Ph.D. Qualifying Examination (26th May 2015)

Fluid Mechanics (TFE-1); Maximum marks: 100; Time: 9:30AM-12:30 PM

This is a **closed book, open hand-written notes** examination. Answer all questions. Make suitable assumptions if required and state them clearly. Assume fluid as Newtonian and flow as incompressible for all problems given below.

Problem 1: Fundamentals of Theoretical Fluid Dynamics [30 Marks]:

a) What are the conservation laws used in fluid mechanics? Discuss them in-detail specially the rate of change term for the Eulerian system used in fluid mechanics as compared to the Lagrangian system used in solid mechanics. **(5 Marks)**

b) Present a CV based derivation of unsteady form of continuity equation, in 2D Cartesian coordinate system. The derivation should be based on mass-flux (mass flow rate per unit area; acting at the various surfaces of the CV) and surface-area (of the CV) as a vector quantity. Represent the mass flow across the surface of the CV as a cyclic surface integral. **(5 Marks)**

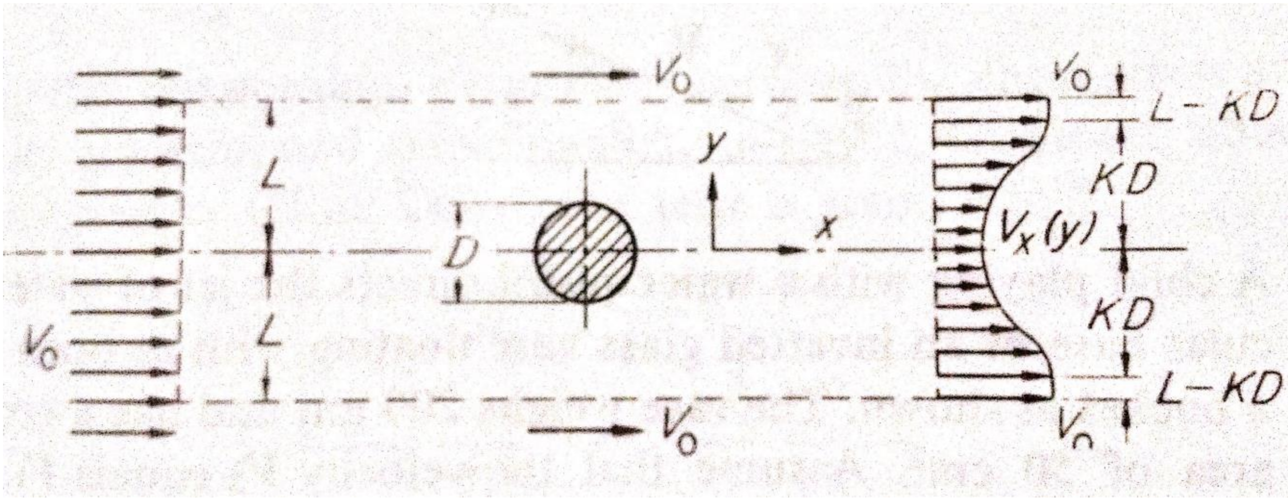
c) Present a CV based derivation of unsteady form of x- and y-momentum equation, in 2D Cartesian coordinate system. The derivation should be based on momentum-flux $\vec{m}\,\vec{u}$ as well as viscous-stress $[\sigma]$ (acting at the various surfaces of the CV) as a second order tensor and surface-area (of the CV) as a vector quantity. Avoid flow kinematics and directly use simplified form of expressions (for incompressible flow) for viscous stresses: $\sigma_{xx} = \mu\, \partial u/\partial x$; $\sigma_{yx} = \mu\, \partial u/\partial y$; $\sigma_{yy} = \mu\, \partial v/\partial y$ and $\sigma_{xy} = \mu\, \partial v/\partial x$. <u>*Hint:*</u> Take the dot product of the flux and stress with unit vector $\hat{i}$ and $\hat{j}$ to derive x- and y-momentum equation, respectively. **(15 Marks)**

d) What is fluid and flow property? What is initial and boundary condition? Discuss the role of IC and BC in fluid dynamics. **(5 Marks)**

Problem 2: External Fluid Dynamics [25 Marks]:

a) The above governing equations and the initial/boundary conditions results in a solution for velocity and pressure. However, for engineering application, the results are used to compute certain parameters - called as engineering parameters. What are the engineering parameters considered for flow across an aeroplane? Using the velocity and pressure distribution of the *surface* of the aeroplane, discuss the procedure for the calculation of the parameters.**(5 Marks)**

b) An experimental method of measuring the force exerted on a solid body consists of placing it in a uniform stream of fluid and studying the velocity pattern downstream. Using the CV show in the next page, obtain the dimensionless equation for the drag force on a cylinder of diameter D:



$$C_D = \frac{F_x}{0.5\rho V_0^2 (WD)} = \int f(V_x^*) dy^*$$

$$\text{where } V_x^* = V_x / V_0 \text{ and } y^* = y / D$$

where F_x is force on the cylinder in x-direction and W is width of the cylinder. Determine the integrand which is shown as a functional relationship $f(V_x^*)$ in the above equation. Assume that the pressure far upstream and downstream are equal. (**<u>10 Marks</u>**)

c) For flow across a circular cylinder, draw the variation of pressure distribution on the surface of the cylinder for inviscid as well as viscous flow. Discuss the fluid dynamics phenomenon which leads to the difference in the result for the two types of flow. **(5 Marks)**

d) What is a wake and vortex shedding for an external fluid flow over an object? Discuss their effect on the forces acting on the object. **(5 Marks)**

Problem 3: Internal Fluid Dynamics [25 Marks]:

a) Consider 2D developing flow in a plane channel (flow between parallel plate), with uniform flow at the inlet and fully developed flow at the outlet. Present the velocity boundary conditions for the channel flow. Draw the growth of boundary layer below the top and above the bottom wall of the channel. Also draw the stream-wise velocity profile $u=f(y)$ at the inlet ($x=x_0=0$), at three different cross-sections ($x=x_1$, x_2 & x_3 with $x_0<x_1<x_2<x_3$) in the developing region and one cross-section in the fully developed region ($x_{fd}>x_3$). Note that the length of the horizontal arrows in the velocity profile – for the various cross section – should be as per scale. Discuss the streamwise acceleration of the fluid in the developing and fully-developed region. Draw a representative figure for the variation of cross-sectional-averaged pressure of the fluid along the length of the channel; and discuss the variation of pressure gradient along the axial direction. (**<u>10 Marks</u>**)

b) For an internal flow, write down the equation for definition of Fanning and Darcy's friction factor; and discuss the physical interpretation of both the friction factor. (**<u>5 Marks</u>**)

c) Turbulent flow in a pipe of non-circular cross-section is often approximated by flow in an equivalent circular pipe of hydraulic diameter $\boldsymbol{D_h}$. The diameter of such a pipe is determined from the fact that the shear stress at walls in turbulent flow depends essentially on the Reynolds number, independent of the cross-sectional shape (provided the cross-section is not too thin). Show by considering the balance of pressure force and shear forces on the walls (using a control volume analysis) that an "equivalent" circular pipe having a diameter $\boldsymbol{D_h=4A/P}$ (where $\boldsymbol{A}$ is the cross-sectional area and $\boldsymbol{P}$ is the perimeter of the non-circular pipe) has the same pressure drop across a given length as the non-circular pipe when the shear stress at the walls is identical in the two cases. The diameter $\boldsymbol{D_h}$ is termed as *equivalent diameter* and is used as the characteristic length in flow through non-circular pipes. (**<u>5 Marks</u>**)

d) Find the pressure drop - across a ***30 m*** length - for the flow in a steel conduit (shown below) formed in-between a square rod of dimension $\boldsymbol{a=1\ cm}$ and a pipe of inner diameter $\boldsymbol{D=2\ cm}$. The flow rate of water (density $\boldsymbol{\rho=1000\ kg/m^3}$ & dynamic viscosity $\boldsymbol{\mu=10^{-3}\ Pa\ s}$) is $\boldsymbol{2.5\times10^{-3}\ m^3/s}$. Assuming the pipe to be smooth, use the equation for Darcy friction factor $\boldsymbol{f=0.184/Re^{0.2}}$, where Re is Reynolds number based on $\boldsymbol{D_h}$. *Show all the steps involved in the calculation.* (**<u>5 Marks</u>**)

Problem 4: Boundary Layer Theory [20 Marks]:

a) Discuss the role and importance of boundary layer theory in the historical development of fluid dynamics. Highlight the limitations of the theoretical results which were obtained before the boundary layer theory. (**<u>5 Marks</u>**)

b) What is flow separation? Discuss the necessary and sufficient condition for the flow separation. (**<u>5 Marks</u>**)

c) The stream-wise velocity component of a steady, incompressible, laminar, flat plate boundary layer (of boundary layer thickness δ) is approximated by the simple linear expression $\boldsymbol{u= u_\infty y/\delta}$ for $\boldsymbol{y<\delta}$, and $\boldsymbol{u= u_\infty}$ for $\boldsymbol{y>\delta}$. For the linear approximation, generate expressions for displacement thickness $\boldsymbol{\delta^*}$ and momentum thickness $\boldsymbol{\delta^{**}}$ as functions of $\boldsymbol{\delta}$. Use the definition of local skin friction coefficient ($\boldsymbol{C_{f,x}=2\times\tau_w/\rho u_\infty^2}$) and the Kármán integral equation ($2\boldsymbol{d\delta^{**}/dx=C_{f,x}}$) to generate an expression for $\boldsymbol{\delta/x}$. (**<u>10 Marks</u>**)