Department of Mechanical Engineering, IIT Bombay

Ph.D. Qualifying Examination: Fluid Mechanics (TFE-1)

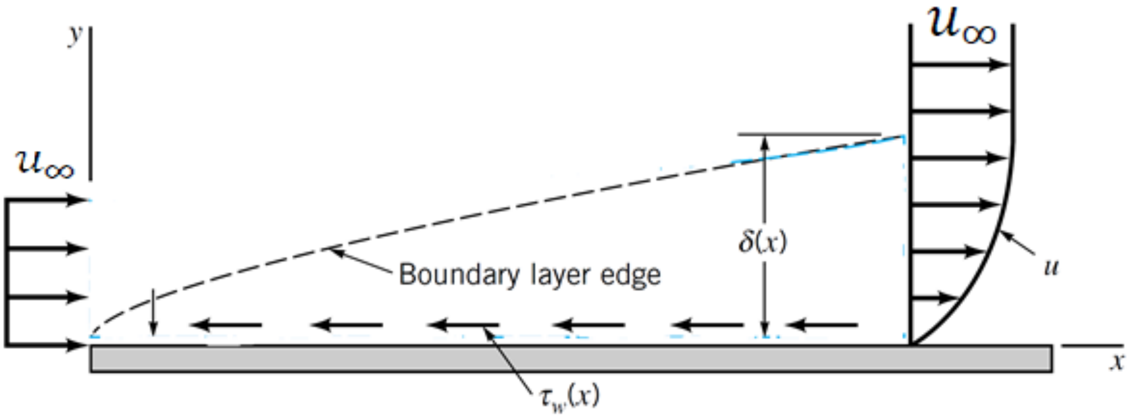
Maximum marks: 100 **1 August, 2022** **Time: 14:30-17:30**

Instructions

1. All questions are compulsory.
2. Make suitable assumptions, if necessary, and state them clearly.
3. Please provide figures wherever necessary.
4. Please start each question in a separate page.
5. Please do not skip steps. Please put all the steps in arriving at the answer for the given problem.
6. Duration of the exam is three hours.
7. Maximum marks is 100. Pass marks for this question paper is 40.
8. This is a closed book, closed-notes examination. All the information required is provided either in the questions or in the Appendix.
9. Calculator is allowed.

Problem 1: [17 marks]

1. Consider turbulent flow of an incompressible fluid past a flat plate as shown in the figure below.



The boundary layer velocity is assumed to be

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad for \quad \frac{y}{\delta} \le 1 \; and \; u = u_\infty \; for \; \frac{y}{\delta} > 1$$

This is a reasonable approximation of experimentally observed profiles. Assume that the shear stress agrees with the experimentally determined relation

$$\tau_w = 0.0225 \rho u_\infty^2 \left(\frac{\nu}{u_\infty \delta}\right)^{\frac{1}{4}}$$

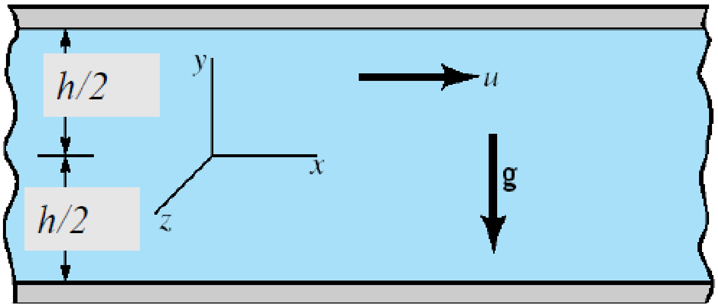
Determine the following parameters in terms of Reynolds number

a) Boundary layer thickness (δ)
b) Displacement thickness (δ_1)
c) Momentum thickness (δ_2)
d) Skin friction coefficient (C_{fx})

$\delta_1 = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy$	$\delta_2 = \int_0^\infty \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$
Von Karman Integral Momentum equation $\frac{d\delta_2}{dx} = \frac{C_{fx}}{2}$	$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2}$

Problem 2: [16 marks]

Consider steady, incompressible, laminar fully developed flow between two fixed parallel plates.



Starting from the continuity and momentum equations in the differential form, derive the relations for the following parameters

a) velocity profile in terms of centre line velocity and distance between the plates (h)
b) Average velocity in terms of centre line velocity
c) Shear stress distribution on either of the walls in terms of average velocity, dynamic viscosity and distance between the plates (h)
d) Pressure drop over a length L in terms of dynamic viscosity, average velocity, length L and distance between the plates (h).
e) Skin friction coefficient in terms of Reynolds number based on distance between the plates (h) as the characteristic length

Problem 3: [16 marks]

a) Consider the surface flow of a river near a bend that may be assumed to be described by the streamfunction $\psi = \alpha xy$, where α is a positive constant. The temperature at any point (x, y) on the surface is described by $T(x,y,t) = \beta x^2 y e^{-\alpha t}$ for $y > 0$ and β is also a constant.

i) Sketch the flow for $x, y > 0$. In your sketch also draw the lines of constant pressure. **[5]**
ii) Determine whether the temperature of *a fluid element/particle* will change with time. **[2]**
iii) In the Lagrangian description, a particle's position $\boldsymbol{x} = (x, y)$ at time t can be described in terms of the position $\boldsymbol{X} = (X, Y)$ at $t = 0$. For this flow, find $\boldsymbol{x}$ as a function of $\boldsymbol{X}$ and t. **[4]**
iv) Find T as function of (X, Y, t). Comment on your answer in (ii). **[2]**

b) According to your understanding of Eulerian and Lagrangian approaches in fluid mechanics, which of the two approaches is more suitable to study

i) the flow of blood in human artery
ii) the flow in the ionosphere (the topmost layer of the Earth's atmosphere).

Answer in brief with appropriate reasoning. **[3]**

Problem 4: [**18 marks**]

An incompressible fluid with velocity V_1 and density ρ approaches a row of long, horizontal, flat plates as shown in the figure below. The flat plates may be assumed to have zero thickness and are spaced at a distance w apart at an angle β to the incoming flow direction. Between each adjacent pair of blades there is a steady region of separated flow within which there is negligible velocity. Thus, the flow area is reduced causing the incoming flow to form a "free-stream jet" between the plates. Towards the end of the plates, the free-stream jet is parallel and occupies a fraction $1/b$ of the geometrical area between the adjacent plates.

a) Explain why, towards the end of the plates, the pressure p_j in the free-stream jet and the pressure in the separated region are nearly the same. **[2]**

b) Find the velocity within the free-stream jet, V_j, in terms of the given parameters. **[3]**

c) If p_1 is the pressure in the incoming flow, find the pressure p_j in terms of $p_1, \rho, b, w, V_1, \beta$. Choose an appropriate control volume. Ignore the shear stress on the plates and assume the flow to be steady. Clearly mention all the assumptions necessary to solve this problem. **[6]**

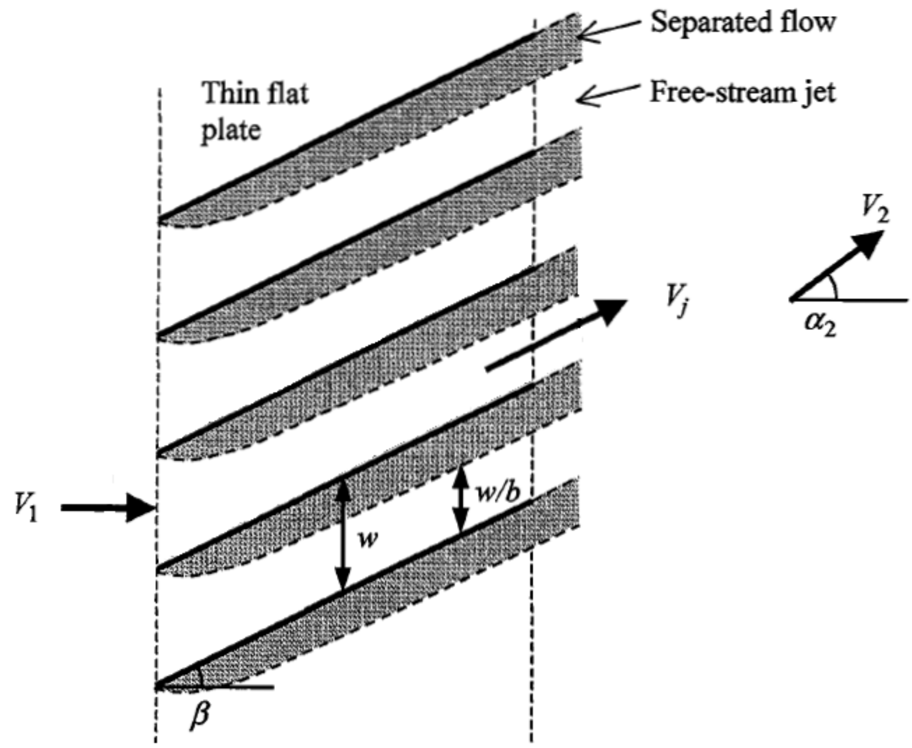
d) In this problem, can the Bernoulli's equation be applied

i) along a streamline between the incoming flow into the free-stream jet?

ii) on the boundary between the separated flow and the free-stream jet (shown in dashed lines)?

Explain with reasoning. **[4]**

e) Far downstream of the row of the plates, the free-stream jets and the wakes formed by the separated regions mix to form a uniform flow. Calculate the flow angle α_2 far downstream of the row of flat plates. **[3]**



Problem 5: [**17 marks**] **Inviscid and viscous flow across a circular cylinder:**

a) Consider 2D inviscid flow across a cylinder, obtained by a superposition of the uniform flow and doublet, with the complex potential $= u_\infty z + \mu/z = u_\infty(z + a^2/z)$. Derive an expression for the non-dimensional pressure distribution, $C_p = (p - p_\infty)/0.5\rho u_\infty^2$ on the surface of a cylinder as a function of θ. **[7]**

b) Draw the variation of the non-dimensional pressure distribution, C_p, along the surface of a cylinder (as a function of θ) for (a) inviscid-flow, and (b) viscous *laminar* flow (at a Reynolds number *Re*=100). **[4]**

c) What is the fluid dynamics phenomenon which leads to the difference in the result (C_p as a function of θ) for the inviscid and viscous flow? Also, present the *necessary* and *sufficient* conditions for the occurrence of the flow phenomenon. **[3]**

d) Finally, present a comparative discussion on the drag and lift forces acting on the cylinder—for the inviscid flow as compared to viscous flow. **[3]**

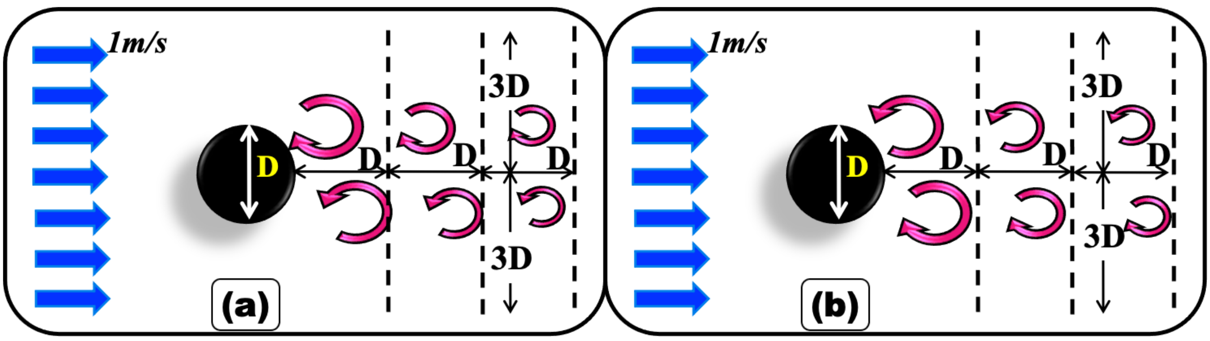
Problem 6: [16 marks] Vorticity and Interaction of Vortices:

a) Take a 2D Cartesian (x-y) element or control volume (CV) of fluid of dimension Δx and Δy. Then, evaluate the right-hand side of the Stokes's theorem $\int \vec{\boldsymbol{\omega}} \cdot d\vec{\boldsymbol{A}} = \oint \vec{\boldsymbol{u}} \cdot d\vec{\boldsymbol{s}}$, where s is the direction tangential to a surface of the CV. Finally, derive the expression for the vorticity ω_z in the z-direction. Here, $\int \vec{\boldsymbol{\omega}} \cdot d\vec{\boldsymbol{A}} = \omega_z \Delta x \Delta y$ and consider u and v as the velocity in the x, y directions, respectively. **[6]**

b) For a 2D free-stream flow (with a horizontal velocity u=1 *m/s* and $v=0$) across a cylinder of diameter D, the figure below shows vortex shedding phenomenon for two-cases: case (a) and case (b) for the *forward* and *reverse* von Karman vortex street, respectively. For the case (a), the figure shows a top-row of clockwise (CW) and bottom-row of counter-clockwise (CCW) vortices; vice-versa for the case (b). Note that the strength of the shed-vortices reduces as they move downstream Considering the *u-velocity induced* by the *interaction* of counter-rotating *vortices* for both the cases, present as follows:

   i. Draw an arrow-based direction of the induced u-velocity, in-between the counter-rotating vortices; separately for the two cases. **[4]**

   ii. Draw a schematic-representation of the qualitative trend of variation of streamwise u-velocity as a function of y-coordinate (along the dashed lines in the figure, varying from *3D* above and *3D* below the horizontal centreline of the cylinder) at a horizontal distance *D, 2D,* and 3*D* behind the cylinder. Draw the three u-velocity profiles for both the cases separately, and discuss the characteristics of resulting velocity profiles behind the cylinder. **[6]**



APPENDIX:

<u>Conservation Equation in Integral Form</u>

$$\boxed{\frac{d}{dt}\int_V \rho\phi\, dV + \int_A \rho\phi\mathbf{u}\cdot d\mathbf{A} = S}$$

where (i) V and A are the control volume and control surface, respectively (ii) $\phi = 1$ and $S = 0$ for the conservation of mass, (iii) $\phi = \mathbf{u}$ and $S = \mathbf{F}$ for the conservation of momentum, $\mathbf{F}$ being the net force acting on the control volume.

Mass conservation

$$\boxed{\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

Momentum conservation (incompressible)

x direction – $$\boxed{\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}$$

y direction – $$\boxed{\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)}$$

z direction – $$\boxed{\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)}$$