Department of Mechanical Engineering, IIT Bombay

Ph.D. Qualifying Examination: Fluid Mechanics (TFE-1)

Maximum marks: 60 **16th January, Wednesday, 2019** **Time: 9:30AM-12:30 PM**

This is a **closed book, closed notes** examination. ***Use of calculator is allowed.*** Answer all questions. Make suitable assumptions if required and state them clearly. Simplify the answers to the extent possible. Useful equations are given in the Appendix. Please write legibly. Start the solution of each of the seven problems below in a NEW page *Best of luck for the exam!*

Problem 1: Bernoulli Equation [10 marks]: Water of density ρ flows steadily through a two-dimensional contraction whose narrowest section is 40 mm wide as shown in the Figure 1 below. From A to B (both points lie on the narrowest section), the radii of curvature, R, of the streamlines is inversely proportional to the distance n from the centre streamline. The radius of curvature of the wall is 50 mm. The flow of water can be assumed to be inviscid and irrotational everywhere (any boundary layers may be ignored).

a) Along the narrowest cross-section AB, where will the pressure be lowest and where will it be the highest, and why? **[2mark]**

b) If p is the pressure at any point along AB, and $\boldsymbol{u}$ is the velocity, then which of these equations about $\partial p/\partial n$ are correct? Explain. **[3 marks]**

$$\frac{\partial p}{\partial n} = -\rho u \frac{\partial u}{\partial n} \qquad (1) \qquad\qquad \frac{\partial p}{\partial n} = -\rho \frac{u^2}{R} \qquad (2)$$

(a) equation (1) only, (b) equation (2) only, (c) both (1) and (2).

c) If the velocity magnitude at A is $u_A = 0.5\ m/s$ then what is the velocity magnitude at B, u_B? Comparing the magnitudes of u_A and u_B comment on your answer in (a) ? **[5 marks]**

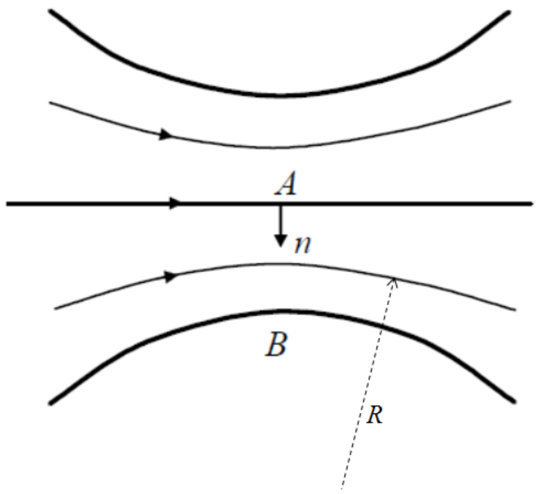

Figure 1

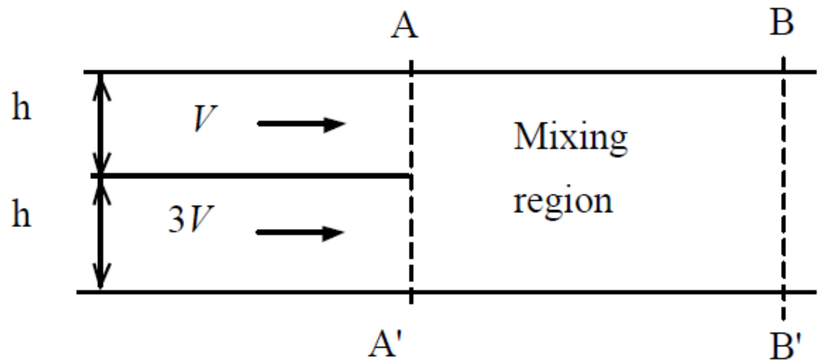

Figure 2

Problem 2: Control Volume Analysis [5 marks]: Two parallel streams of an incompressible fluid of density ρ flowing in horizontal rectangular ducts of height h and depth d come together at the location AA′ as shown in the Figure 2 above. They have the same pressure p_A and speeds V and $3V$, respectively. The two streams mix over a short distance due to the turbulence generated by the shear layer between them. The viscous shear stress on the solid surfaces can be assumed to be absent and other body forces be ignored.

Starting with the integral form of the appropriate conservation equations, calculate the velocity and pressure at the location BB' where the mixing is complete. State clearly all the assumptions you have made. Comment on the relative magnitudes of the pressures at BB' and AA'.

Problem 3: Non-dimensionalization [5 marks]: For an incompressible fluid, the momentum equation in the rotating reference frame under the Boussinesq approximation can be written as

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla\bar{p} - 2\boldsymbol{\Omega}\times\boldsymbol{u} - \alpha T\mathbf{g} + \nu\nabla^2\boldsymbol{u}$$

where, $\bar{p} = p/\rho_0$, p is the dynamic pressure including centrifugal force, ρ_0 is the reference density, ν is kinematic viscosity, $\boldsymbol{\Omega} = \Omega\hat{\mathbf{e}}_\mathbf{z}$ is the rotation vector, $\mathbf{g} = -\mathrm{g}\hat{\mathbf{e}}_\mathbf{y}$ is gravity, α is the coefficient of thermal expansion and T is the temperature perturbation. Use the following transformations to convert the above equation in the non-dimensional form: $\boldsymbol{x}^* = \boldsymbol{x}/l$, $t^* = t\nu/l^2$ and $T^* = T/\Delta T$ where l is a characteristic length scale and ΔT is the temperature difference (for example it can be temperature difference between the cold and hot boundaries in Rayleigh–Bénard convection). Show all the steps. The non-dimensional equation you will obtain should contain the following dimensionless numbers: $E = \nu/\Omega l^2$ (Ekman number), $Ra = \alpha \mathrm{g}\Delta T l^3/\nu\kappa$ (Rayleigh number) and $Pr = \nu/\kappa$ (Prandtl number), where κ is the thermal diffusivity. By writing $E = \Omega^{-1}/(l^2/\nu)$, can you predict what is the physical significance of the Ekman number ?

Problem 4: Exact solutions [10 marks]: Consider steady, incompressible, viscous, **fully-developed** flow of a thin liquid (density ρ, viscosity μ) film flowing over a wall in ambient, making an angle θ with the horizontal, under the influence of gravity g. Consider two dimensional coordinates in which x-axis is along the wall, pointing downwards, and z-axis is normal to x axis such that it points from wall to the fluid. The velocities u and w are in the respective x and z directions.

a) Simplify the governing equations for the fluid flow in the film. **[1 mark]**
b) Assume that the film thickness (say e) is uniform and pressure at the liquid-gas interface is atmospheric pressure (P_{atm}), derive pressure profile along the thickness of the film as function of P_{atm}, g, z and θ. Draw the pressure profile in the film. **[2 + 1 marks]**
c) Using equations derived in (a) and results obtained in (b), solve for velocity profile $u(z)$. Assume the dynamic viscosity of liquid is extremely large than that of air; and use appropriate boundary condition at the liquid-gas interface. Draw velocity profile in the film. **[3 + 1 marks]**
d) Find the average flow rate Q (per unit length along y) in the film. **[2 marks]**

Problem 5: Potential flows [**10 marks**]

a) Develop complex potential for free stream horizontal flow (free stream velocity $= U_\infty$) past a circular cylinder of radius a mounted closed to a wall, as shown in Figure 3. You may use complex potentials for elementary potential flows given in the table in the Appendix. <u>*Hint:*</u> Using method of images, the effect of the wall can be accounted for by placing identical potential entities below the wall, *i.e*, assuming wall as a mirror. [**3 marks**]
b) Using answer in (a), find the magnitude and direction of velocities of the points at A, B, C and D. [**7 marks**]

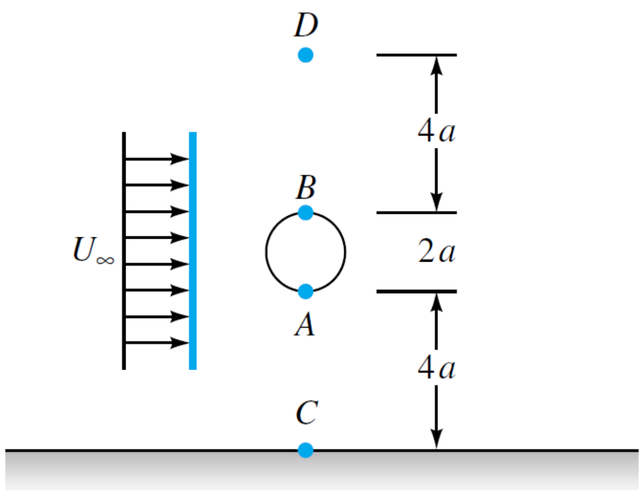

Figure 3

Problem 6: Vorticity Dynamics [10 Marks]:

a) **Vorticity:** For a free-stream flow over a flat plate, plot a boundary layer; and draw a streamline $\mathbf{S_1}$ inside boundary layer (BL) and another one $\mathbf{S_2}$ outside the BL. Thereafter, ***plot and discuss*** the shape of a fluid particle (represented as a smiling-face ) as it moves along the streamlines - plot the particle at five different locations (corresponding to five different increasing time instants t_1-t_5) on each the two streamlines. Assume that there is is no change in size of the particle. [**2 marks**]

b) **Vorticity Transport Equation:** Show **derivation** of the equation for vorticity in z-direction ω_z from 2D unsteady momentum equations in Cartesian Coordinate (x-y). Considering this equation, discuss the ***mechanism of vorticity transport*** during fluid flow across a stationary solid surface; and draw an ***analogy*** with mechanism of energy transport during forced convective fluid flow across a heated solid surface. [**4 marks**]

c) **Interaction of Vortices:** Consider a pair of line-vortices, separated by a distance L and rotating in (i) same and (ii) opposite direction; shown in the Figure 4. Also consider the circulation of first-vortex (acting at A) as Γ_1 and second-vortex (acting at B) as Γ_2. ***Write down*** the equation for the velocity induced by the first-vortex Γ_1 at B ($V_{1,B}$) and by the second vortex Γ_2 at A ($V_{2,A}$). What is the ***net effect*** of the these induced velocities on the pair of vortices – discuss separately for case (i) and case (ii). [**4 marks**]

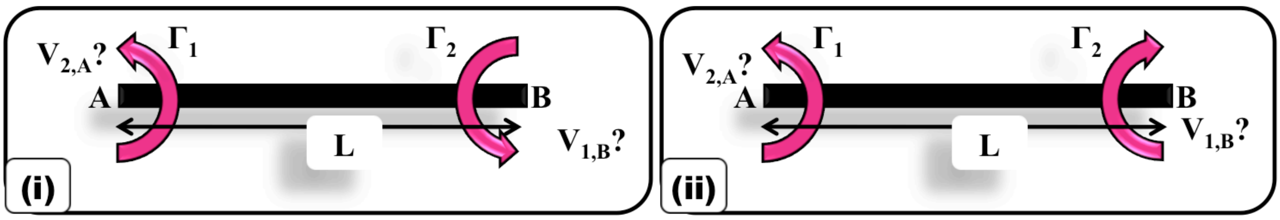

Figure 4

Problem 7: Laminar Boundary Layer Theory [10 Marks]:

a) **Scaling Analysis:** Consider a free stream flow (with velocity $\boldsymbol{u_\infty}$) over a flat plate (of length L), with x-axis along and y-axis normal to the plate. For the flow over a flat plate, consider 2D Prandtl boundary layer equation as

Continuity Eq.: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ X-Momentum Eq.: $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$ Y-Momentum Eq.: $\frac{dp}{dy}=0$

and wall shear stress $\tau_w = \mu\left(\partial u / \partial y\right)_{y=0}$. ***Using scaling analysis, show*** that the non-dimensional ***boundary layer thickness*** ε (= δ/L) as well as ***skin friction coefficient*** ($C_f = 2\tau_w / \rho u_\infty^2$, where $\boldsymbol{\tau_w}$ is the wall shear stress) varies as inverse of square root of Reynolds number $\mathrm{Re}_L \left(\equiv \boldsymbol{\rho u_\infty L / \mu}\right)$. [**4 marks**]

b) **Flow Separation:**
   a. What is flow separation? What is the (a) necessary and (b) sufficient condition for flow separation? [**3 marks**]
   b. For incompressible fluid flow, consider three classical internal flow problems: flow inside (a) plane channel, (b) nozzle, and (c) diffuser. For all the three problems, ***write down*** (in a tabular form) the ***sign*** (positive or negative) of pressure gradient over their length and ***discuss*** the possibility of flow-separation (no-separation or likely-to-separate or certainly-separate). [**3 marks**]

APPENDIX:

Integral Form of conservation equations: A general conservation law for a property, f can be written as

$$\int_V \frac{\partial f}{\partial t} dV \; + \; \int_A f\boldsymbol{u} \cdot d\boldsymbol{A} \; = \; F_B \; + \; F_S$$

where, the quantities F_B and F_S represent the rate of change in f through a control volume (with volume V and surface area A) by body and surface effects, respectively. For example, if f represents momentum then F_B is a body force and F_S is a surface force.

Continuity Equation for steady, incompressible flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier Stokes Equations for incompressible flow in Cartesian coordinates:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\}$$

Elementary complex potentials:

Case	Complex potential
Uniform flow at an angle α,	$w = Ue^{-i\alpha} z$
Source(+) / sink(-)	$w = \pm \dfrac{m}{2\pi} \ln z$
Flow in a sector	$w = Cz^n$
Doublet	$w = \mu / z$