

ME 338
Manufacturing Processes II
HW#1

1. Consider the Merchant's Cutting Force diagram and tool-chip interface presented in the lecture notes. Consider the directions of the cutting force and the thrust force. Will the F_c , cutting force, be always positive and why? Is the thrust force, F_t , also positive at all times? If not, explain why. Show from force expressions how you can make $F_t = 0$ for a given friction coefficient between the tool and the work piece. Please provide physical explanations of your suggestions/recommendations.

$$F_t = R \sin(\beta - \alpha) \text{ and } F_t = F_c \tan(\beta - \alpha)$$

The magnitude of F_c is always positive (since the direction of the feed is always towards the material); the sign of F_t can be either positive or negative, depending on the relative magnitudes of β and α . When $\beta > \alpha$, the sign of F_t is positive (downward) and when $\beta < \alpha$, it is negative (upward). Therefore, in order to make the thrust force = 0, we make $\beta = \alpha$. This can be achieved in two ways:

- 1) Change the rake angle of the tool, α .
- 2) By adding lubricant, effectively changing the friction angle β .

It is the second step that we most commonly employ in machining when something is "wrong" in the cut.

2)

Given,

$$F_c = 950 \text{ N}$$

$$w = 2.5 \text{ mm}$$

$$t_o = 0.25 \text{ mm}$$

$$F_t = 475 \text{ N}$$

$$\alpha = 0^\circ$$

$$t_c = 0.75 \text{ mm}$$

we have.

$$F_t = F_c \tan(\beta - \alpha)$$

$$\Rightarrow \frac{475}{950} = \tan \beta = \mu$$

$$\Rightarrow \boxed{\mu = 0.5}$$

$$\frac{t_o}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

cutting ratio

we have

$$\Rightarrow \frac{0.25}{0.75} = \tan \phi$$

$$\Rightarrow \phi = 18.43^\circ$$

$$\begin{aligned} \text{we have } \tau &= \frac{F_c \sin \phi \cos(\phi + \beta - \alpha)}{w t_o \cos(\beta - \alpha)} = \frac{950 \times \sin(18.43) \cos(18.43 + 26.56)}{2.5 \times 0.25 \times 10^{-6} \times \cos 26.56} \\ &= 379.951 \text{ MPa} \end{aligned}$$

$$\text{By Tresca Criterion } \frac{\sigma_2}{2} = \tau \Rightarrow \text{US stress} = 759.9 \text{ MPa}$$

$$\text{By VonMises Criterion } \frac{\sigma_2}{\sqrt{3}} = \tau \Rightarrow \text{US stress} = 658 \text{ MPa}$$

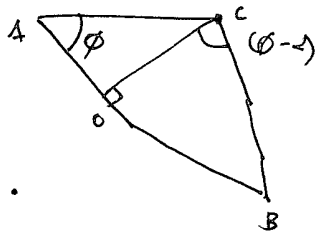
(3). $V = 280 \text{ m/min}$
 $\alpha = 10^\circ$
 $t_0 = 2 \text{ mm}$
 $\text{feed} = 0.2 \text{ mm/rev.}$
 $\mu = 0.5$
 $T_s = 400 \text{ N/mm}^2$

(a) Merchant's equation $2\phi + \beta - \alpha = \pi/2$
 $\beta = \tan^{-1}(0.5) = 26.565^\circ$
 $2\phi + 26.565 - 10 = 90$
 $\phi = 36.7175^\circ$

(b) $F_c = \frac{T_s \omega t_0 \cos(\beta - \alpha)}{\sin\phi \cos(\phi + \beta - \alpha)} = \frac{T_s \omega t_0 \cos(\pi/2 - 2\alpha)}{\sin\phi \cos(\pi/2 - \phi)}$
 $= \frac{2T_s \omega t_0 \cos 2\alpha}{2\sin\phi} = 429 \text{ N}$

$F_t = F_c \tan(\beta - \alpha)$
 $= F_c \tan(16.565^\circ)$
 $= 127.56 \text{ N}$

4. The shear plane diagram for the orthogonal cutting is



a) Shear strain in chip

$$\gamma = \frac{AB}{OC} = \frac{AO}{OC} + \frac{OB}{OC} = \cot \phi + \tan(\phi - \alpha)$$

Cutting ratio $r = \frac{\sin \phi}{\cos(\phi - \alpha)}$ from $r = \frac{t_0}{t_c}$

b)

For minimum strain condition

$$\frac{d\gamma}{d\phi} = 0 \Rightarrow \sec(\phi - \alpha) \tan(\phi - \alpha) (-1) \neq 0$$

$$\Rightarrow \phi = \sec^{-1}(\phi - \alpha)$$

$$-\cot \phi \sec(\phi) + \tan(\phi - \alpha) \sec(\phi - \alpha) = 0$$

$$\Rightarrow \frac{\cos(\phi)}{\sin^2 \phi} = \frac{\sin(\phi - \alpha)}{\cos^2(\phi - \alpha)}$$

$$+ \cos^2 \phi = \sec^2(\phi - \alpha)$$

$$\Rightarrow \cos^2(\phi - \alpha) = \sin^2(\phi)$$

$$\rightarrow \sin \phi = \cos(\phi - \alpha)$$

$$(or) \sin \phi = -\cos(\phi - \alpha)$$

~~2nd not possible as it gives~~

1st gives

$$\pi/2 - \phi = \phi - \alpha$$

$$\Rightarrow \pi/2 = 2\phi - \alpha$$

[Taking basic solution only and avoiding

$n\pi/2$ needd]

2nd gives

$$\pi/2 + \phi = \phi - \alpha$$

$$\Rightarrow \alpha = -\pi/2 \text{ (not possible)}$$

$$\Rightarrow \phi = \left(\frac{\pi/2 + \alpha}{2} \right) = \left[\frac{\pi/4 + \alpha/2}{2} \right]$$

c)

we have $r = \frac{\sin \phi}{\cos(\phi - \alpha)}$

$$\text{put } \phi = \pi/4 + \alpha/2$$

$$\Rightarrow r = \frac{\sin(\pi/4 + \alpha/2)}{\cos(\pi/4 - \alpha/2)}$$

d)

Expected value is 1 for $\alpha = \pi/2$

It is physically realisable understanding that the tool just slices off the workpiece and no shear strain is needed to move the chip.



5 Rake angle, $\alpha = 0^\circ$

$$t_c = t_c.$$

$$\tau_f = k \tau_s.$$

$$\Rightarrow \frac{t_o}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)} = \frac{\sin \phi}{\cos \phi} = \tan \phi.$$

$$\Rightarrow \boxed{t_c = \frac{t_o}{\tan \phi}}$$

Contact area of tool chip interface.

$$\boxed{A = \frac{w t_o}{k \tan \phi}}$$

$$\text{Shear force } F_s = F_c \cos \phi - F_f \sin \phi \quad (1)$$

$$\text{Friction force } F = F_c \sin \alpha + F_f \cos \alpha.$$

$$\therefore F = F_f.$$

$$\Rightarrow \underline{F \sin \phi = F_f \sin \phi} \quad (2)$$

Adding (1) & (2)

$$F_s + F \sin \phi = F_c \cos \phi.$$

$$\Rightarrow \tau_s A_s + \tau_f A \sin \phi = F_c \cos \phi.$$

$$\Rightarrow F_c = \frac{\tau_s A_s + \tau_f A \sin \phi}{\cos \phi}$$

$$= \frac{\tau_s A_s + k \tau_s A \sin \phi}{\cos \phi}$$

$$= \tau_s \left[\frac{A_s + k A \sin \phi}{\cos \phi} \right]$$

$$\Rightarrow F_c = \tau_s \left[\frac{\omega t_0}{\sin \phi \cos \phi} + \frac{k \omega t_0}{\tan \phi \cos \phi} \cdot \sin \phi \right]$$

$$= \tau_s \left[\frac{2 \omega t_0}{\sin 2\phi} + k \omega t_0 \right]$$

$$\Rightarrow \boxed{F_c = \tau_s \omega t_0 \left[\frac{k \sin 2\phi + 2}{\sin 2\phi} \right]}$$

Since $\frac{F}{A} = k \cdot \frac{F_s}{A_s} \quad (\tau_f = k \cdot \tau_s)$

$$\Rightarrow \frac{F}{\omega t_0} \tan \phi = \frac{k F_s}{\omega t_0} \sin \phi$$

$$\Rightarrow \boxed{F = k F_s \cos \phi}$$

Now, $F = F_t$

$$\Rightarrow k F_s \cos \phi = F_t$$

$$\Rightarrow F_s = \frac{F_t}{k \cos \phi}$$

From (1)

$$F_c \cos \phi - F_t \sin \phi = \frac{F_t}{k \cos \phi}$$

$$\Rightarrow F_c \cos \phi = F_t \left[\frac{1 + k \sin \phi \cos \phi}{\cos \phi} \right]$$

$$\Rightarrow F_t = \left[\frac{F_c \cos^2 \phi}{1 + k \sin \phi \cos \phi} \right]$$

$$= \left(\frac{2 \omega t_0 \cos^2 \phi}{k \sin 2\phi + 2} \right) \times \tau_s \omega t_0 \left(\frac{k \sin 2\phi + 2}{\sin 2\phi} \right)$$

$$\boxed{F_t = \tau_s \omega t_0 \cot \phi}$$

7. The top view of a tube being turned orthogonally is shown in the Figure 1. Rake angle is $+10^\circ$ and the dynamometer shows $F_x = 1259 \text{ N}$, $F_y = 0$ and $F_z = 1601 \text{ N}$. The axial feed is 100 mm/min and the rotational speed of the spindle is 1000 rpm . The chip thickness is 0.28 mm . Estimate the shear strength of the workpiece material and list the assumptions underlying the theory you are using.

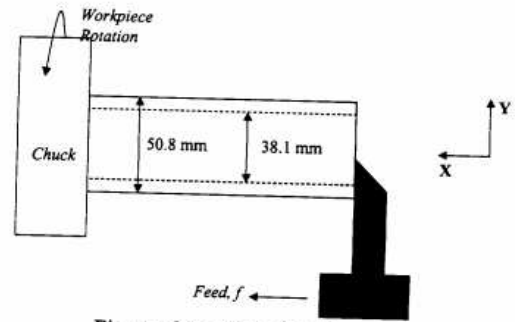


Figure. Top view of cutting process

$$q_{\text{cut}} \quad \alpha = 10^\circ, \quad F_y = F_t = 1259 \text{ N}, \quad F_z = F_c = 1601 \text{ N}$$

$$t_o = \frac{100}{1000} = 0.1 \text{ mm/rev}; \quad \omega = 6.35 \text{ m/s}$$

$$R = \sqrt{F_c^2 + F_t^2} = \sqrt{1601^2 + 1259^2} \text{ N}$$

$$\frac{t_o}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

$$\frac{0.1}{0.28} = \frac{\sin(\phi)}{\cos(\phi - 10^\circ)} \Rightarrow \phi = 0.35 \text{ rad}$$

$$F_c = R \cos(\beta - \alpha) \Rightarrow \beta = 0.84 \text{ rad}$$

$$\tau_s = \frac{F_s}{\omega t_o / \sin \phi} = \frac{R \cdot \cos(\phi + \beta - \alpha)}{\omega t_o / \sin \phi}$$

$$\tau_s = 584 \text{ MPa}$$