Transfer Function & Chatter

Transfer Function

- Transfer function expresses the relationship between the periodic force F and the vibration X it produces.
- Consider the shown system. Harmonic force F is acting on the mass in the direction of *g*.

$$F = F_o \exp(j\omega t)$$

- Differential equation of motion of the system

$$m\ddot{x} + c\dot{x} + kx = F_o \exp(j\omega t)$$

- Steady state solution has the form

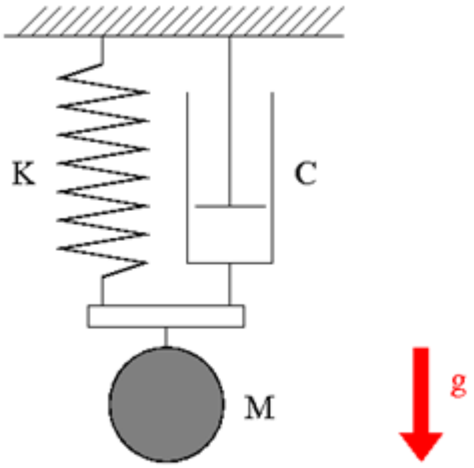
$$x = X \exp(j\omega t)$$

- Eq. 1 solves for the complex amplitude X,

$$\mathbf{X} = F_o/(k - m\omega^2 + jc\omega)$$

- Transfer function G, being the ratio of the output amplitude X over the inpurt amplitude F_o

$$G = X/Fo = 1/(k - m\omega^2 + jc\omega)$$



Transfer Function

- Introducing the following notation,
- $k/m = \omega_n^2$, which is the square of the natural frequency of the system
- $c/2\sqrt{km} = \zeta$, which is the damping ratio of the system
- $p=f/f_n$

$$G = \frac{1/k}{1-p2+2j\zeta p}$$

- Special features of the TF are obtained at several frequencies,

1. At $f/f_n = 0$, the value of the TF is $G=1/k$. Is is purely real and it is the static flexibilty.
2. At $f/f_n = 1$, the phase shift between X and F_o is π/2. This is called resonance. The value of G is purely imaginary and it is very close to being the maximum negative imaginary. Its value is $G= -j/2k\zeta$
3. The maximum of the magnitude |G| occurs close to resonance; to be precise, it occurs at

$$f/f_n = \sqrt{(1-\ \zeta 2)}$$

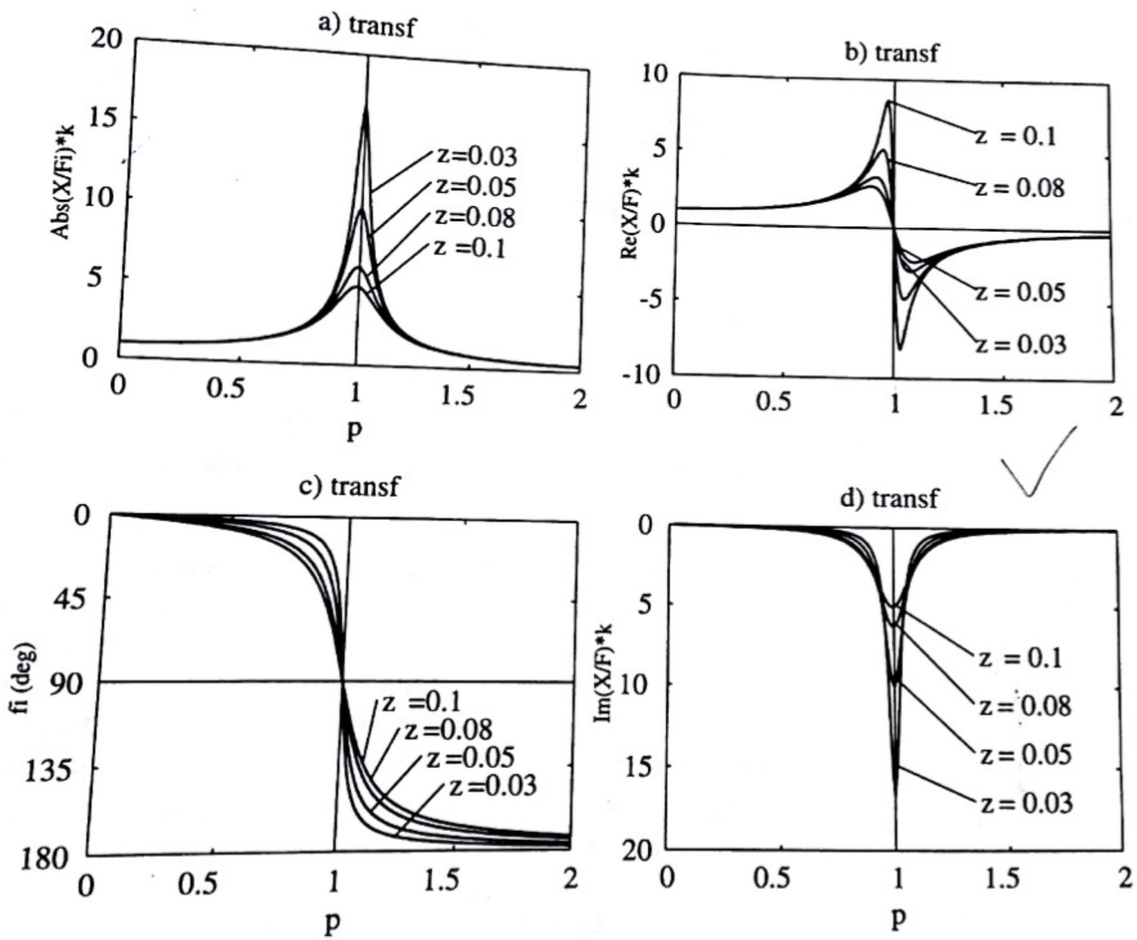
   and it is approximately, $|G| = 1/2k\,\zeta$

4. The real part of Re[G] has a special significance for the limit of stability of chatter. It has value of G = 1/k at $f/f_n = 0$, the value of Re[G] = 0 at $f/f_n = 1$, and it has two extremes. Approximately these occurs at frequencies $f/f_n = 1 + \zeta$ and 1- ζ and their values are,

$$Re[G]_{max} = \frac{1}{4k\zeta\,(1-\zeta)} \qquad Re[G]_{min} = \frac{1}{4k\zeta\,(1+\zeta)}$$

Typical Plots of Transfer Function

Absolute Real



Phase Imaginary

Chatter

- Chatter is a self-excited type of vibration that occurs in metal cutting if the chip width is too large with respect to the dynamic stiffness of the system.
- Under such conditions these vibrations start and quickly grow. The cutting force becomes periodically variable, reaching considerable amplitudes, the machined surface becomes undulated and the chip thickness varies in the extreme so much that it becomes dissected.
- The most significant cutting parameter, which is decisive for the generation of chatter, is the width of cut (width of chip) b.
- For sufficiently small chip widths, cutting is stable, without chatter.
- By increasing b chatter starts to occur at a certain width b_{lim} and becomes more energetic for all values of $b > b_{lim}$.
- The value of b_{lim} depends on the dynamic characteristics of the structure, on the workpiece material, cutting seed and feed and on the geometry of the tool.
- In milling, the cumulative chip width b_{cum} has to be considered, which is the sum of the chip widths of all the teeth cutting simultaneously.

Limit of Stability of Chatter

- **Simplifications**

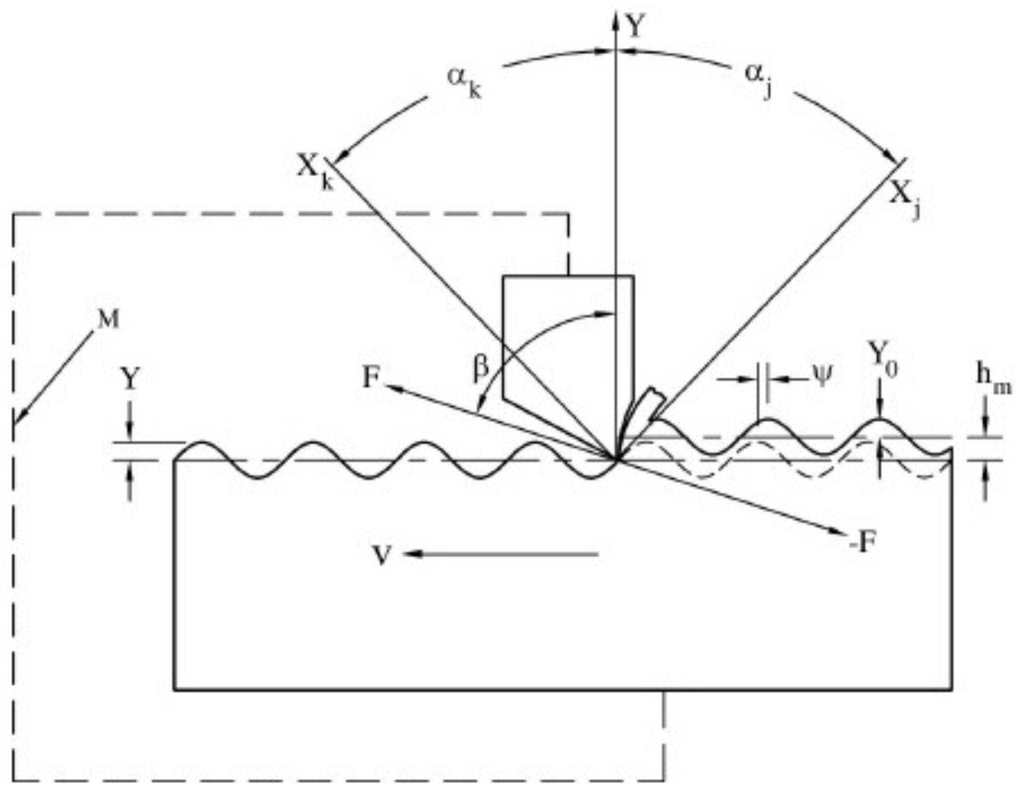
a) The vibratory system of the machine is linear.

b) The direction of the variable component of the cutting force is constant.

c) The variable component of the cutting force depends only on vibration in the direction of the normal to the cut surface *(Y)*.

d) The value of the variable component of the cutting force varies proportionally and instantaneously with the variation of chip thickness.

e) The frequency of the vibration and mutual phase shift of undulations in subsequent overlapping cuts are not influenced by the relationship of wavelength to the length of cut.



Stability Limits

- The structure is a vibratory system that is characterized by the individual mode of vibration, each of which represents a freedom of the relative motion between the tool and workpiece and has a particular direction. Directions X_k and X_j of two such modes are indicated.
- The vibration component, which is normal to the cut surface, produces undulations with an amplitude Y_o in one cut, and in the subsequent cut this component has amplitude Y.
- The process of self-excitation is a closed-loop one in which the vibrations cause a force variation and the variable force in turn produces vibrations.
- The force depends on vibrations in two subsequent passes:

$$F = K_s b h$$

- Chip thickness h consists of a steady-state value h_m (the mean chip thickness)

$$h = h_m + (Y_o - Y)\ exp\ (j\omega t)$$

- Correspondingly, the force also has a mean component Fm and a variable component. Because we are considering a linear system, we may neglect the mean components and write,

  $F = K_s b(Y_o - Y)$: b = chip width, $(Y_o - Y)$ = chip thickness variation…………………………………………..(1)
- The feedback relationship of vibrations caused by this force is, in general $Y = FG(\omega)$……………………..(2)
- $G(\omega)$ is the oriented transfer function of the system

Stability Limit Formulation

- Oriented TF is obtained as a sum of all the direct TFs of the modes G_i multiplied by the directional factors u_i:

$$ui = cos\ \alpha_i \cos(\alpha_i - \beta)$$

$$G = \sum u_i G_i$$

- Combining equations (1) and (2) so as to eliminate the force:

$$Y = K_s bG\ (Yo - Y)$$

- After modification,

$$Y_o / Y = [1/(K_s b) + G] / G \qquad (3)$$

- The condition for the stability may be formulated so that vibrations do not decay nor increase from pass to pass, or so that the magnitudes $|Y_o|$ and $|Y|$ are equal:

$$|Y_o / Y| = 1 \qquad (4)$$

- From Eqs. (3) and (4)

$$|\ 1/K_s b + G\ | = |G|$$

- This condition has two parts:

$$\mathrm{Im}(G) = \mathrm{Im}(G) \text{ and } 1/K_s b + \mathrm{Re}(G) = + \text{ or } - \mathrm{Re}(G)$$

Results

- Negative sign gives

$$1/Ksb = -2\mathrm{Re}(G)$$

which is the actual condition for the limit of stabilty.

- Of all the values b that satisfy the above wquation, there is a minimum one, the smallest chip width at which chatter can occur. This is the actual critical limit of stability.

$$b_{\mathrm{lim,cr}} = -1 / 2K_s\mathrm{Re}(G)_{\mathrm{min}}$$

- For chip widths $b < b_{lim}$ cutting is stable; there is no self-excited vibration.
- For $b > b_{lim}$ chatter will occur and grow.