Department of Mechanical Engineering, IIT Bombay

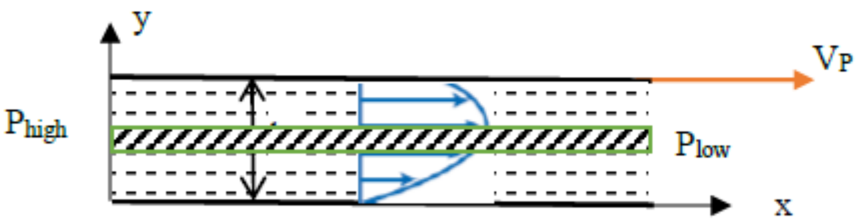
Ph.D. Qualifying Examination: Fluid Mechanics (TFE-1)

2022

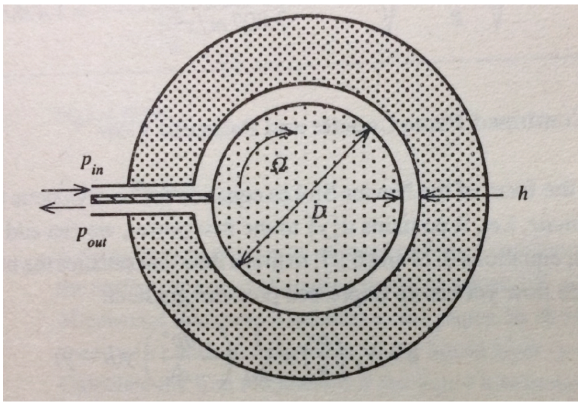
This is a **closed book** examination. Answer all questions. Make suitable assumptions if required and state them clearly. Simplify the answers to the extent possible. Please write legibly. Start the solution of each of the problems below in a NEW page

Best of luck for the exam!

1. Consider the combined Planar Couette-Poiseuille flow wherein the top plate is moving with velocity V_P, while the bottom plate is anchored and an external pressure difference is applied as shown in Fig (a) below. If the flow can also be assumed to be fully developed, then, the u velocity (x-direction) distribution, as shown in the figure, can be expressed as the sum of the velocity distribution of Couette and Poiseuille flows. Proceed as directed.
   (a) Assuming that the velocity in x-direction is fully developed, show systematically using continuity, y-momentum and x-momentum equations that the velocity distribution is only a function of y and the pressure gradient is constant. Using the stated boundary conditions, derive an expression for the same in terms of y, V_p, $\frac{dP}{dx}$, h and μ. (4 marks)
   (b) Express the shear stress on the moving plate in terms of V_p, $\frac{dP}{dx}$, h and μ. (2 marks)
   (c) Now proceed to compute the total flow rate per unit width in terms of V_p, $\frac{dP}{dx}$, h and μ. (2 marks)
   (d) Express the location y_{max}, where the maximum velocity occurs in terms of V_p, $\frac{dP}{dx}$, h and μ. (2 marks)
   (e) Now consider a case of a friction pump shown in the Fig (b) below, which consists of a solid cylinder of diameter D and length W perpendicular to the paper rotating clockwise at an angular velocity of Ω inside a co-axial hollow cylinder of inside diameter D+2h. Due to this, fluid is pulled into the pump and moved around the cylinder and sent out. As a result of the movement of fluid, a pressure gradient is established. The partitioning plate seals leakage of fluid from the high-pressure side to the low-pressure side. If one can assume that the results arrived in parts (a)-(c) above can be directly extrapolated to the present case, compute the volumetric flow rate Q, the pump drives in terms of Ω, D, W, h, ΔP and μ. The pressure drop in the horizontal portions shown can be neglected. Note that $\Delta P = P_{out} - P_{in}$. (2 marks)
   (f) Express the power that needs to be supplied to the rotating shaft to generate the necessary flow and pressure gradient in terms of Ω, D, W, h, ΔP and μ. (2 marks)

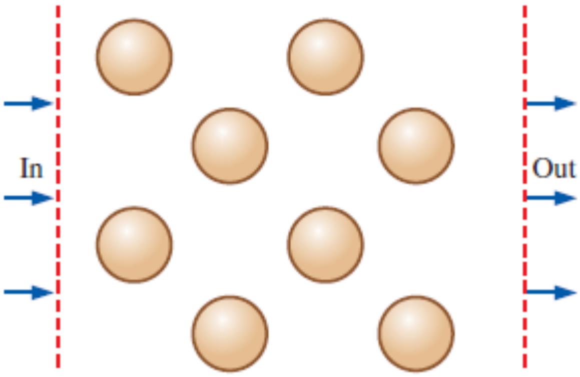


(a)



(b)

2. Consider a cross-sectional slice through an array of heat exchanger tubes shown in the figure below. For each desired piece of information (a) to (d), choose which kind of flow visualization plot out of the ones listed below [(i) to (iv)] would be appropriate.



a) Visualisation of the location of maximum fluid speed in the domain.
b) Visualisation of the flow separation at the rear of the tubes

(i) Velocity vector plot, (ii) Contour plot of resultant velocity, and (iii) Contour plot of a component of vorticity. (3 marks)

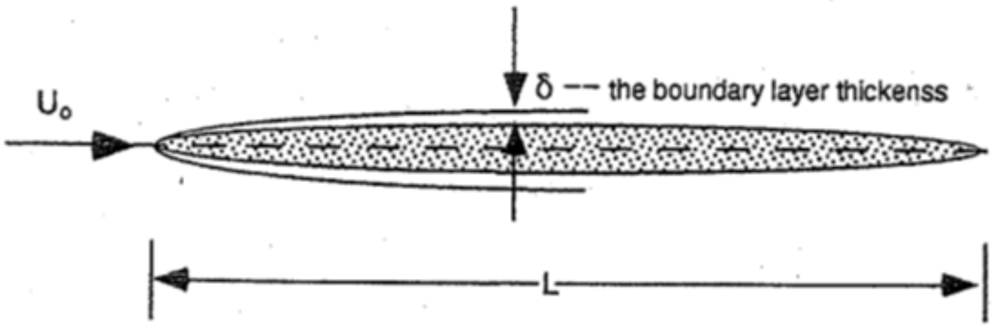
<u>Note</u>: In order to get full credit, for each part (a), (b)and (c) you should list all the possible visualisation methods that can be used from (i), (ii) and (iii) along with proper justification

3. Consider a slender, infinitely wide object of length L with axis placed parallel to a uniform fluid stream having axial velocity U_∞.

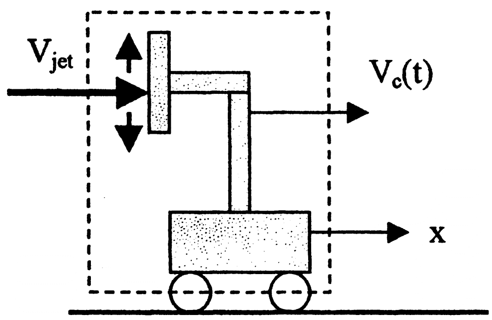
   a) Beginning with the appropriate forms of the continuity and Navier-Stokes equations, derive the boundary-layer equations using an order of magnitude analysis. Assume incompressible steady flow. Gravitational forces may be neglected. (8 marks)

b) Discuss the effect of the pressure gradient on the tendency for the boundary layer to separate. In your discussion use the boundary-layer equations which were developed in part (a). (3 marks)

c) Now, assume that the viscosity varies with temperature. Discuss the effect of wall temperature on the tendency for the boundary layer to separate if the fluid is a gas and if the fluid is a liquid. (2 marks)



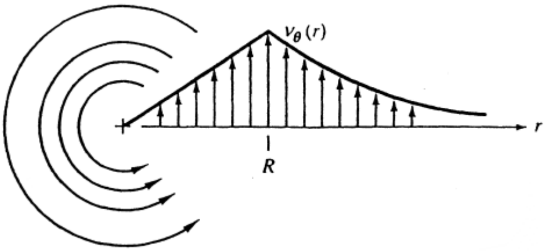
4. A cart with frictionless wheels carries a vertical plate. A jet of water with a velocity V_j hits the plate and causes it to move with a velocity V_c. A_j is the area on which the jet impinges and ρ_j is the density of water. Use the control volume shown by dotted lines in the figure below.



   a. (5 marks) Find $V_c(t)$ using the relevant form of conservation of momentum and mass.
   b. (3 marks) Find the time required for the cart to accelerate to 90% of jet velocity.
   c. (2 marks) Can the cart reach the jet velocity? Please explain.

5. A tornado may be modelled as a circulating flow as shown in the figure below. Reassuringly, only the tangential velocity component exists and the formulation is provided in the figure.

$$v_\theta = \begin{cases} \omega r & r \le R \\ \dfrac{\omega R^2}{r} & r > R \end{cases}$$



a. (3 marks) Determine whether this flow pattern is irrotational in either the inner or outer region.

b. (5 marks) Determine the pressure distribution in the tornado, assuming pressure tends to p_0 as r tends to infinity.

c. (2 marks) Find the location and magnitude of the lowest pressure.

6. Paintball is a game in which guns filled with paint pellets and propelled by a pressurized gas are used to fight a mock battle. A typical carbon dioxide tank for a paintball gun holds about 12 oz of liquid CO_2. The tank is filled no more than one-third with liquid, which, at room temperature, maintains the gaseous phase at about 850 psia (5.86 MPa).

   a. (5 marks) If a valve is opened that simulates a converging nozzle with an exit diameter of 0.050 in (0.00127 m), what mass flow and exit velocity result?

   b. (3 marks) Repeat the calculation for helium.

   c. (2 marks) As an engineer in a paintball design company, which gas would you choose to create the system with?

   For CO_2: R = 189 J/kg-K and k = 1.30.

   For Helium: R = 2077 J/kg-K and k = 1.66.

Required data and equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial(v_\theta)}{\partial\theta}+\frac{\partial(v_z)}{\partial z}=0$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{{v_\theta}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\rho f_r-\frac{\partial p}{\partial r}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)-\frac{v_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\rho f_\theta-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=\rho f_z-\frac{\partial p}{\partial z}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right\}$$

Curl of a vector

$$\nabla\times\mathbf{u}=\mathbf{i}_R\left(\frac{1}{R}\frac{\partial u_x}{\partial\theta}-\frac{\partial u_\theta}{\partial x}\right)+\mathbf{i}_\theta\left(\frac{\partial u_R}{\partial x}-\frac{\partial u_x}{\partial R}\right)+\mathbf{i}_x\left[\frac{1}{R}\frac{\partial(Ru_\theta)}{\partial R}-\frac{1}{R}\frac{\partial u_R}{\partial\theta}\right].$$