Department of Mechanical Engineering, IIT Bombay
Ph.D. Qualifier Examination in Heat Transfer (March 2022)

Time Duration: 3 hours **Max. marks: 100**

Note:

1. All questions are compulsory. This paper consists of 6 questions over 4 pages.
2. The maximum marks per question are given in parenthesis.
3. **Please begin each question on a new page and keep ALL subparts of a question together.** This is important as multiple faculty are involved in grading.
4. Strike out all unwanted work neatly, else, the work that appears first will be taken up for evaluation.
5. This is an open book examination. Only hard copy of the text book **'Fundamentals of Heat and Mass Transfer'** by Incropera, DeWitt, Bergman and Lavine is allowed. Hand written notes, or any other text are not allowed.
6. Make suitable assumptions when needed and clearly state them.

(16 marks Total) Question 1: Consider a cylindrical nuclear fuel rod of 25 mm diameter that sees uniform internal heat generation of 50 MW/m^3. It is clad with another material of thickness 5 mm (in which there is no heat generation). An annular space is present outside the cladding surface through which cooling water (at high pressure of 100 bar) flows. The thermal conductivity of the nuclear fuel rod material can be considered as 20 W/m-K, while that of the cladding can be considered as 150 W/m-K. The fuel rod length is 3 meters. Assume only 1-D radial conduction. For simplicity, assume that the outer surface of the cladding has uniform temperature throughout its length (in line with the 1-D conduction assumption). For heat transfer calculation from the cladding surface, the temperature of the flowing water can be assumed to be the average of the inlet and outlet water temperatures. The heat transfer coefficient on the outer cladding surface can be assumed as 10000 W/m^{2}K, the mass flow rate of water can be assumed as 2 kg/s, C_p of water can be assumed 4200 J/kg-K, and the inlet temperature of water can be assumed as 50°C.

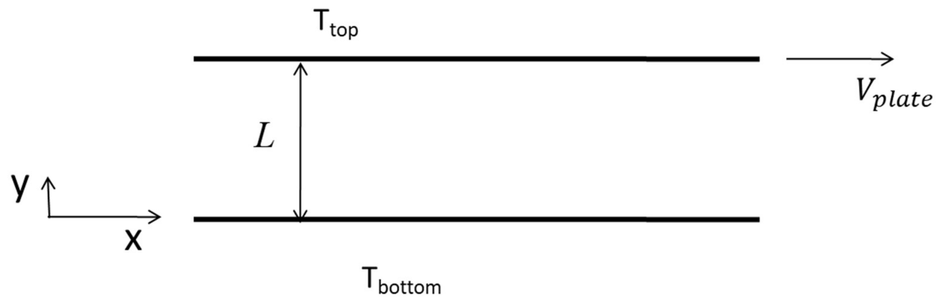
(a) What is the maximum temperature inside the fuel rod?
(b) What is the temperature at the interface between the fuel rod and the cladding?
(c) What is the temperature on the outer surface of the cladding?
(d) Draw a temperature profile as a function of radius and mark critical points.

(18 marks Total) Question 2: Consider a ball made of copper at 1000 K with diameter equal to 10 mm. It is kept in ambient environment at 300 K. The ball loses heat by convection and radiation (assume emissivity of 1). Use proper convective heat transfer correlations and other thermal properties for copper and air from the text book (Incropera and DeWitt). Try to linearize the radiation equation to obtain an equivalent heat transfer coefficient for radiation. Also assume convective and radiative heat transfer coefficient do not change significantly over the time period.

Check if it is possible to use a lumped capacitance model. Justify its use and then write an equation for obtaining the temperature at the surface of the ball as a function of time. What is the temperature after 20 s?

How much do both coefficients change after 20 s? How will you provide a correction for this? Is the lumped capacitance model valid during this time frame?

(16 marks Total) Question 3: Consider two parallel plates of very long dimensions in the X and Z directions. The top plate moves at a velocity, V_{plate}, and is at a temperature T_{top}. The bottom plate is stationary, i.e., velocity is zero, and is maintained at a temperature T_{bottom}. Distance between plates is L. Assume steady, laminar, Newtonian, incompressible flow with constant viscosity. In addition, assume fully developed conditions prevail with zero pressure gradient and no body forces. However, viscous dissipation **cannot** be ignored.



Starting from continuity, momentum and energy equations (given below) listing all assumptions, simplify the equations to obtain the following:

a. Non-dimensionalize the simplified momentum equations and obtain the velocity distribution between the plates. **[6 marks]**
b. Non-dimensionalize energy equation (after simplifications) and identify the non-dimensional numbers you obtain. **[6 marks]**
c. Obtain non-dimensional temperature distribution based on the boundary conditions provided above. **[4 marks]**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} - \rho g + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$

$$\rho c_P\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2$$

(10 marks Total) Question 4: Consider steady, incompressible flow through a circular tube with uniform heat flux specified at the walls. Assume the flow to be plug (or slug) flow (i.e., velocity is uniform across the cross-sectional area along the entire length of the tube). Assume thermally fully developed flow and no slip conditions at the walls.

a. List all assumptions and simplify the governing equations. **[4 marks]**

b. Calculate the temperature distribution and Nusselt number. **[6 marks]**

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{\partial}{\partial z}(\rho u) = 0$$

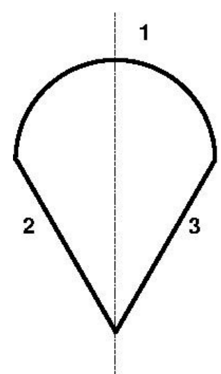
$$\rho\left[u_r\frac{\partial u_r}{\partial r} + u\frac{\partial u_r}{\partial z}\right] = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_r}{\partial r}\right] + \frac{\partial^2 u_r}{\partial z^2} - \frac{u}{r^2}\right]$$

$$\rho\left[u_r\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{\partial^2 u}{\partial z^2}\right]$$

$$\rho c_p\left[u_r\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right] = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right]$$

(20 marks Total) Question 5: Consider an infinite enclosure that is formed by three opaque surfaces with cross-section as shown in the figure. The enclosure can be considered evacuated. The diameter of the semi-circular surface is 2 m, while the length of each of the two straight surfaces is also 2 m. The three surfaces are all opaque, diffuse and gray with emissivity equal to 0.5.

Surface 1 is maintained at 1000 K, surface 2 at 800 K and surface 3 at 800 K. Find the net rate of radiative heat transfer between each of the surfaces per unit length of the enclosure.



(20 marks Total) Question 6:

a. **(4 marks)** Consider the situation depicted below. To the left of the fin base, isothermal processing of materials happens, and to the right, ambient temperature is maintained at a constant value. Assume that fins effectively improve heat transfer from the fin base to ambient. Assume one-dimensional heat flow in the fin as well. You are asked to draw one-dimensional temperature profiles in the base (heat flowing from $T_{process}$ to $T_{ambient}$) for the following two cases:

i) Case 1: Without fins attached to the fin base **(1 mark)**

ii) Case 2: With fins attached to the fin base **(2 marks)**

Ensure that you have both the cases captured in a single graph and explain the qualitative features briefly. **(1 mark)**

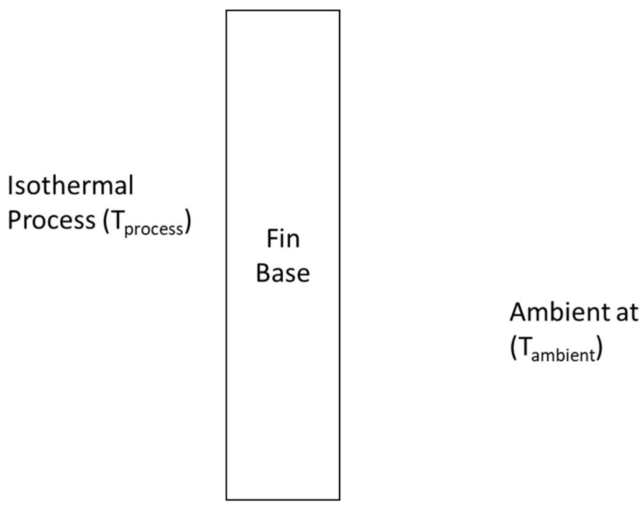


Figure: Sketch for Question 6a

b. **(16 marks)** Consider a wall made of low-carbon steel that separates two tanks of liquid maintained at different temperatures, T_{hot} = 500 K and T_{cold} = 400 K. The thickness of the low-carbon steel wall, whose cross-sectional area is 1 m^2, is 8 mm. The average heat transfer coefficient between the wall and the liquid is 5000 W/m^{2}K (on both sides). The wall is at a steady state initially because the wall has been exposed to the fluid in the tanks for a long time. At some time, t = 0, both tanks are drained, and then both sides of the wall are exposed to gas at T_{gas} = 300 K. The average heat transfer coefficient between the walls and the gas is h_{gas} = 100 W/m^{2}K (on both sides). Assume that the process of draining liquid and exposure to gas occurs instantaneously.

Sketch the temperature distribution in the wall at t = 0 and also at t = 0.5 s, 5 s, 50 s, 500 s, and 5000 s. Ensure you have captured the qualitative features correctly and provide an order of magnitude estimate of time to reach a steady-state condition after exposure to gas.

(Order of magnitude estimate and temperature calculations: (4+3 = 7) marks.
Sketches at 0, 0.5, 5 seconds: 5 marks,
Sketches for 50, 500 and 5000 seconds: 4 marks)

===THE END===