(1)

Chvorinov's Eqn

$$\left(\frac{V}{A}\right)^2_{riser} > \left(\frac{V}{A}\right)^2_{casting}$$

Heat transfer in riser is slower than mold therefore we can have smaller riser

$$n\left(\frac{V}{A}\right)^2_{riser} > \left(\frac{V}{A}\right)^2_{casting}$$

$$3\left(\frac{V}{A}\right)^2_{riser} \geq \left(\frac{V}{A}\right)^2_{casting}$$

$$3\left(\frac{V}{A}\right)^2_{riser} = (30)^2$$

$$\left(\frac{V}{A}\right)_{riser} = \frac{30}{\sqrt{3}} = 17.32$$

$V = \pi r^2 h$

$A = 2\pi r h$ (Top is insulated ∴ heat transfer is only @ curved surface

$$\frac{V}{A} = \frac{\pi r^2 h}{2\pi r h} = 17.32$$

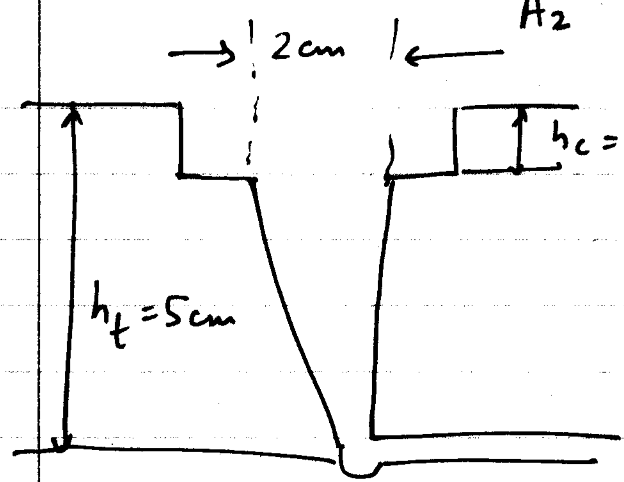
$r = 34.64$ $d = 69.28$

A common mistake will be to equate

$$\left(\frac{V}{A}\right)^2_{riser} = 3\left(\frac{V}{A}\right)^2_{casting}$$

but this will result in over design because heat transfer is slower in riser to begin with even if you equate $\left(\frac{V}{A}\right)^2_{riser} = \left(\frac{V}{A}\right)^2_{casting}$ you will be safe and therefore because of this condition you can get a much smaller riser to serve the purpose.

②



For no aspiration,

$$\left(\frac{A_3}{A_2}\right) = \left(\frac{V_2}{V_3}\right) = \sqrt{\frac{h_c}{h_t}} = \sqrt{\frac{2}{5}}$$

$$\left(\frac{A_3}{A_2}\right) = \sqrt{\frac{2}{5}}$$

$$A_3 = \frac{\pi}{4}(2)^2 \cdot \sqrt{\frac{2}{5}} = 1.99$$

$$d_3 = 1.59 \text{ cm}$$

$$V_3 = \sqrt{2gh_t} = \sqrt{2 \times 9.8 \times \frac{5}{100}} = 0.99 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{2500 \times 0.99 \times 0.0159}{2.25 \times 10^{-3}}$$

$$Re = 17{,}490$$

It is not laminar but mixture of laminar & turbulent but less than < 20,000. Higher than 20,000 is unacceptable.

(3)

$T_o = 28\,^\circ C$

Metal Casting

$T_m = 1540\,^\circ C$

$H_f = 272\ KJ/Kg \cdot K$

$\rho_{Casting} = 7850\ Kg/m^3$

$K_{casting} = 83\ W/m \cdot K$

Sand

$\rho_m =$ ~~7850~~ $1600\ Kg/m^3$

$K_m = 0.8655\ W/m \cdot K$

$C_m = 1.17\ KJ/Kg$

Test if the case qualifies for insulating mold:

$k_m << K_{casting}$

$0.8655 << 83$ (True)

$\alpha_m << \alpha_{casting}$

$$\frac{k_m}{\rho_m \cdot C_m} << \frac{K_{casting}}{\rho_{casting} \cdot C_{casting}}$$

This mold is an insulating mold.

$$t_s = \left[\frac{\pi}{4}\left(\frac{\rho_{casting}\,\Delta H_f}{T_m - T_o}\right)^2 \frac{1}{K_m \rho_m C_m}\right]\left(\frac{V}{A}\right)^2$$

For slab,

$V = lbh$

$A = 2lb + bh + lh$

$l, b >> h$

$\therefore \quad 2lb + bh + lh \simeq 2lb$

$V = lbh$

$A = 2lb$

$\therefore \left(\frac{V}{A}\right) = \left(\frac{h}{2}\right)$

$$t_s = \left(\frac{V}{A}\right)^2 \left[\frac{\pi}{4}\left(\frac{7850\ \frac{kg}{m^3} \cdot 272\ KJ/kg}{(1540-28)\ K}\right)\left(\frac{1}{\frac{0.8655}{1000}\ \frac{KJ/s}{m \cdot K} \cdot 1600\ kg/m^3 \cdot \left(\frac{1.175}{kg \cdot K}\right)}\right)\right]$$

$$t_s = 966697\ \frac{s}{m^2}\left(\frac{0.1\,m}{2}\right)^2$$

$$= \frac{966697 \times 0.01}{2 \times 2} = 2417\ sec = 0.67\ hrs$$

For Circle,

$$\left(\frac{V}{A}\right)^2 = \left[\frac{4/3\pi r^3}{4\pi r^2}\right]^2 = \left[\frac{4/3\pi 5^3}{4\pi 5^2}\right]^2 cm^2$$

$$= \left[5/3\right]^2 cm^2$$

$$= 2.78 \times 10^{-4}$$

$$\therefore\ ts = 966697 \times 2.78 \times 10^{-4} \text{ secs}$$

$$= 269\,s = 0.0745 \text{ hrs.}$$

(4) Shrinkage is avoided in figure 6 because the rounded corners eliminate the thicker region.
Shrinkage is avoided with the chill because it increases the rate of cooling in the thicker area.