

Que 1

Specific grinding energy is 35 W-s/mm³ in Homework
but here que solved by taking it as 25 W-s/mm³

Sheet 1 of 4

Given

$$u = 25 \text{ W-s/mm}^3$$

$$\text{RPM} = 3000$$

$$D = 120 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$C = 5 \text{ grains/mm}^2$$

$$P = 2 \text{ kW}$$

$$V = 1.5 \text{ m/min} \quad \lambda = 10 \quad K_2 = 0.2 \text{ K mm/N}$$

$$P = u \cdot v \cdot d \cdot b$$

$$2000 \text{ W} = \frac{25 \cdot \text{W-s}}{\text{mm}^3} \times \frac{1.5 \times 10^3 \text{ mm}}{60 \text{ s}} \times d \times 20 \text{ mm}$$

$$d = \frac{2000 \times 60}{25 \times 1.5 \times 10^3 \times 20} = 0.16 \text{ mm}$$

Grinding force \times Peripheral velocity = Power

$$\Rightarrow F_g \left(\frac{\pi D N}{60} \right) = \text{Power}$$

$$F_g (\text{N}) \times \frac{\pi \times 120 \times 10^{-3} \times 3000}{60} = 2000 \text{ W}$$

$$F_g = \frac{2000 \times 60}{\pi \times 120 \times 10^{-3} \times 3000} = \frac{1000}{3\pi} = 106.10 \text{ N}$$

Sheet 2 of 4

$$F/\text{grain} = u \times \frac{1}{2} \omega \times t$$

$$= u \times \frac{1}{2} r t^2 \omega$$

$$t = \sqrt{\frac{4v}{V.c.r.}} \sqrt{\frac{d}{D}}$$

$$= \sqrt{\frac{4 \times 1.5 \times 10^3 / 60 \text{ mm/s}}{\left(\frac{\pi D H}{60}\right) \times \frac{5 \times 10}{\text{mm}^2}}} \sqrt{\frac{0.16 \text{ mm}}{120 \text{ mm}}}$$

$$t = \sqrt{\frac{4 \times 1.5 \times 10^3}{\pi \times 120 \times 3000 \times 5 \times 10}} \sqrt{\frac{0.16}{120}} \text{ mm}$$

$$t = 1.97 \times 10^{-3} \text{ mm}$$

$$F/\text{grain} = \frac{25 \text{ W-s}}{\text{mm}^3} \times \frac{1}{2} \times 10 \times (1.97 \times 10^{-3})^2$$

$$= \frac{25 \times 1000 \text{ N} \cdot \text{mm}}{\text{mm}^3} \times 0.5 \times 10 \times (1.97 \times 10^{-3})^2$$

$$= 0.485 \text{ N}$$

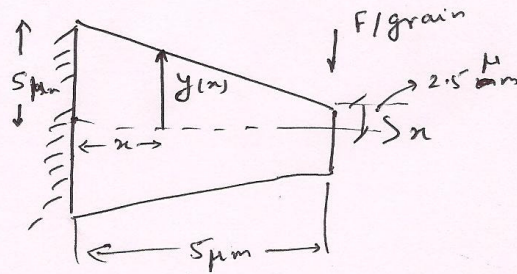
$$K_1 = 49.25 \times 10^3 \frac{\text{N}}{\text{mm}^2 \cdot \text{m}}$$

$$\delta T = K_2 \cdot K_1 \cdot \frac{1}{t} \cdot d = 0.2 \text{ K} \cdot \text{mm} \times 49.25 \times \frac{1}{1.97 \times 10^{-3}} \times 0.16 \text{ mm}$$

$$= 0.2 \times \text{N} \times 49.25 \times 0.16 \times \frac{1}{1.97 \times 10^3} = 800 \text{ K}$$

$$y(0) = 5 \text{ mm}$$

Sheet 3 of 4



$$\frac{y(x) - 5}{x - 0} = \frac{2.5 - 5}{5 - 0}$$

$$y(x) - 5 = \frac{-2.5 \cdot x}{5}$$

$$y(x) - 5 = -\frac{x}{2}$$

$$y(x) = 5 - \frac{x}{2}$$

$$\sigma(x) = \frac{M y(x)}{I(x)}$$

$$= \frac{F \cdot (5-x) y(x)}{\frac{\pi y(x)^4}{4}}$$

$$= \frac{4 F (5-x) \left(\frac{5-x}{2}\right)}{\pi \left(\frac{5-x}{2}\right)^4 \left(\frac{5-x}{2}\right)^3}$$

$$\sigma(x) = \frac{4 F (5-x)}{\pi \left(\frac{5-x}{2}\right)^3}$$

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$$\frac{d\sigma(x)}{dx} = \frac{d}{dx} \left[\frac{4 F (5-x)}{\pi \left(\frac{5-x}{2}\right)^3} \right]$$

Sheet 4 Q4

$$\frac{d}{dn} \left[\frac{4F}{\pi} \frac{(s-x)}{(s-x/2)^3} \right] = 0$$

$$\Rightarrow x = s/2$$

$$\sigma(x) = \frac{4F}{\pi} \left[\frac{(s-s/2)}{(s-\frac{s}{2})^3} \right] \mu\text{m}^2$$

$$= \frac{4F}{\pi} \times \frac{2.5}{52.743} \times \frac{1}{(10^{-6})^2}$$

$$= \frac{4 \times 0.4851}{\pi} \times \frac{2.5}{52.743} \times 10^{12} \frac{\text{N}}{\text{m}^2}$$

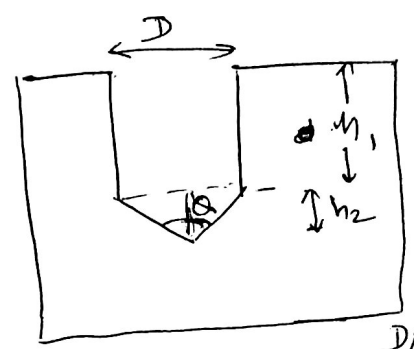
$$\sigma(x) = 29.2 \text{ GPa}$$

Note: Brittle materials do not fail in shear they always fail in tension. Failure in shear is not applicable even though they may show failure in shear as well.

From Matweb, Sintered-SiC has compressive strength 4.6 GPa. Therefore the grain will fail from the center.

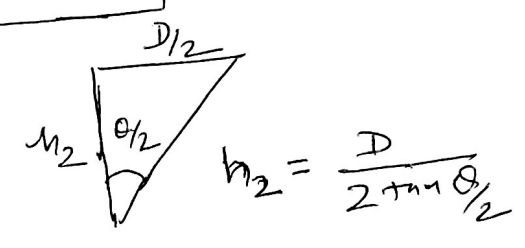
Q.2

Spindle speed \uparrow
 $F = \text{feed rate} = \cancel{v} \times \cancel{N} \times f$
 (mm/min) \downarrow \downarrow \downarrow
 mm/rev



feed = f

Distance travelled
 $d = h_1 + h_2$
 \downarrow \downarrow
 depth of hole Depth due to conical region



Taylor's eqⁿ $\Rightarrow VT^n = C$

$T =$ Time Total for 40 make holes
 $=$ no. of holes \times Time taken for one hole
 $= n_0 \times \frac{d}{F}$

$V = \pi DN$

$VT^n = C$

put V_1, T_1 & V_2, T_2
 using two set of experiments data
 & calculate n & C

Three distinct possibilities exist for the values of ζ [32, 33]:

1. $\zeta > 1$ yields real roots and an over-damped system (Fig. 3.75a)

$$x_{trans}(t) = (c_1 e^{\omega_d t} + c_2 e^{-\omega_d t}) e^{-\zeta \omega_n t}$$

where c_1 and c_2 are unknown coefficients to be determined from the boundary conditions and where ω_d is the frequency of the vibration given by

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n$$

2. $\zeta = 1$ yields two identical roots and a critically damped system (Fig. 3.75b):

$$x_{trans}(t) = (c_1 + c_2 t) e^{-\omega_n t}$$

3. $\zeta < 1$ yields imaginary roots and an under-damped system (Fig. 3.75c)

$$x_{trans}(t) = A e^{-\zeta \omega_n t} \cos(\omega_d t - \psi)$$

where ψ is the phase angle.

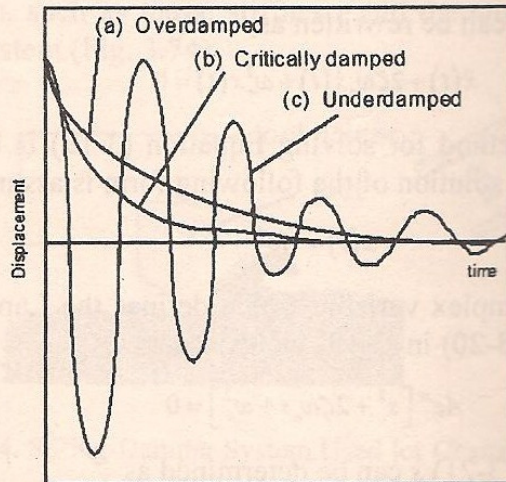


Fig. 3.75. Transient Response of (a) Overdamped, (b) Critically Damped, (c) Underdamped Single Mass Vibrator.

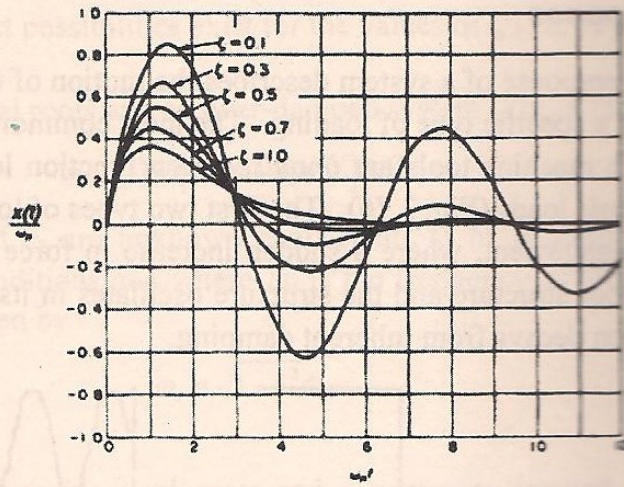


Fig. 3.77. Unit Impulse Response Curves [32]

The unit step response (Fig. 3.78) of the single mass vibrator is

For $\zeta < 1$,

$$x(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin \left[\omega_d t + \tan^{-1} \left(\frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] \quad (t \geq 0)$$

For $\zeta = 1$,

$$x(t) = 1 - e^{-\omega_n t} (1 + \omega_n t) \quad (t \geq 0)$$

For $\zeta > 1$,

$$x(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2-1}} \left(\frac{e^{s_1 t}}{s_1} - \frac{e^{s_2 t}}{s_2} \right) \quad (t \geq 0)$$

where $s_1 = (\zeta + \sqrt{\zeta^2-1})\omega_n$ and $s_2 = (\zeta - \sqrt{\zeta^2-1})\omega_n$.

In designing machine tools, tool vibration should be minimized as a force impulse or step has been encountered. However, critical damping ($\zeta=1$) conditions should not be exceeded since critical damping will minimize the detrimental effects of vibration and chatter, bringing the system to equilibrium conditions without overshoot or oscillation.

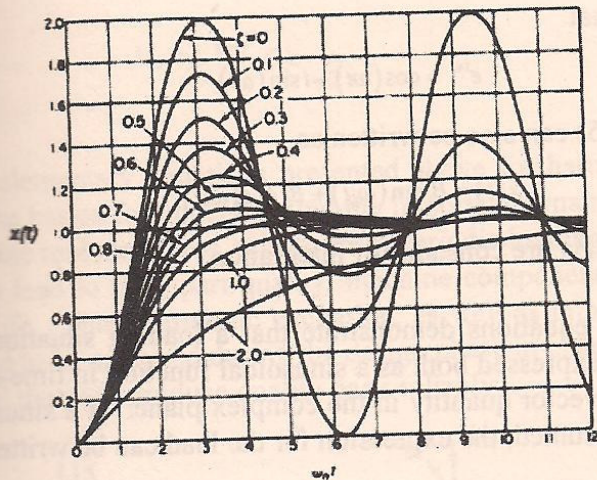


Fig. 3.78. Unit Step Response Curves [32]

al rotation and the reciprocation of other masses during the ma-
 process may result in dynamic instabilities due to the periodic na-
 excitation. This type of load will cause unacceptable structural
 when the forcing frequency coincides with one of the natural fre-
 of the structure. This phenomenon is known as *resonance*. Dur-
 esigning of a machine tool, it is critical to consider the dynamic
 of the machine due to harmonic loading.

er to better introduce the resonance notion, the response of a
 system owing to periodic excitations, is analyzed mathematically
 dering the response of a single mass vibrator. The equation of
 or such a system is given by:

$$\ddot{x}(t) + \omega_n^2 x(t) = 0 \quad (3-33)$$

acing the general solution for the response Equation (3-20) into
 (3-33), the following characteristic equation is obtained,

$$s^2 + \omega_n^2 = 0 \quad (3-34)$$

ields two complex roots $\lambda_{1,2} = \pm i\omega_n$. This gives the following solu-

$$x(t) = A_1 e^{i\omega_n t} + A_2 e^{-i\omega_n t} \quad (3-35)$$

A_1 and A_2 are the constants of integration. For $x(t)$ to be real, A_1
 the complex conjugate of A_2 . This consideration allows the solu-
 Equation (3-35) to be expressed alternatively as a sinusoidal func-