Fixturing/Workholding

What is a fixture?

- A fixture is a device used to "fix" (constrain all degrees of freedom) a workpiece in a given coordinate system relative to the cutting tool.
- Primary functions of a fixture:
  - *Location*: to accurately position and orient a part relative to the cutting tool
  - *Support*: to increase the stiffness of compliant regions of a part
  - *Clamping*: to rigidly clamp the workpiece in its desired location (relative to the cutting tool)

Fixtures

- Routinely used in machining, welding, and manual/robotic assembly operations.
- Types of fixtures:
  - *General purpose*: mechanical vise, lathe chucks
  - *Permanent/Dedicated*: specially designed to hold one part for a limited number of operations; commonly used in high volume production.
  - *Flexible*/*Reconfigurable*: can be used for more than one part and for multiple operations e.g. modular fixtures, pin array, phase change, etc.

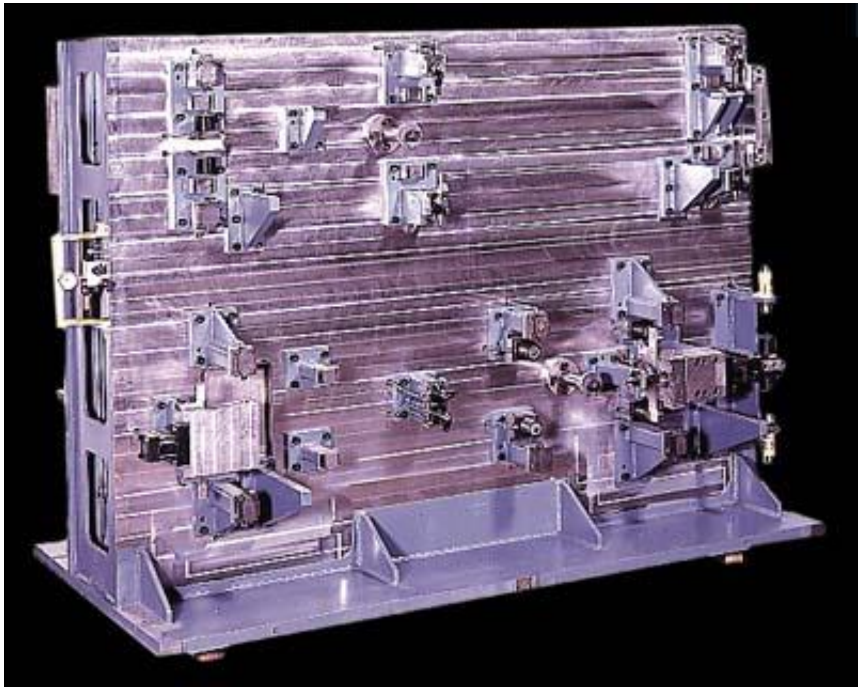
General Purpose Fixtures

Vise

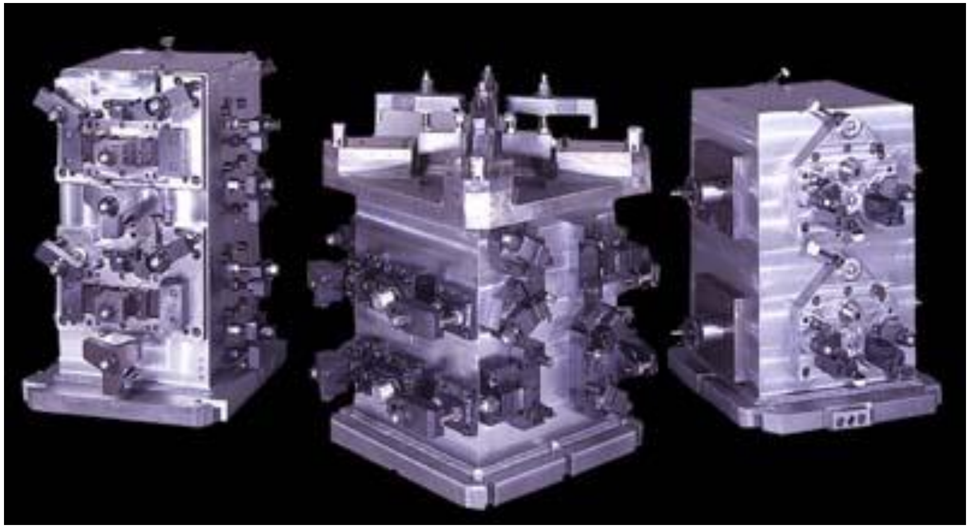
3-Jaw Chuck

6-Jaw Chuck

Dedicated Fixtures

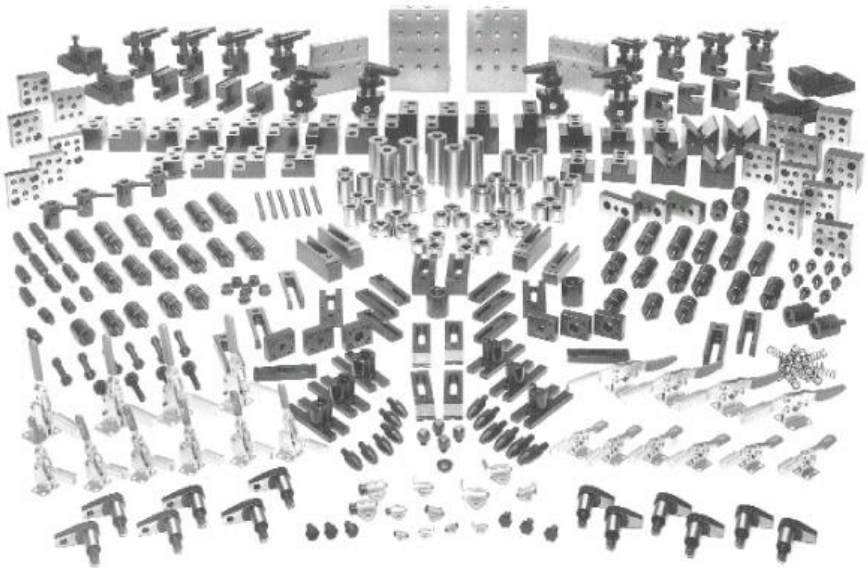


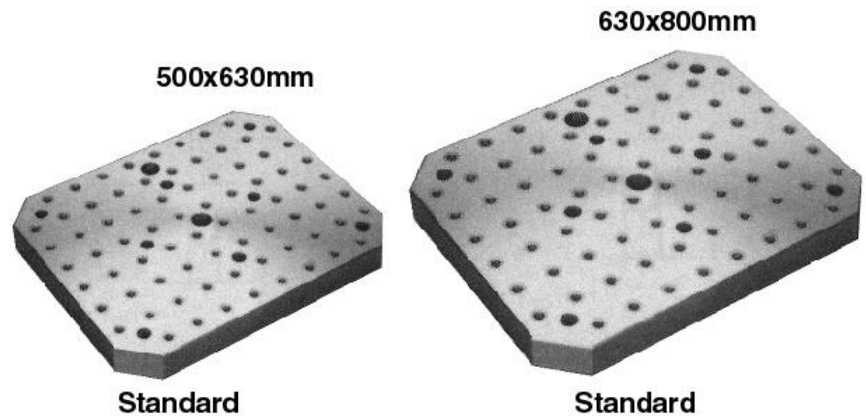
Milling Fixture



Tombstone Fixture

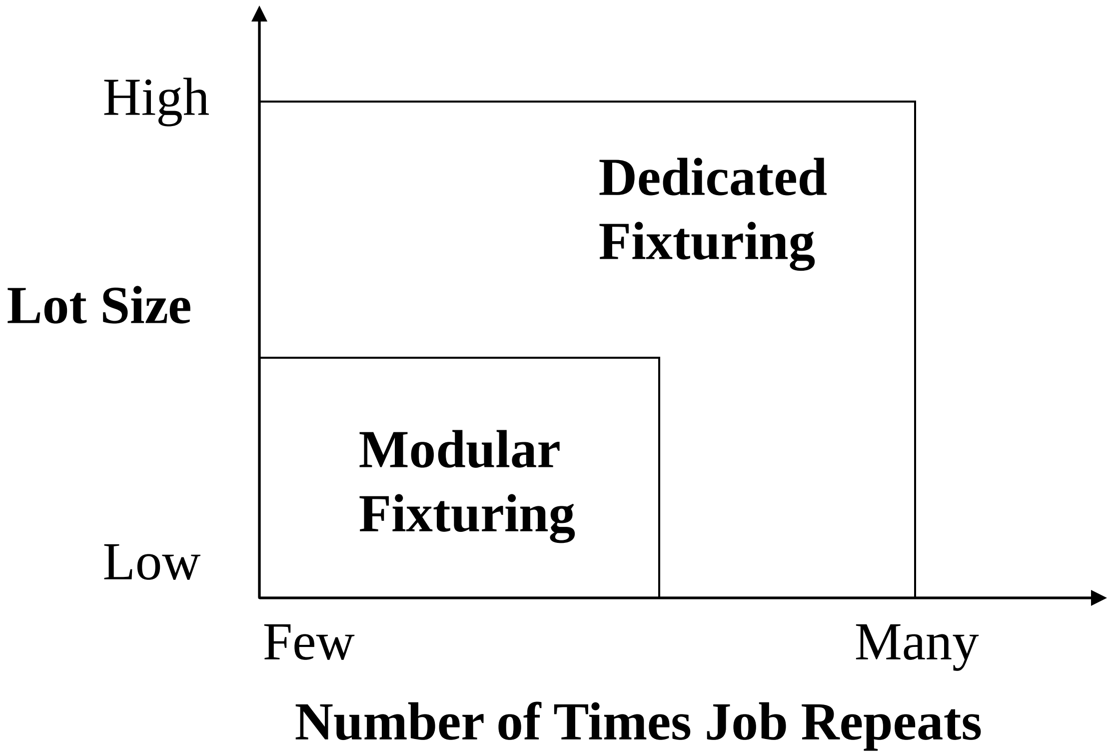
Flexible Fixture: Modular Fixture



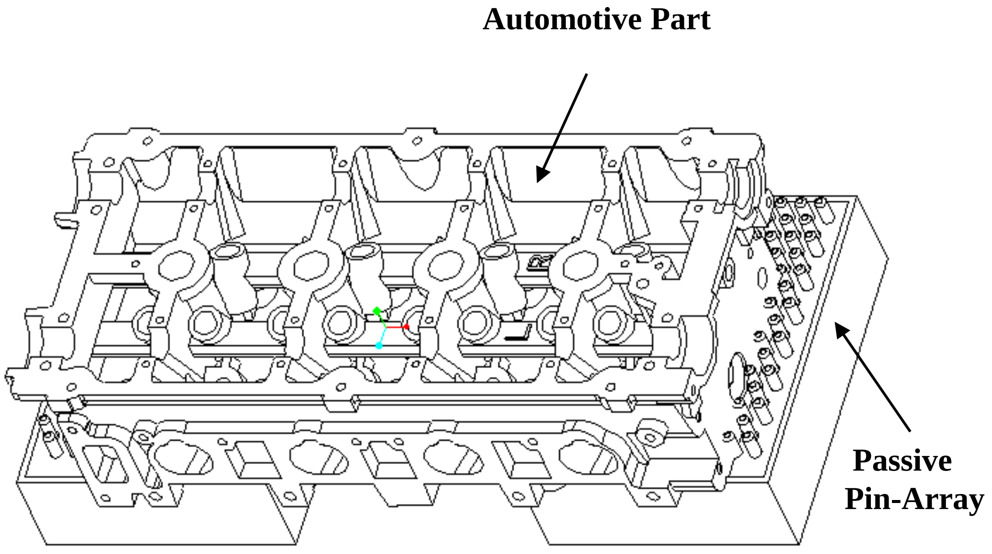


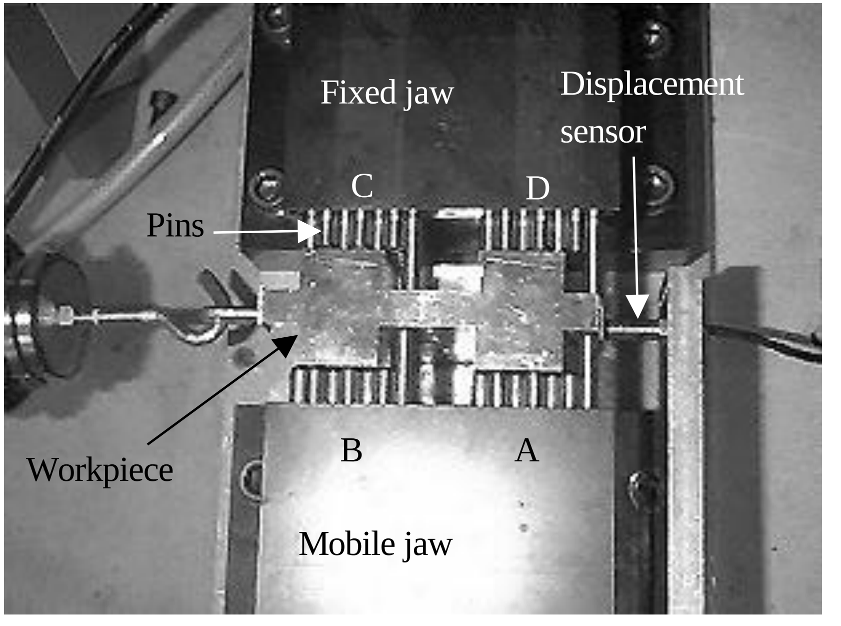
Modular Fixture Kit

Flexible Fixture: Modular Fixture



Flexible Fixture

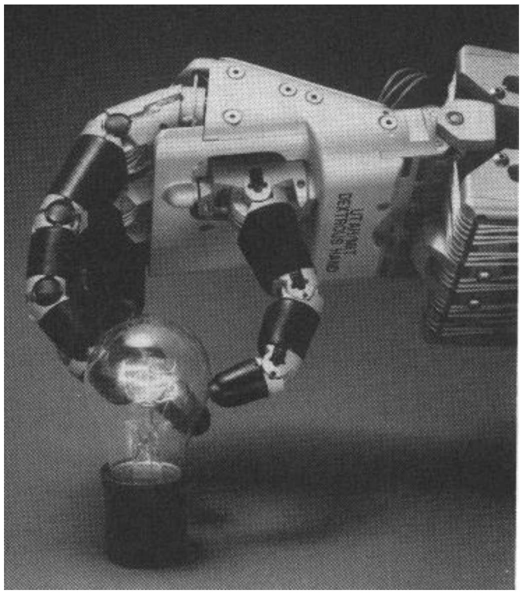




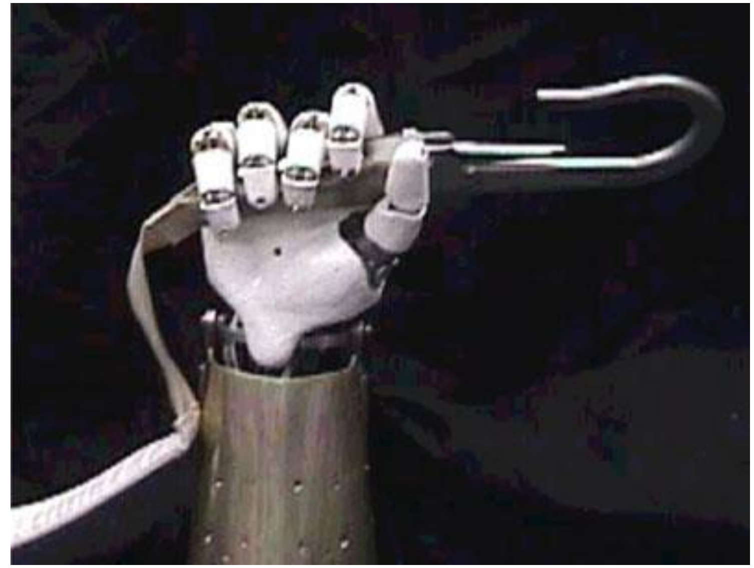
Active Pin-Array Vise
Holding a Complex Part

Bed of Nails Fixture

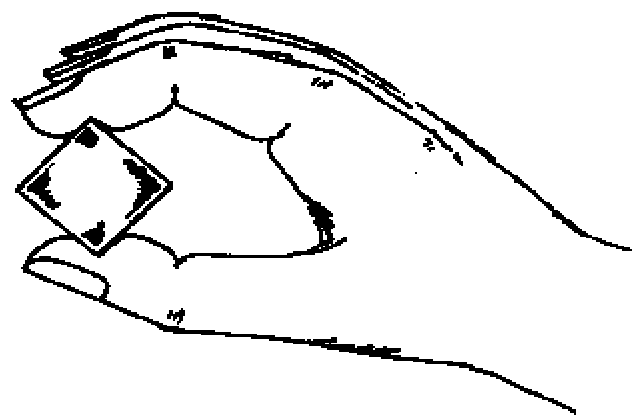
Grasping



Utah / M.I.T. Hand



Robonaut Nasa Hand



Human Object Handling

Axiom-based Workpiece Control

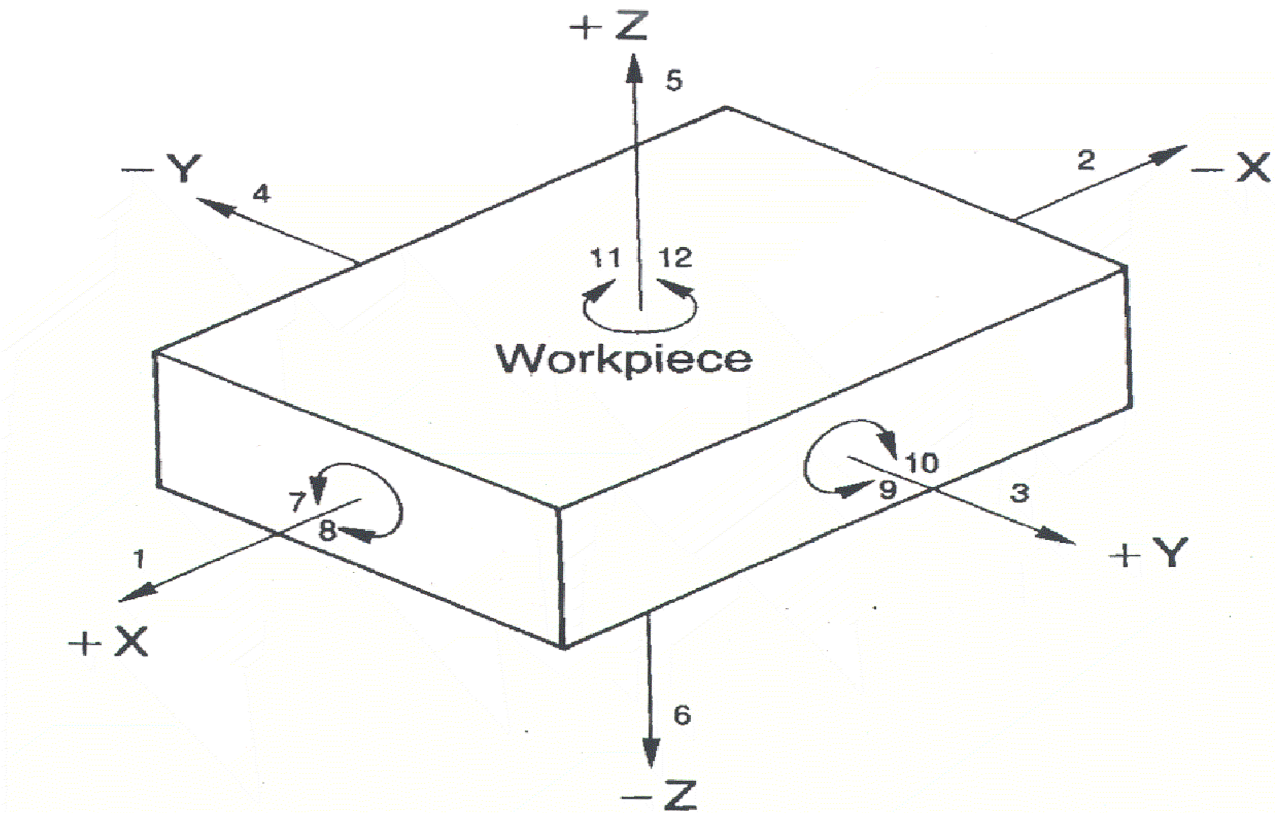
- Geometric control
- Dimensional control
- Mechanical control

Geometric Control Axioms

1. Only six locators are necessary to completely locate a rigid prismatic workpiece. More locators are redundant and may give rise to uncertainty
2. Three locators define a plane
3. Only one direction of each degree of freedom is located
4. Each degree of freedom has one locator
5. The six locators are placed as widely as possible to provide maximum workpiece stability and

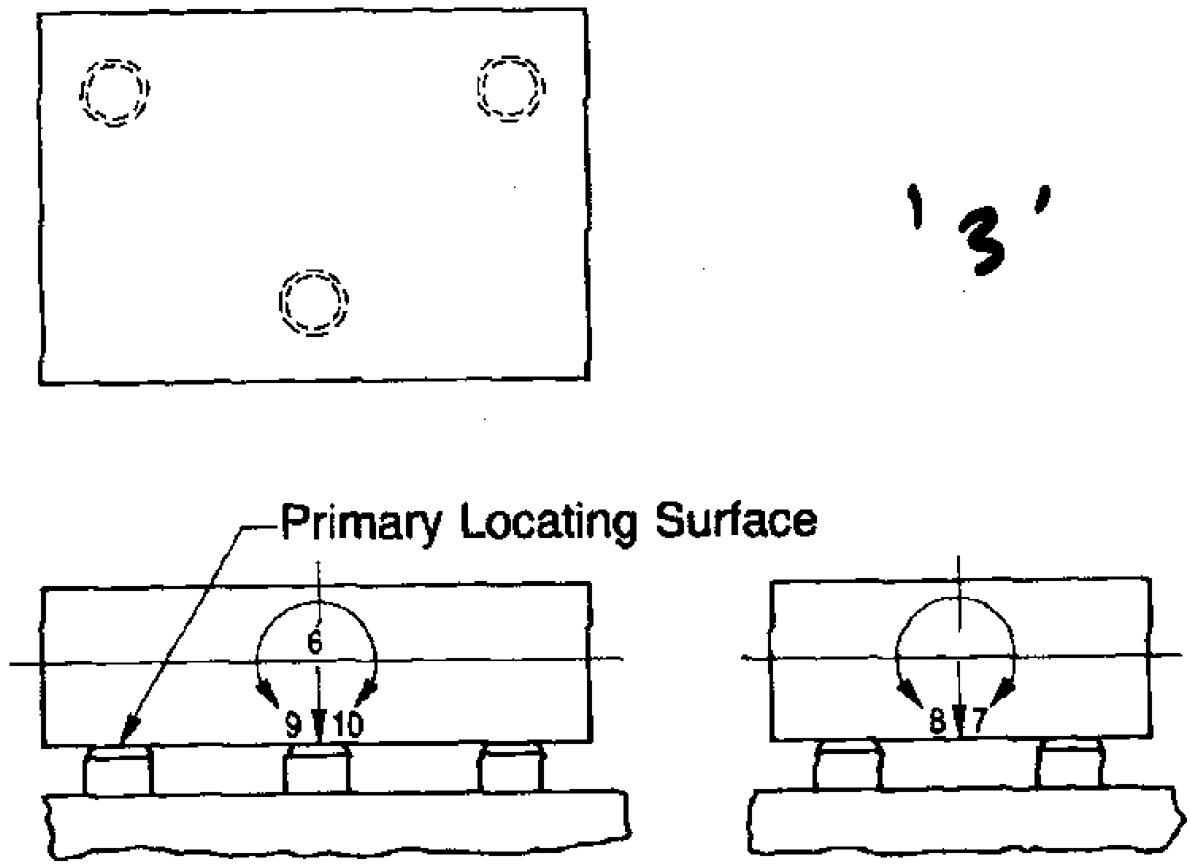
Fixture Design/Planning In Practice (1)

- Many dedicated fixtures for prismatic parts are designed using the “3-2-1” locating principle.



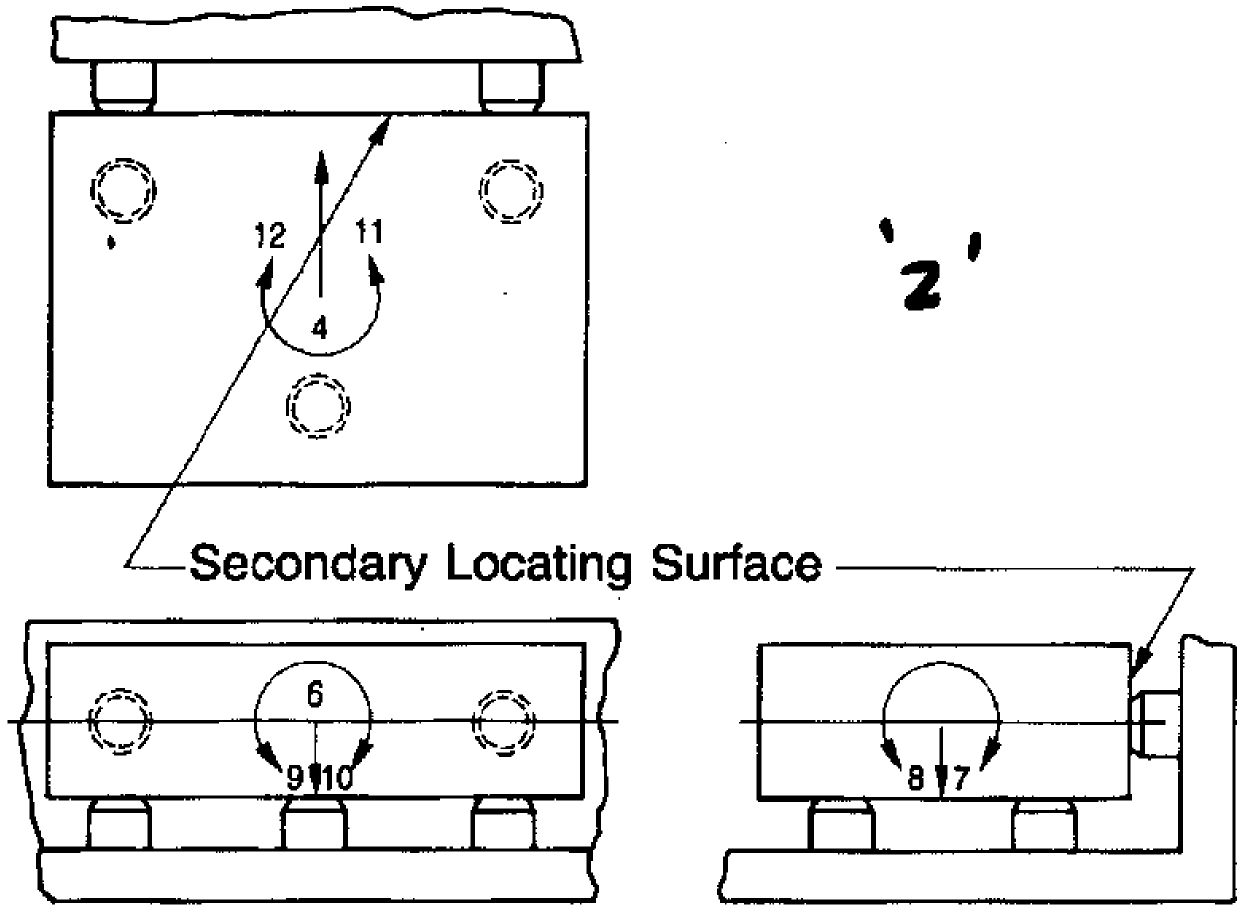
Fixture Design/Planning In Practice (2)

- The “3” in 3-2-1 refers to 3 locators (passive fixture elements) on the primary locating/datum surface.



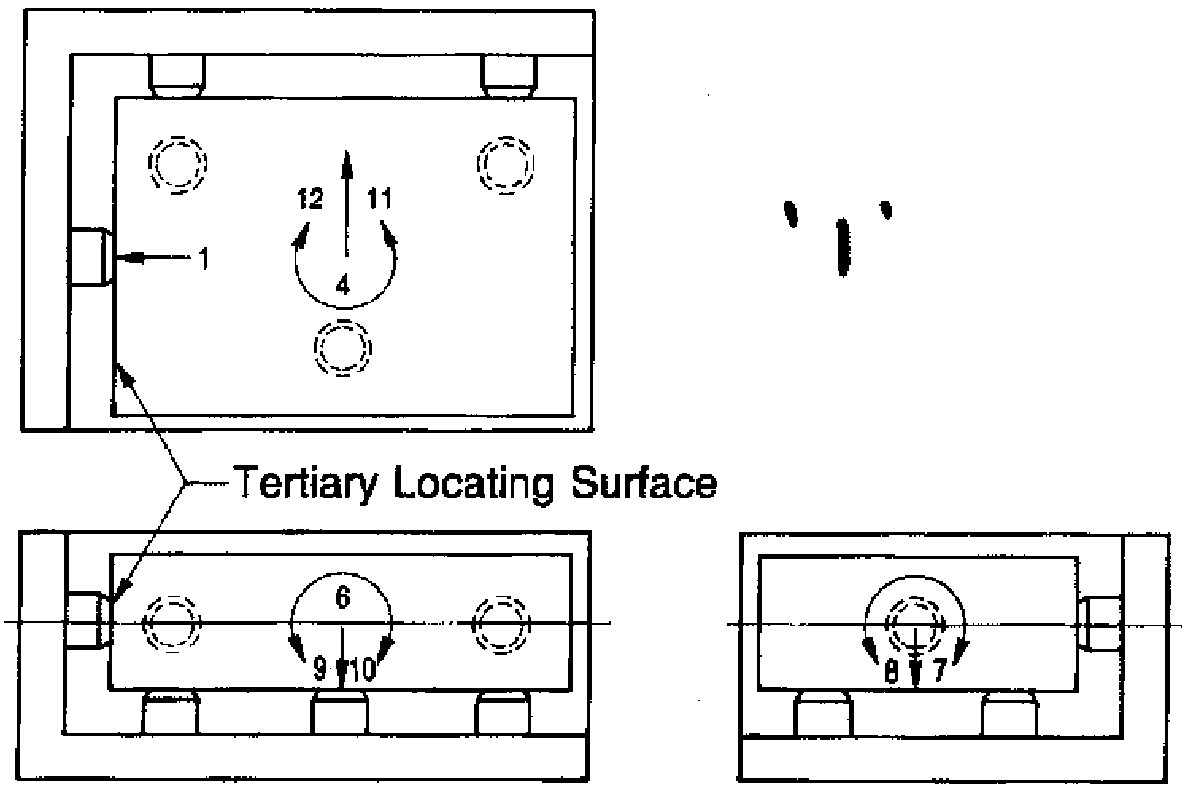
Fixture Design/Planning In Practice (2)

- The "2" in 3-2-1 refers to 2 locators on the secondary locating/datum surface.



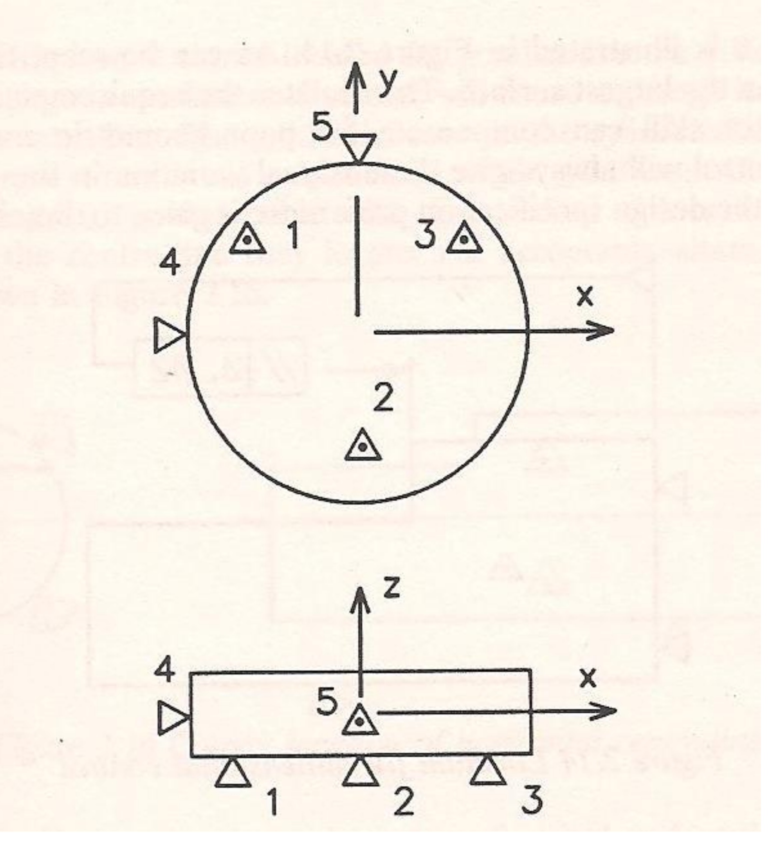
Fixture Design/Planning In Practice (3)

- The “1” in 3-2-1 refers to 1 locator on the tertiary locating/datum surface.



Cylindrical Workpiece

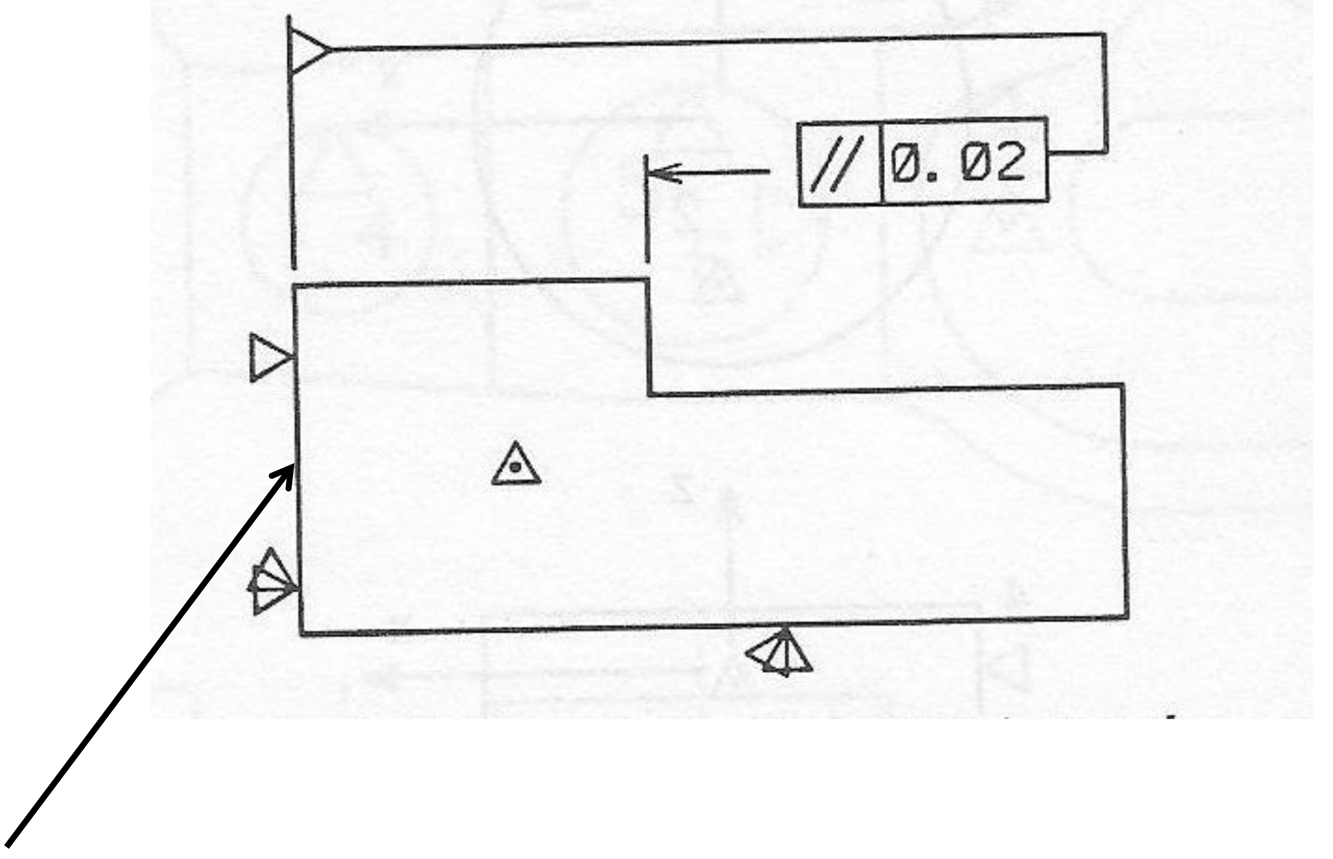
6. Only five locators are required for locating cylindrical workpiece



Dimensional Control

7. To prevent tolerance stacks locators must be placed on one of the two surfaces which are related by the dimension on the workpiece
8. When two surfaces are related by geometrical tolerance of parallelism or perpendicularity, the reference surface must be located by three locators
9. In case of conflict between geometric and dimensional control, precedence is given to dimensional control.
10. To locate the centerline of the cylindrical surface the locators must straddle the centerline
11. Locators should be placed on machined surface for better dimensional control

Example of Reference



Three locators on reference side

Mechanical Control

12. Place locators directly opposite to cutting forces to minimize deflection/deformation
13. Place locators directly opposite to clamping or holding forces to minimize deflection/deformation
14. If external forces cannot be reacted directly via the locators , limit the deflection and distortion by placing fixed supports opposite to applied force
15. Fixed supports should not contact the workpiece before the load is applied
16. Holding forces must force the components to contact the locators

Mechanical Control (contd.)

17. The moment of the clamping forces about all possible centers of rotation must be sufficient to overcome the effect of tool forces and restrict any movement away from locators
18. Tool forces should aid the workpiece to remain in contact with locators

Fixture Design/Planning In Practice

- Current approach to fixture design and planning relies on experience and trial-and-error methods → leads to expensive fixtures.

- Thumb rules are often used to design fixtures in practice.

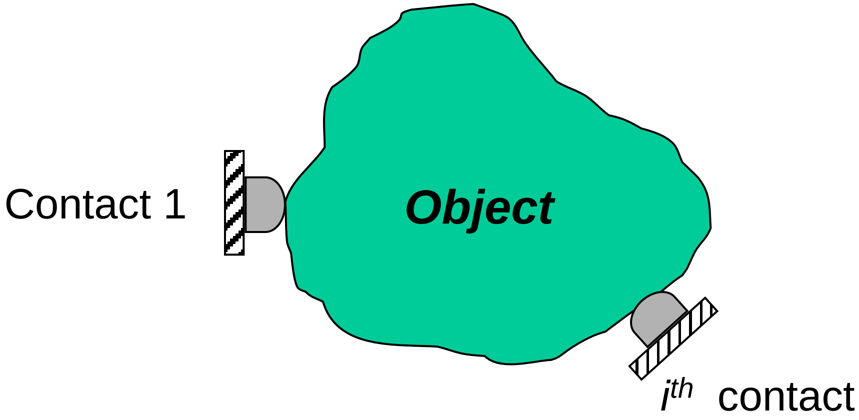
- Need for more scientific methods in fixture analysis and design.

Typical Questions in Fixture Design

- Does the fixture accurately locate the part relative to the cutting tool?
- Does the fixture ensure that the part is totally constrained?
- Can the part be easily loaded into/unloaded from the fixture?
- What is the role of type and number of contacts on fixturability of the part?
- What is the minimum clamping force needed to restrain the part during machining?
- ….

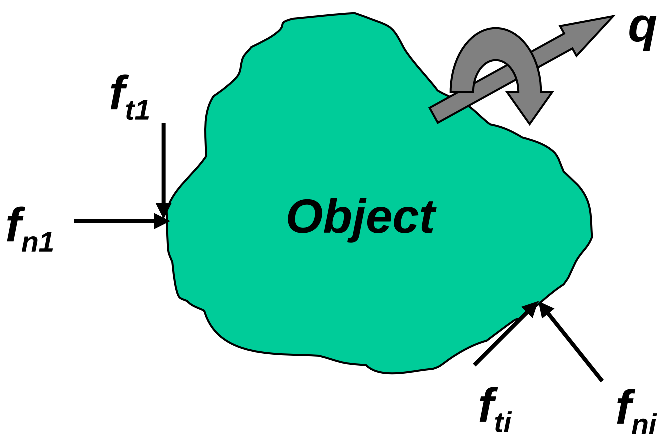
Kinematic Analysis of Fixtures (1)

- The fixture can be treated as a set of rigid contacts that are fixed in a reference frame.
- The workpiece is treated as a rigid body whose motion is restricted by the contacts.



Kinematic Analysis of Fixtures (2)

- It is of interest to determine the possible motions of the object constrained by the contacts → instantaneous motion properties of the rigid body → displacements, velocities that the object undergoes.



Kinematic Analysis of Fixtures (3)

- The instantaneous motion properties of the object are influenced by the following factors:
  - Shape and number of contacts
  - Relative location of the contacts
  - Relative orientation of the contacts
  - Friction

Types of Fixture-Workpiece Contact

- Common contact geometries include:
  - Point contact e.g. point-on-plane, plane-on-point, line on non-parallel line
  - Line contact e.g. line-on-plane, plane-on-line
  - Planar contact e.g. plane-on-plane
- Assuming that the contact between the object and fixture element (locator pin, clamp, etc.) is always maintained, freedom of motion allowed by each contact depends on the presence/absence of friction.

Effect of Friction

- Impact of friction on the degrees of freedom allowed by different contact types is as follows:

Contact Type	Friction	No Friction
Point	3	5
Line	1	4
Planar	0	3

Form vs. Force Closed Fixtures (1)

- Fixtures (and grasps) can be also characterized in terms of their “closure” properties.
- ***<u>Form Closure</u>*****:** if the contacts with the object are arranged such that they can resist arbitrary *disturbance forces and moments*, then the object is said to be form closed (or equivalently, the fixture is said to provide form closure).
- Equivalent statement: a set of contacts provides form closure if it eliminates all *degrees of freedom* of the object purely on the basis of the geometrical placement of the contacts.

Form vs. Force Closed Fixtures (2)

- ***<u>Force Closure</u>*****:** the fixtured object is said to be force closed if it relies on disturbance forces and moments to maintain contact.

- In practice, most machining fixtures are force closed fixtures because they rely on frictional forces to totally constrain part motion.

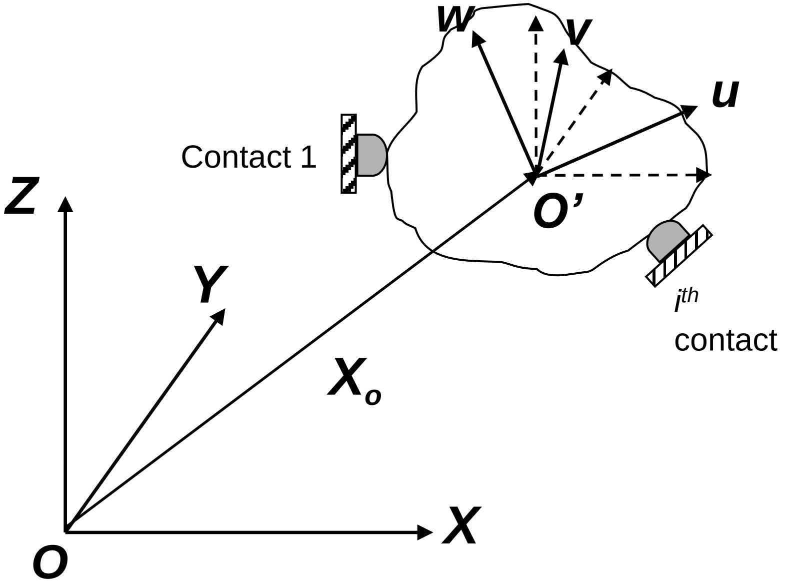
Kinematic Analysis of Fixtures (3)

- What are the necessary and sufficient conditions for a fixture to guarantee the following:
  - Accurate location → *Deterministic Positioning*
  - No movement → *Total Constraint*
  - Ease of loading/unloading → *Accessibility/Detachability*

Assumptions of Analysis

- The main assumptions are as follows:
  - The object (workpiece) and contacts (fixture elements/fingers) are perfectly rigid
  - Point contacts
  - Frictionless contacts
  - The object surface is piecewise differentiable

Modeling Basics



Deterministic Positioning

The points that lie on surface defined by the piecewise differentiable function, g(u,v,w) :

$g(u,v,w) = 0$ or could be represented as $g(U) = 0$, where U is the vector containg all the three axes, u, v and w

All the points that lie outside that surface are defined by

$g(U) > 0$

Based on the figure it can be seen that there are two co-ordinate systems :

Fixed coordinate system of the assembly station/machine: O(X,Y,Z)

The coordinate fixed to the workpart: O'(u,v,w)

Deterministic Positioning

The origin of coordinate system O' (u,v,w) is related to O(X, Y,Z) by radius vector X_0 = *col* $[X,Y,Z]$ and orientation Θ = *col* $[\theta, \phi, \psi]$

The coordinate transformation from U to X is given by

$$X = A(\Theta)U + X_0$$

Consider a system of fixture elements 1 through m,

the ith element is in contact with workpiece

$$g\left[A(\Theta)^T\{X_i - X_0\}\right] = 0$$

$$g_i[q] = g\left[A(\Theta)^T\{X_i - X_0\}\right]$$

Transformation matrix for a 2-D system

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X - X_0 \\ Y - Y_0 \end{bmatrix}$$

q is located O(X, Y, Z)

q = *col* $[X,Y,Z,\theta, \phi, \psi]$

Deterministic Positioning

Let q* be the unique position where we want to position all the fixture elements 1 to m are in contact

$$g_i\left[q^*\right]=0,\ for 1\le i\le m$$

Assuming that workpart can be placed in the vicinity q* + Δq,

uniqueness of the solution in the vicinity of q* need to be considered

$$g_i\left[q^*+\Delta q\right]=g_i\left[q^*\right]+h_i\Delta q=0,\ for 1\le i\le m$$

where hi is the 1x6 gradient vector consisting of

partial derivatives of function gi wrt q

$$h_i=\left[\frac{\partial g_i}{\partial X},\frac{\partial g_i}{\partial Y},\frac{\partial g_i}{\partial Z},\frac{\partial g_i}{\partial \theta},\frac{\partial g_i}{\partial \varphi},\frac{\partial g_i}{\partial \psi}\right]$$

In order to write m simultaneous equation in matrix form,

$$G=\begin{bmatrix} h_1 \\ \bullet \\ \bullet \\ h_m \end{bmatrix}$$

$$G\Delta q=0$$

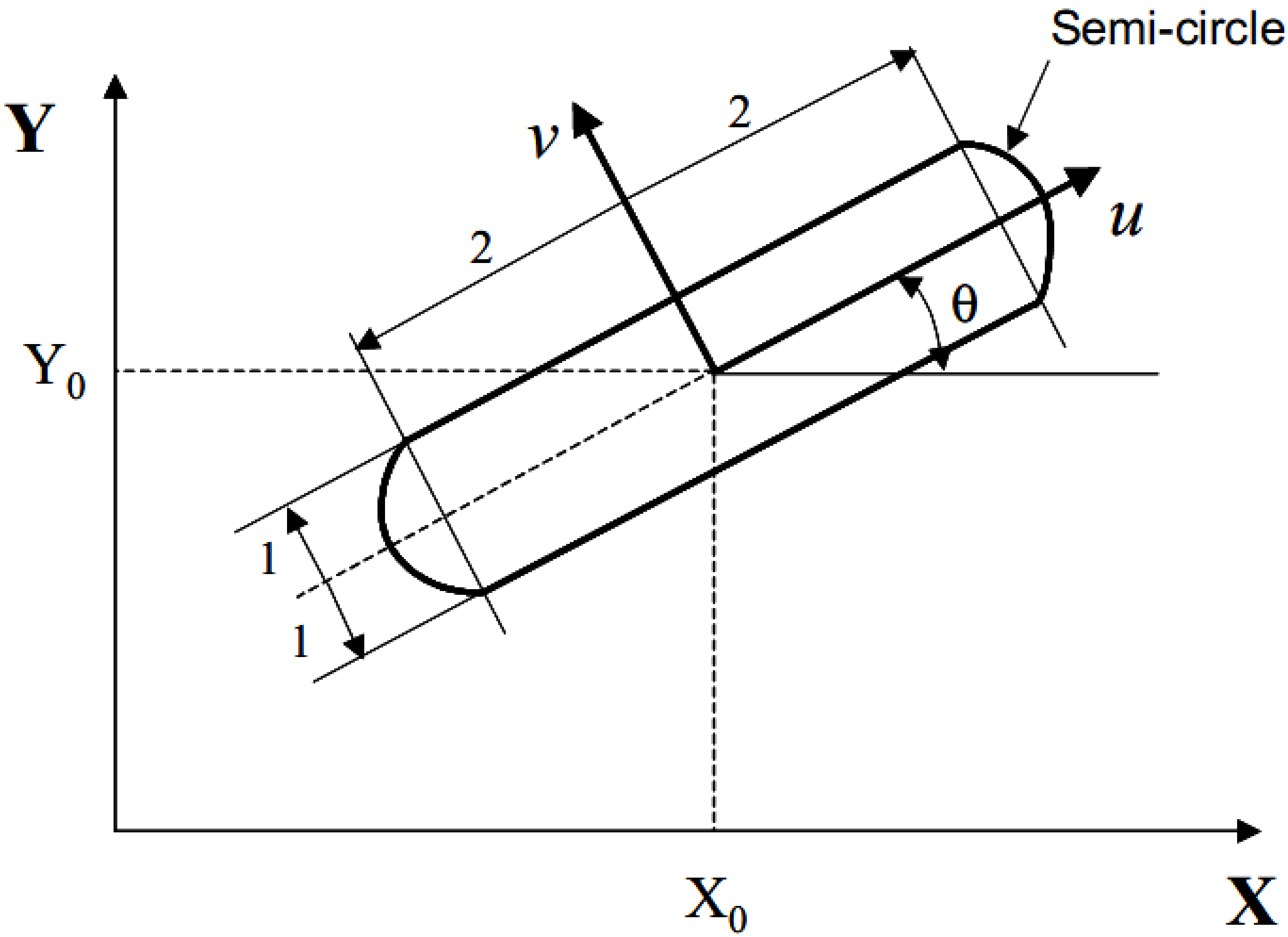
For a 2-D system, a sample G matrix

$$G=\begin{bmatrix} \frac{\partial g_1}{\partial X} & \frac{\partial g_1}{\partial Y} & \frac{\partial g_1}{\partial \theta} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_m}{\partial X} & \frac{\partial g_m}{\partial Y} & \frac{\partial g_m}{\partial \theta} \end{bmatrix}$$

Deterministic Positioning

- For unique solution where $G\Delta q = 0$, the Jacobian matrix, G must have full rank.

Deterministic Positioning Example

- 

Example

The 2-D object is to be clamped in a fixture with the following layout consisting of locators (denoted by L) and clamps (denoted by C):

L1: $(u_{L1}, v_{L1}) = (-1, -1)$
L2: $(u_{L2}, v_{L2}) = (1, -1)$
L3: $(u_{L3}, v_{L3}) = (-3, 0)$
C1: $(u_{C1}, v_{C1}) = (0, 1)$
C2: $(u_{C2}, v_{C2}) = (3, 0)$

In addition, $X_0 = Y_0 = 6$, and $\theta = 45^{\circ}$.

(a) Prove that the fixture layout given above guarantees *deterministic positioning*. Show all steps clearly.

Solution

(a) Deterministic positioning requires that the Jacobian matrix G for the fixture layout (determined solely by the locators) is full rank. Note that, for a 2D problem, G has the following general form:

$$G = \begin{bmatrix} \frac{\partial g_1}{\partial X} & \frac{\partial g_1}{\partial Y} & \frac{\partial g_1}{\partial \theta} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_m}{\partial X} & \frac{\partial g_m}{\partial Y} & \frac{\partial g_m}{\partial \theta} \end{bmatrix}$$

where m refers to the number of locators.

We first need to determine G. This, in turn, requires that we establish the equations of the curve/line segment of the object surfaces on which the locators make contact.

Solution (Contd.)

Looking at the coordinates of locators L1 and L2, we can see that they both contact the lower straight segment of the object surface. If we denote the equation of the object surface (or curve) in the object coordinate system as $g_i(u,v)$, by definition (see lecture slides on deterministic positioning) this function must satisfy the following requirements:

$g_i(u,v)=0$ for a point (u, v) on the object's boundary

$g_i(u,v)>0$ for a point (u, v) outside the object's boundary, and

$g_i(u,v)<0$ for a point (u, v) inside the object's boundary

Solution (Contd.)

We can relate the object coordinate system (***u*, *v***) to the fixture (or fixed) coordinate system as follows:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X - X_0 \\ Y - Y_0 \end{bmatrix} \quad \ldots (1)$$

$$u = (X - X_0)\cos\theta + (Y - Y_0)\sin\theta$$

$$v = -(X - X_0)\sin\theta + (Y - Y_0)\cos\theta$$

For locators L1 and L2, the relevant object boundary equation can be written in the object coordinate system as follows:

$g_{L1,L2}(u,v) = -v - 1 = 0$, or, using (1) we can re-write this in the fixture coordinate system as:

$$g_{L1,L2}(X,Y) = (X - X_0)\sin\theta - (Y - Y_0)\cos\theta - 1 = 0 \quad \ldots (2)$$

Solution (Contd.)

We can now compute the partial derivatives of g as follows:

$$\frac{\partial g_{L1,L2}}{\partial X} = \sin\theta,\ \frac{\partial g_{L1,L2}}{\partial Y} = -\cos\theta,\ \frac{\partial g_{L1,L2}}{\partial \theta} = (X - X_0)\cos\theta + (Y - Y_0)\sin\theta = u$$

… (3)

Note that in the last partial derivative with respect to θ, Eq. (1) has been used to express the result in terms of the object coordinates.

In a similar fashion, we can determine the equation of the object boundary for locator L3 (which makes contact with the semi-circular portion of the boundary on the left) as:

$g_{L3}(u,v) = (-u-2)^2 + v^2 - 1 = 0$, or, using (1) we can re-write this in the fixture coordinate system as:

$$g_{L3}(u,v) = \left[-\left\{(X - X_0)\cos\theta + (Y - Y_0)\sin\theta\right\} - 2\right]^2 + \left[(Y - Y_0)\cos\theta - (X - X_0)\sin\theta\right]^2 - 1 = 0$$

… (4)

Solution (Contd.)

$$\frac{\partial g_{L3}}{\partial X} = 2\left[-\left\{(X-X_0)\cos\theta + (Y-Y_0)\sin\theta\right\} - 2\right](-\cos\theta) +$$

$$2\left[(Y-Y_0)\cos\theta - (X-X_0)\sin\theta\right](-\sin\theta) = -2(-u-2)\cos\theta - 2v\sin\theta$$

$$\frac{\partial g_{L3}}{\partial Y} = 2\left[-\left\{(X-X_0)\cos\theta + (Y-Y_0)\sin\theta\right\} - 2\right](-\sin\theta) +$$

$$2\left[(Y-Y_0)\cos\theta - (X-X_0)\sin\theta\right](\cos\theta) = -2(-u-2)\sin\theta + 2v\cos\theta$$

$$\frac{\partial g_{L3}}{\partial \theta} = 2\left[-\left\{(X-X_0)\cos\theta + (Y-Y_0)\sin\theta\right\} - 2\right]\left[(X-X_0)\sin\theta - (Y-Y_0)\cos\theta\right] +$$

$$2\left[(Y-Y_0)\cos\theta - (X-X_0)\sin\theta\right]\left[-(Y-Y_0)\sin\theta - (X-X_0)\cos\theta\right]$$

$$= -2v(-u-2) - 2uv$$

… (5)

You can also work in X and Y by finding corresponding values

Solution (Contd.)

By substituting $X_0 = Y_0 = 6$, $\theta = 45^\circ$, and the coordinates of the locators (given in the problem statement) L1, L2 into Eqs (3), and for L3 into Eqs. (5), we get the following *G* matrix:

$$G = \begin{bmatrix} \frac{\partial g_{L1}}{\partial X} & \frac{\partial g_{L1}}{\partial Y} & \frac{\partial g_{L1}}{\partial \theta} \\ \frac{\partial g_{L2}}{\partial X} & \frac{\partial g_{L2}}{\partial Y} & \frac{\partial g_{L2}}{\partial \theta} \\ \frac{\partial g_{L3}}{\partial X} & \frac{\partial g_{L3}}{\partial Y} & \frac{\partial g_{L3}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & -1 \\ 0.707 & -0.707 & 1 \\ -1.414 & -1.414 & 0 \end{bmatrix}$$

Checking the rank of *G* (using MATLAB), we get a value of 3 => the Jacobian matrix is full rank => the fixture layout provides deterministic positioning.

Total Constraint (1)

- Total constraint is a concept that applies to a fixture after clamps are actuated.
- An object is *totally constrained* if the fixture layout (or grasp) allows no geometrically admissible (small) motion of the object from the desired location.

Total Constraint (2)

If an object is totally constrained then at least one of the following inequalities is not satisfied for an arbitrary infinitesimal displacement $\delta\boldsymbol{q}$:

$$\nabla g_i \big|_{q^*} \delta\boldsymbol{q} \geq 0, \qquad 1 \leq i \leq (m+C) \qquad \ldots (1)$$

where C is the number of clamps in the fixture; m is the number of locators.

In other words, we can write:

$\forall\ \delta\boldsymbol{q},\ \exists\ i$ such that $\nabla g_i \big|_{q^*} \delta\boldsymbol{q} < 0$ → fixel penetrates the object surface!

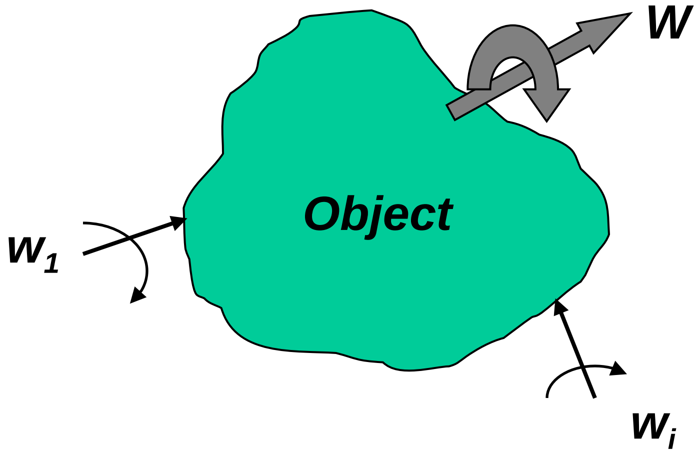
<u>∴ The fixture layout provides total constraint if there exists no non-zero solution to Eq. (1) above.</u>

Total Constraint (3)

The total constraint analysis just presented was from a *motion* point of view. One can also formulate a condition for total constraint from a *force* point of view.

Total Constraint (4): Wrenches & Twists

A system of forces and moments acting on a rigid body can be replaced by a *wrench*, **w**, which consists of a force (***f***) acting along a unique axis in space and a moment (***m***) about that axis.

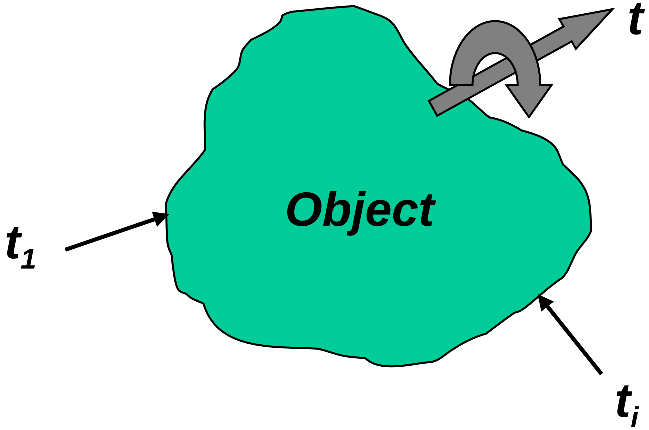


$$\mathbf{w}_i = [\mathbf{f}\ \ \mathbf{m}]^T$$

$$= [f_x\ f_y\ f_z\ m_x\ m_y\ m_z]^T$$

Total Constraint (5): Wrenches & Twists

The motion of a rigid body can be described by a *twist*, ***t***, which consists of a translation along a unique axis in space and a rotation about that axis.



$$\boldsymbol{t}_i = [\boldsymbol{d} \;\; \boldsymbol{\omega}]^T$$

$$= [d_x \; d_y \; d_z \; \omega_x \; \omega_y \; \omega_z]^T$$

Total Constraint (6): Screw Theory Basics

Twists and *wrenches* are forms of *screws*, which have a principal axis (unique axis in space) and a pitch.

For a *wrench* $\boldsymbol{w} = [f_x\ f_y\ f_z\ m_x\ m_y\ m_z]^T = [\boldsymbol{f}\ \ \boldsymbol{m}]^T$:

pitch, $p = \dfrac{f \bullet m}{f \bullet f}$

magnitude or "intensity" of wrench = $\| \boldsymbol{f} \|$

For a *twist* $\boldsymbol{t} = [d_x\ d_y\ d_z\ \omega_x\ \omega_y\ \omega_z]^T = [\boldsymbol{d}\ \ \boldsymbol{\omega}]^T$:

pitch, $p = \dfrac{d \bullet \omega}{\omega \bullet \omega}$

magnitude or "intensity" of twist = $\| \boldsymbol{\omega} \|$

Total Constraint (7): Twist Approach

Each contact limits the object to executing a particular system of *twists*. For multiple contacts, the net motion of the object is given by the intersection of the individual twist systems.

For total constraint of the object, it is necessary and sufficient that the intersection of all twist systems be equal to the *null set*.

Total Constraint (8): Wrench Approach

The static equilibrium of a rigid object that has been clamped in the fixture can be written in wrench form as follows:

$$\mathbf{W}\mathbf{c} = -\mathbf{w}_c \qquad \ldots (2)$$

where **W** is a (6 x *m'*) contact wrench matrix that is full-rank, **c** is a (*m'* x 1) vector of contact wrench intensities and $\mathbf{w}_c$ is a (6 x 1) wrench of external disturbances (e.g. object's weight, cutting forces, etc.)

Total Constraint (9): Wrench Approach

The necessary and sufficient condition for total constraint is that the system of equations in (2) should have a <u>non-negative solution</u> (for **c**).

The general solution to (5) is of the form:

$$\mathbf{c} = \mathbf{c}_p + \mathbf{c}_h \qquad \ldots (3)$$

where $\mathbf{c}_p$ (a m'x1 vector) is the particular solution and $\mathbf{c}_h$ (also a m'x1 vector) is the homogenous solution to (2).

Total Constraint (10): Wrench Approach

In general, the elements of $\mathbf{c}_p$ can be > 0, < 0, or equal to 0.

In general, $\mathbf{c}_h$ is of the form:

$$\mathbf{c}_h = \lambda_1 c_{h,1} + \lambda_2 c_{h,2} + \cdots + \lambda_q c_{h,q} \qquad \ldots(4)$$

where λ_i's are arbitrary free variables and $q = m' - \text{rank}(\mathbf{W})$; $c_{h,i}$ are (m' x 1) vectors.

Total Constraint (11): Wrench Approach

For frictionless contact, total constraint requires that all elements of **c** be non-negative.

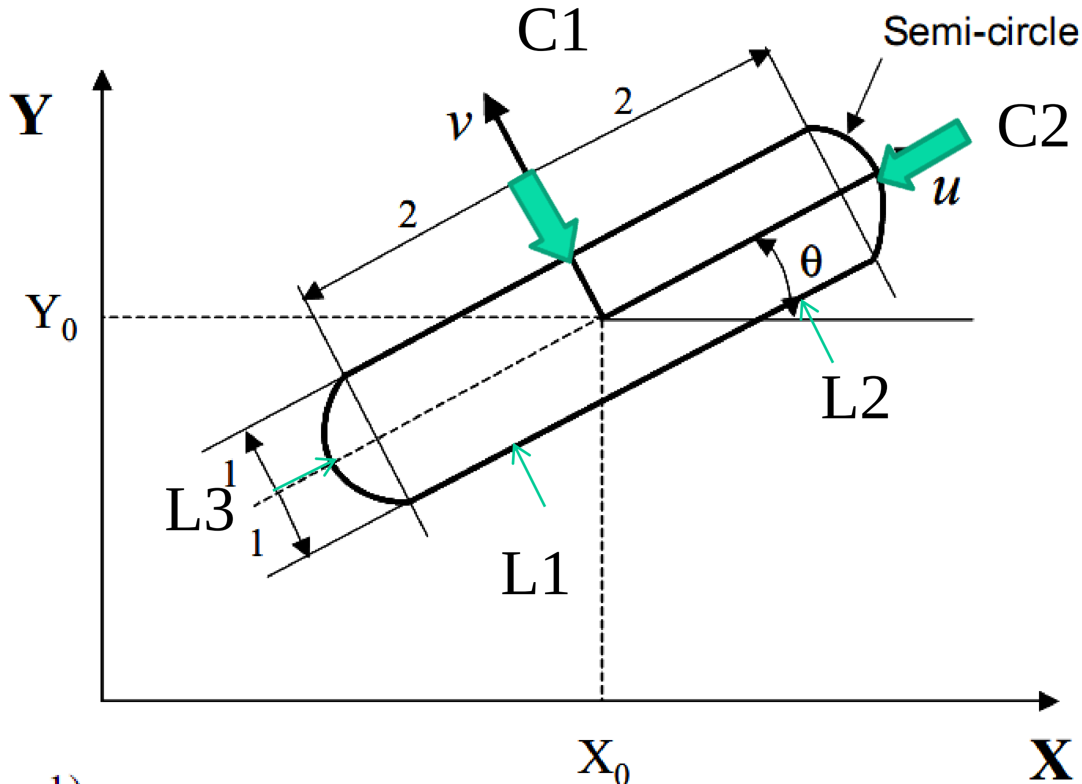
In order to meet this requirement, it is sufficient that $\mathbf{c}_h$ be positive since **c** can always be made non-negative by selecting appropriate values for the free variables $\lambda_1, \ldots, \lambda_q$. Mathematically, this can be stated as:

$$\lambda_1 c_{h,1} + \lambda_2 c_{h,2} + \cdots + \lambda_q c_{h,q} > [0] \quad \ldots(5)$$

Total Constraint (12): Wrench Approach

In general, total constraint can be verified by checking for the existence of a solution to the set of inequalities in (5). (how?)

However, for simple problems it is easy to determine total constraint from force equilibrium considerations.



L1: $(u_{L1}, v_{L1}) = (-1, -1)$
L2: $(u_{L2}, v_{L2}) = (1, -1)$
L3: $(u_{L3}, v_{L3}) = (-3, 0)$
C1: $(u_{C1}, v_{C1}) = (0, 1)$
C2: $(u_{C2}, v_{C2}) = (3, 0)$

Total Constraint Example

For total constraint, a force (or wrench-based) approach is easier to work with. Denoting the normal forces exerted by the two clamps as C1 and C2 and the reaction forces acting on the object at the locators as L1, L2, and L3, we can write the force and moment equilibrium equations for the object (in the object coordinate system) as follows:

$$\sum F_u = 0 \Rightarrow L3 - C2 = 0$$

$$\sum F_v = 0 \Rightarrow L1 + L2 - C1 = 0$$

$$\sum M_{o'} = 0 \Rightarrow (L2)(1) - (L1)(1) = 0 \Rightarrow L2 - L1 = 0$$

Matrix vector form

$$\begin{bmatrix} 0 & 0 & 1 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L1 \\ L2 \\ L3 \\ C1 \\ C2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The (3x5) matrix is the wrench matrix (W), the (5x1) vector is the wrench intensity vector (c) and the right hand size null vector is the disturbance wrench (wp). Note that W is full rank as required by the total constraint condition (you can verify this using MATLAB).

Solution

- where Cp is the particular solution and Ch is the homogeneous solution

$$c = c_p + c_h$$

$$c_h = \lambda_1 c_{h,1} + \cdots + \lambda_q c_{q,1}$$

where, q = (total number of fixture elements, including clamps) – rank(W) = 5 – 3 = 2 (in this problem). This means that there are 2 arbitrary (or free) variables (λ's) in this problem.

Solution

Since $q = 2$, we can choose any two of the 5 variables (L1, L2, L3, C1, C2) as the two free variables. Physically, it makes sense to choose C1 and C2, the two clamping forces, as the free variables because we can adjust the clamping forces (through mechanical/hydraulic/pneumatic means) (Note that, from a theoretical standpoint, it does not matter which two you select). Let's select C1 and C2 for the free variables λ_1 and, respectively.

Converting L1, L2, L3 in terms of C2

$$c = \begin{bmatrix} L1 \\ L2 \\ L3 \\ C1 \\ C2 \end{bmatrix} = \begin{bmatrix} 0.5C1 \\ 0.5C1 \\ C2 \\ C1 \\ C2 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 0.5C1 \\ 0.5C1 \\ C2 \\ C1 \\ C2 \end{bmatrix} = c_p + \lambda_1 c_{h,1} + \lambda_2 c_{h,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + C1 \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 1 \\ 0 \end{bmatrix} + C2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

The constraint is guaranteed when C1 and C2 > 0 which is met
In this case

Accessibility/Detachability (1)

- Accessibility/detachability relate to ease of part loading/unloading into/from the fixture.
- The object is *detachable* from the desired location in the fixture, $\boldsymbol{q}^*$, if there exists at least one admissible motion $\delta\boldsymbol{q}$ from $\boldsymbol{q}^*$ to a neighboring location where the object is detached from one or more fixels.
- If the object is detachable from $\boldsymbol{q}^*$, the desired location $\boldsymbol{q}^*$ is also *accessible*.

Accessibility/Detachability (2)

- The object is said to be *weakly detachable* from the fixture if there exists a non-zero solution to the following system of equations:

  $$G\,\delta\boldsymbol{q} \geq \mathbf{0} \qquad \text{... (1)}$$

  where G is the Jacobian matrix of full-rank.

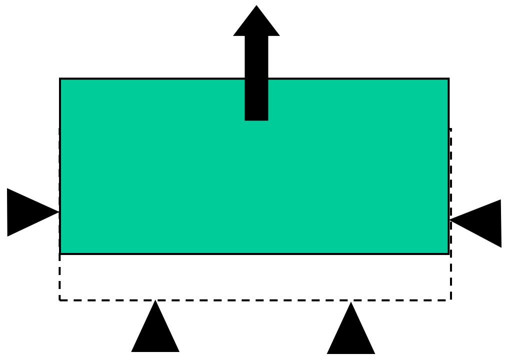
- The object is said to be *strongly detachable* from the fixture if there exists a solution to the following system of equations:

  $$G\,\delta\boldsymbol{q} > \mathbf{0} \qquad \text{... (2)}$$

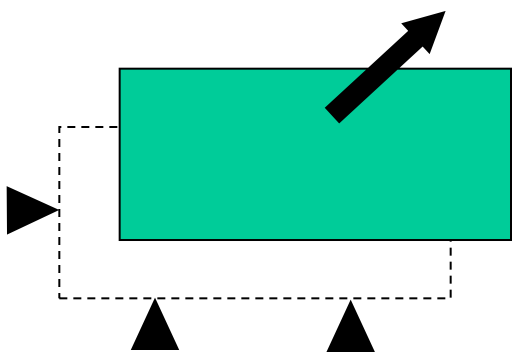
  where G is the Jacobian matrix of full-rank.

Accessibility/Detachability (3)

- Geometrical interpretation:



Weakly detachable



Strongly detachable

Summary

- Functions of a fixture
- Types of fixtures
- Kinematic/force analysis of fixtures
  - Deterministic positioning
  - Total constraint
  - Accessibility/Detachability