1. Temperature raise for infinite line heat source:-

$$dT(x,y,z,t) = \frac{\delta q}{\rho c(4\pi\alpha(t-t'))^{3/2}} \exp\left[-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4\alpha(t-t')}\right]$$

If q_e is watt per unit thickness. Then:-

$$dT(x,y,z,t) = \frac{q_e\,dt'\,dz}{\rho c(4\pi\alpha(t-t'))^{3/2}} \exp\left[-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4\alpha(t-t')}\right]$$

Integrating the above expression ($-\infty$ to ∞) along the thickness (z-axis)

$$dT(x,y,z,t) = \frac{q_e\,dt'}{\rho c(4\pi\alpha(t-t'))^{3/2}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4\alpha(t-t')}\right] dz$$

$$dT(x,y,t) = \frac{q_e\,dt'}{4\pi k(t-t')} \exp\left[-\frac{(x-x')^2+(y-y')^2}{4\alpha(t-t')}\right]$$

converting it to fixed co-ordinate system:-

$$dT(x,y,t) = \frac{q_e\,dt'}{4\pi k(t-t')} \exp\left[-\frac{(x-vt'-x')^2+(y-y')^2}{4\alpha(t-t')}\right]$$

Integrating with respect to time

$$T(x,y,z) - T_0 = \int_0^t \frac{q_e}{4\pi k(t-t')} \exp\left[-\frac{(x-vt'-x')^2+(y-y')^2}{4\alpha(t-t')}\right] dt'$$

for steady state $t \to \infty$

$$T(x,y,t) - T_0 = \int_0^{\infty} \frac{q_L}{4\pi k(t-t')} \exp\left[-\frac{(x-vt'-x')^2+(y-y')^2}{4\alpha(t-t')}\right] dt'$$

assume $t - t' = z$

$$\rightarrow T(x,y,t) - T_0 = \int_0^\infty \frac{q_e}{4\pi k z} \exp\left[-\frac{(x+vz)^2 + y^2}{4\alpha z}\right] dt'$$

$$T(x,y,t) = T_0 + \frac{q_e}{2k\pi} \exp\left(-\frac{vx}{2\alpha}\right) \text{BesselK}\left[0, \frac{\sqrt{\frac{v^2}{\alpha}}}{2\sqrt{\frac{\alpha}{x^2+y^2}}}\right]$$

$$\Rightarrow T(x,y,t) = T_0 + \frac{q_e}{2k\pi} \exp\left[-\frac{vx}{2\alpha}\right] \text{BesselK}\left[0, \frac{v\sqrt{x^2+y^2}}{2\alpha}\right]$$

BesselK $[0, 0] \rightarrow \infty$

$\therefore$ Max temperature rise location is at $(0, 0)$.

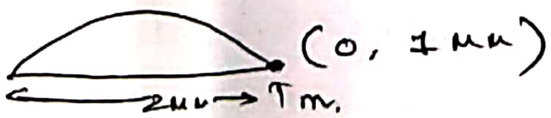
<u>Power</u> :-

$v = 40$ mm/sec. $= 0.04$ m/sec.

bead width $= 2$ mm ; $\rho = 7870 \frac{kg}{m^3}$

$c = 452 \frac{J}{kg\,k}$, $k = 0.073 \times 10^3$ W/m , $T_0 = 300k$

$T_{melt} = 1538°C$ $T_{tran} = 850°C$, $\alpha = \frac{0.073 \times 10^3}{7870 \times 452}$



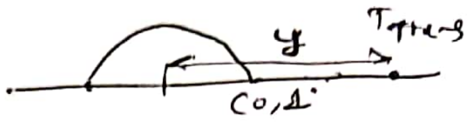
$$\therefore 1538 - 25 = \frac{q_e}{2 \times 0.073 \times \pi \times 10^3} \times \text{BesselK}\left[0, \frac{0.04 \times 0.001}{2\alpha}\right]$$

$$\Rightarrow 1513 = \frac{q_e \times 0.4366}{2 \times 0.073 \times 10^3 \times \pi}$$

$\rightarrow q_l = 1.5895\times10^6 \ \frac{W}{m}$

$\rightarrow \boxed{Q = 7.94\times10^3 \ W}$

Extent of HAZ :-



$T_{trans} = 850°$

$$850° - 25° = \frac{1.5895\times10^6}{2\times0.073\times10^3\times\pi} \times \text{BesselK}\left[0, \frac{0.04\times y}{2\times\alpha}\right]$$

~~800~~

$$825 = \cancel{1.5895\times10^6}\ 3.46\times10^3 \times \text{BesselK}[0, M]$$

where $\boxed{M = \frac{0.04\times y}{2\alpha}}$

$$\text{BesselK}[0, M] = 0.2381$$

$$M = 1.418$$

$$\rightarrow y = \frac{2\alpha\times 1.418}{0.04\times1}$$

$$\boxed{y = 1.4486 \text{ mm}}$$

Q2. Differential eqⁿ for conduction of heat in stationary medium.
(No convention & radiation)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + q = \frac{1}{2}\frac{\partial T}{\partial t}$$

It's satisfied by the solution for infinite body

$$dT(x,y,z,t) = \frac{\delta q}{\delta c\,(4\pi a(t-t'))^{3/2}}\, e^{-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4a(t-t')}}$$

Where, ρ = density
t = time
V_1 = thermal conductivity
δq = instantaneous heat generated
C = specific heat capacity

for moving point source (along x-axis) (in stationary frame)

$$dT(x,y,z,t) = \frac{2\delta q}{\delta c\,(4\pi a(t-t'))^{3/2}}\, e^{-\frac{(x-vt'x')^2+(y-y')^2+(z-z')^2}{4a(t-t')}}$$

for a line source along z axis.

$$\delta q = q_e\, dt' dz$$

$$\therefore dT(x,y,z,t) = \frac{2q_e\, dt'}{\delta c\,(4\pi a(t-t'))^{3/2}} \int_{-\infty}^{\infty} e^{-\left(\frac{(x-vt'x')^2+(y-y')^2+(z-z')^2}{4a(t-t')}\right)}$$

Integrating and also assuming $t' = -\tau_a$ time $t=0$ when line source is at origin

$$\therefore T - T_0\,(x,y,t) = \int_0^t \frac{q_e\, d\tau}{4\pi k \tau}\, e^{\left[\frac{-(x+vt)^2+y^2}{4a\tau}\right]}$$

Temp rise at current time for infinite instantaneous heat source released at time t

$$T - T_0 = \frac{Q}{4\pi k t}\, e^{-r^2/4at}$$

Time for thick welding $v=0$

$$800 - T_0 = \frac{Q}{4\pi k t_8}$$

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$$t_8 = \frac{Q}{4\pi k\,(800 - T_0)}$$

$$500 - T_0 = \frac{Q}{4\pi k t_5} \Rightarrow t_5 = \frac{Q}{4\pi k\,(500 - T_0)}$$

$$t_{8/5} = t_5 - t_8 = \frac{Q}{4\pi k}\left(\frac{1}{(500 - T_0)} - \frac{1}{(800 - T_0)}\right)$$

Q3 Given, plate thickness = 12mm = h

Welding Conditions :- 25V, 300A & $\eta = 0.9$

$T_0 = 25°C$

$8s \leq \tau_{8/5} \leq 40s$

To avoid martensitic zone, let welding speed be v

$$\lambda = h\sqrt{\frac{\rho c\,(550 - T_0)}{Q}} \quad \& \quad Q = \frac{\eta VI}{v}$$

Firstly, considering the plate to be thin,

$$\tau_{8/5} = \frac{(Q/h)^2}{4\pi k\rho c}\left[\left(\frac{1}{500 - T_0}\right)^2 - \left(\frac{1}{800 - T_0}\right)^2\right]$$

Now $Q = \dfrac{0.9 \times 25 \times 300}{v} = \dfrac{6750}{v}$

So $8s \leq \tau_{8/5} \leq 40s \Rightarrow$

$$8 \leq \frac{1}{4\pi \times 0.04 \times 10^3} \times \left(\frac{6750}{v \times 12 \times 10^{-3}}\right)^2 \times \frac{1}{4.8 \times 0.5 \times 10^2}\left[\left(\frac{1}{475}\right)^2 - \left(\frac{1}{775}\right)^2\right] \leq 40$$

$$8 \leq \frac{4.47 \times 10^{-4}}{v^2} \leq 40$$

$$0.75 \times 10^{-2} \geq v \geq 0.33 \times 10^{-2} \text{ m/s} \quad \text{— (1)}$$

If the plate is thick,

$$\tau_{8/5} = \frac{Q}{4\pi k}\left[\frac{1}{(500 - T_0)} - \frac{1}{(800 - T_0)}\right]$$

$$\tau_{8/5} = \frac{6750}{v \times 4\pi \times 0.04 \times 10^3}\left[\frac{1}{475} - \frac{1}{775}\right] = \frac{0.011}{v}$$

$$8s \leq \tau_{8/5} \leq 40s$$

$$8 \leq \frac{0.011}{v} \leq 40$$

$$\Rightarrow \quad 0.1375 \times 10^{-2} \geq v \geq 0.0275 \times 10^{-2} \text{ m/s} \quad -(2)$$

Now putting Q in expression of λ

$$\lambda = h\sqrt{\frac{\rho c(550 - T_0)}{Q}} = 12 \times 10^{-3}\sqrt{\frac{7.8 \times 0.5 \times 525 \times 10^6}{(6750)/v}}$$

$$\lambda = 6.61\sqrt{v} \quad -(3)$$

We know, $\lambda > 0.75 \Rightarrow$ Thick Plate & $\lambda < 0.75 \Rightarrow$ Thin plate

So from (1), (2) & (3)

Assuming thin plate $\lambda = 6.61\sqrt{v} \Rightarrow$

$$6.61\sqrt{0.75 \times 10^{-2}} > \lambda > 6.61 \times \sqrt{3.3 \times 10^{-3}}$$

$$0.57 > \lambda > 0.38.$$

i.e. $\lambda < 0.75$ (satisfied)

So, our assumption can be true.

If thick plate is assumed

$$6.61 \times \sqrt{0.1375 \times 10^{-2}} > \lambda > 6.61\sqrt{0.0275 \times 10^{-2}}$$

$$0.25 > \lambda > 0.11$$

So $\lambda > 0.75$ is not satisfied

Here plate should be thin &

~~0.75 × 10~~

$$0.75 \times 10^{-2} > v > 3.3 \times 10^{-3} \text{ m/s}$$

Q4. $h = 25\,mm$ $\quad Q = \frac{6750}{v}$

$$\lambda = h\sqrt{\frac{\rho c(550 - T_0)}{Q}} = 25\times10^{-3}\sqrt{\frac{7.8\times10^3\times 0.5\times10^3\times(525)v}{6750}}$$

$$= 13.7689\sqrt{v}$$

For thick plate assumption $\lambda = 13.7689\sqrt{v} > 0.75$

$$\sqrt{v} > 0.0545$$

$$v > 2.967\times10^{-3}\,m/s \quad -\ 1$$

Also, $8 < \tau_{8/5} = \frac{Q}{4\pi k}\left[\frac{1}{(500 - T_0)} - \frac{1}{(800 - T_0)}\right] < 40$

$$8 < \frac{6750}{v(4\pi 40)}\left[\frac{1}{475} - \frac{1}{775}\right] < 40 \quad -\ 2$$

$$2.7359\times10^{-4} < v < 1.3679\times10^{-3}\,m/s \quad -\ 2$$

which doesn't satisfy 1, hence wrong assumption.

Hence, assuming thin plate

$$\lambda = 13.7689\sqrt{v} < 0.75$$

$$\therefore v < 2.967\times10^{-3}\,m/s$$

Also $8 < \left(\frac{Q}{h}\right)^2 \frac{1}{(4\pi k\rho c)}\left[\frac{1}{(500 - T_0)^2} - \frac{1}{(800 - T_0)^2}\right] < 40$

$$8 < \left(\frac{6750}{25\times10^{-3}}\right)^2 \frac{1}{(4\pi\times40)\ 7.8\times\frac{1}{2}\times10^6}\left[\frac{1}{(475)^2} - \frac{1}{(775)^2}\right] < 40$$

$$8 < \frac{1.029\times10^{-4}}{v^2} < 40$$

$$1.6039\times10^{-3} < v < 3.5865 \quad -\ 4$$

from 3 & 4

$$1.6039\times10^{-3} < v < 2.967\times10^{-3}\,m/s$$

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