross-section area of the jet remains constant and also that the jet turns instantly at the baffle plate. Determine the torque on the hinge required to keep the plate in equilibrium as shown if jet diameter is D_1 = 2 mm, and flow through the jet is Q_1 = 0.01 lit/s. Ignore gravity and water viscosity effects. Plate width is 1 m normal to plane of figure. Assume surrounding pressure is atmospheric.

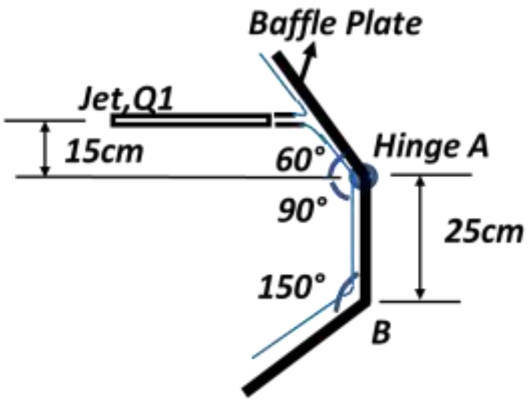

Figure for Problem 3, not to scale

(17 marks) Problem 4: Engineers call the supersonic combustion in a scramjet almost miraculous, "like lighting a match in a hurricane". The figure below is a crude idealization of the engine. Air enters, burns fuel in the narrow section, then exits, all at supersonic speeds. There are no shock waves. Assume air to be diatomic and the ratio of specific heats (k) to be 1.4; and the molecular weight is 29 gm/mol. The universal gas constant is 8.314 J/mol.K.

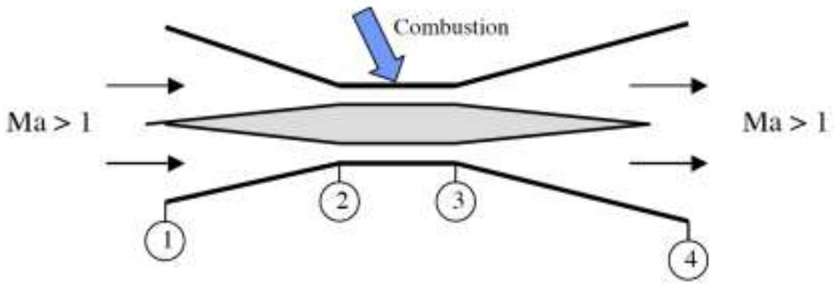

Figure for Problem 4, not to scale

Assume areas of 1 m^2 at sections 1 and 4 and 0.2 m^2 at sections 2 and 3. Let the entrance conditions be Ma_1 = 6, at 10,000 m standard altitude.

The conditions at that altitude are: P_1 = 26,416 Pa, T_1 = 223.16 K, ρ_1 = 0.4125 kg/m^3.

Assume isentropic flow from 1 to 2, frictionless heat transfer from 2 to 3 with Q = 600 kJ/kg, and isentropic flow from 3 to 4.

Calculate the exit conditions, and the thrust produced.

(16 marks) Problem 5: A surface is modeled as a flat plate in an incompressible uniform stream of air of velocity 'U' as shown in the figure below. The stagnation pressure facing the flow direction shown in the figure at the beginning 'A' and end 'B' locations at a distance of 10 mm above the plate are measured to be 100128 Pa and 100050 Pa respectively. The static pressure at a location 'C' on the plate which is in between the two sensors at L/2 from the 'A' is 100010 Pa. Determine the drag force on the top side of the plate assuming the boundary layer is turbulent right from the start to end of the plate and that the velocity profile is well approximated by the 1/7th power law. The standard expressions from the integral analysis can be used. The width of the plate normal to the flow is assumed to be equal to unity. State explicitly any assumptions that you may need under the purview of the boundary layer theory.

Assume air density and viscosity as 1 kg/m^3 and 1.8×10^{-5} Ns/m^2.

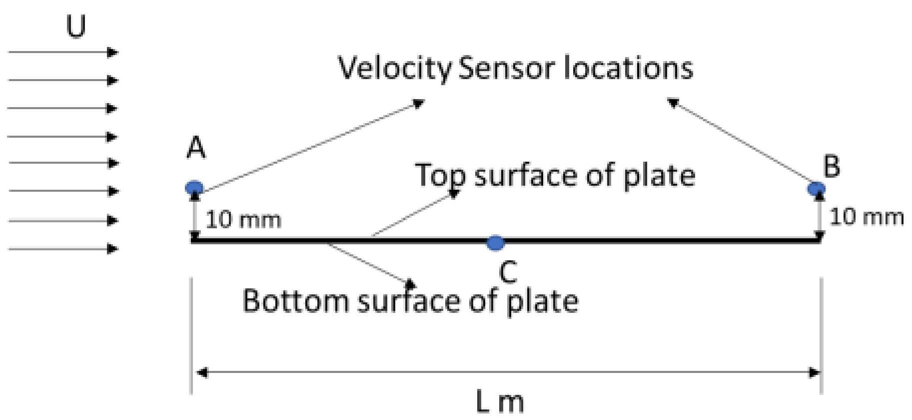

Figure for Problem 5, not to scale

(24 marks) Problem 6: Consider the flow of two distinct immiscible (non-mixing) fluids A and B flowing parallel to each other in a channel (formed by two stationary parallel plates separated by distance $2h$) driven by a pressure gradient. The channel extends infinitely into the plane of the paper (z direction) and the length of the channel is L along the flow direction (x direction), $(L \gg 2h)$ The centre of the channel represents the location y = 0.

The flow rates are so adjusted such that **each fluid occupies exactly half the channel width**. The interface is assumed flat and surface tension effects are negligible. Our aim is to determine the velocity profile for each fluid.

Starting with the full form of the N-S equations in all directions, listing all assumptions, **obtain the expression for the velocity distribution for both fluids. Sketch the velocity profile for both fluids.**

For consistency, let subscript A and B refer to the quantities associated with the fluid in the top half and bottom half of the channel respectively. Let the viscosity ratio be denoted as $\beta = \frac{\mu_B}{\mu_A}$. Also, once constants of integration are obtained, you need NOT substitute it back in the general solution. Make sure your work is systematic, neat and legible. Number your equations appropriately.