Errors in Measurement

ME 338: Manufacturing Processes II
Instructor: Ramesh Singh; Notes: Profs.
Singh/Melkote/Colton

1

Errors in Measurement

❖ Controllable Errors

– Environmental Conditions

– Elastic Deformation Due to Loading

– Alignment Errors

– Parallax Errors

❖ Non-Controllable Errors

– Scale Errors

– Reading Errors

– Linearity

– Hysteresis

– Repeatability & Random Errors

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2

Errors in Measurement

❖ Nothing is perfect

– a 30 mm gage block

◆ measured error in the block = -0.0002 mm

◆ accuracy of determination ±0.0004 mm

– understanding tolerance to imperfection is critical

❖ Error Compounding

– Accuracy of Determination

– Combination of Errors

– Confidence Intervals

Some Simple but Embarrassing Errors

❖ Misreading an instrument

❖ Arithmetic errors

❖ Errors in units

Controllable Errors - Environmental

❖ Temperature

– International Standard of Measurement – 20ºC (68ºF)

– Allow for temperature stabilization after handling

◆ 20 minutes per 25mm length for high precision measurement

– Temperature Compensation.

$$\text{Error} = l\alpha(t - t_s)$$
$$l = \text{nominal length}$$
$$\alpha = \text{thermal coefficient of expansion}$$
$$(t - t_s) = \text{deviation from standard temperature}$$

Controllable Errors - Environmental

❖ Temperature

– Comparative Measurement

◆ The error will be small if both objects are at the same temperature
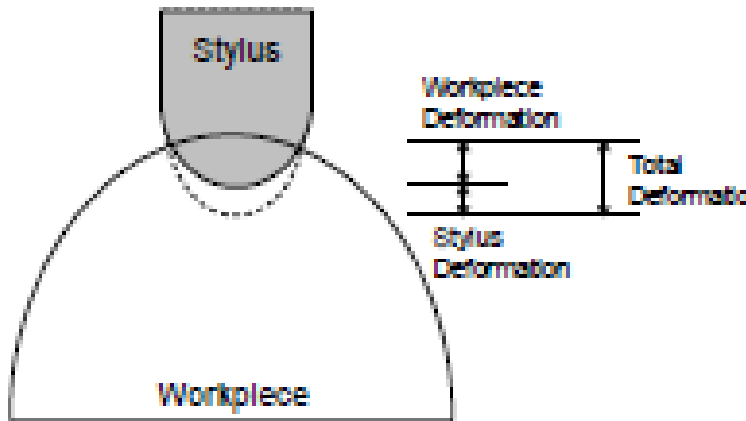
◆ For objects with different expansion coefficients:

$$Error = l(\alpha_1 - \alpha_2)(t - t_s)$$

❖ Pressure

❖ Humidity

❖ Vibration

❖ Dirt.

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Controllable Errors - Loading

❖ Stylus Pressure

$$\text{Deflection} = 1.774W^{2/3}(k_1 + k_2)^{2/3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{1/3}$$

W = Stylus pressure

$$k_1 = \frac{1 - u_1^2}{E_1}$$

$$k_2 = \frac{1 - u_2^2}{E_2}$$

R_1 = Radius of Stylus

R_2 = Radius of Workpiece

E_1 = Stylus Modulus of Elasticity

E_2 = Workpiece Modulus of Elasticity

u_1 = Stylus Poisson's Ratio

u_2 = Workpiece Poisson's Ratio

For comparative measurements, the loads should be the same.

Controllable Errors - Loading

Deflection of a hollow cylinder

$$\beta = \frac{1.7856 R_m \, p}{E L_i t^3}$$

t = Wall thickness

L = Effective length of workpiece

E = Young's Modulus for workpiece

p = Stylus pressure

Rm = Mean radius of the cylinder

Controllable Errors - Loading

- Effects of Supports
  - Large objects will sag under their own weight
  - Optimal placement of supports (Airy)
  - Minimization of deflection

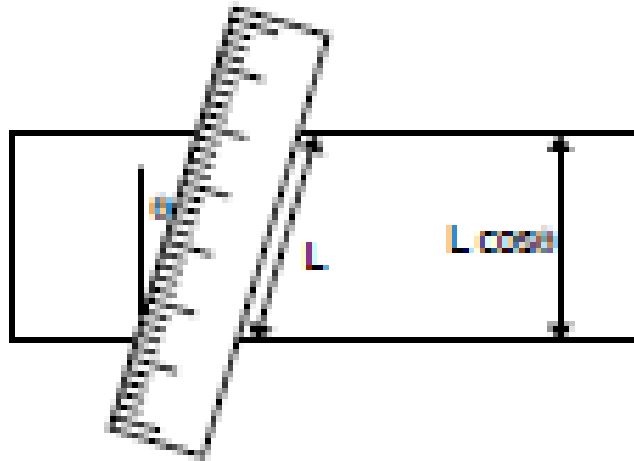Deflection at ends = Deflection at center

Controllable Errors - Loading

- ## Effects of Supports
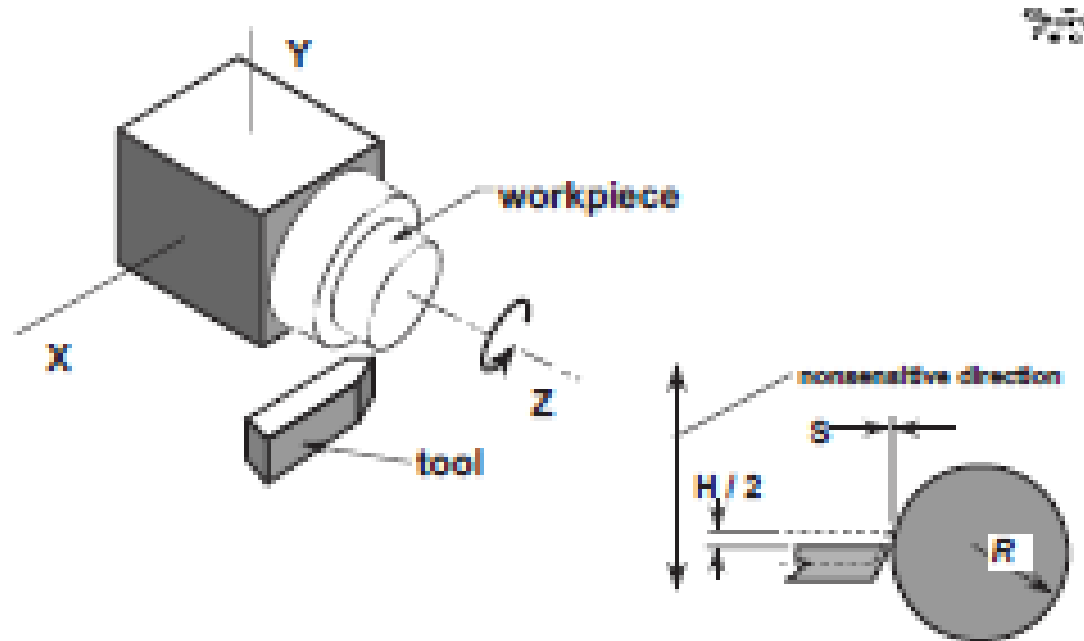 - Placement of Supports
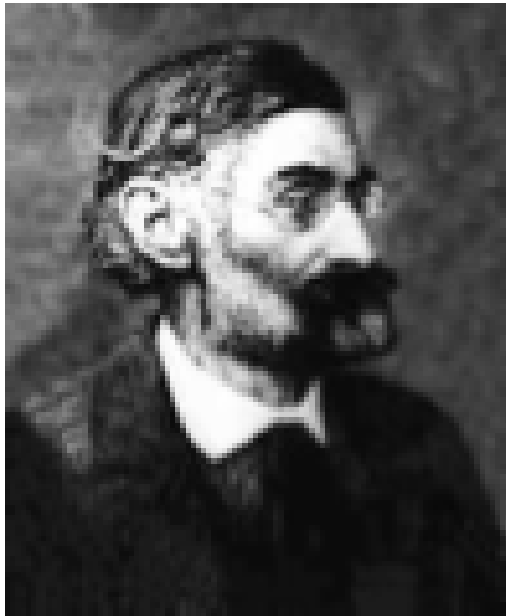 - Zero slope at ends

Controllable Errors – Cosine Errors

- ## Cosine Error –
  - The error generated when the scale and desired dimension are not aligned

Controllable Errors – Sensitive Directions
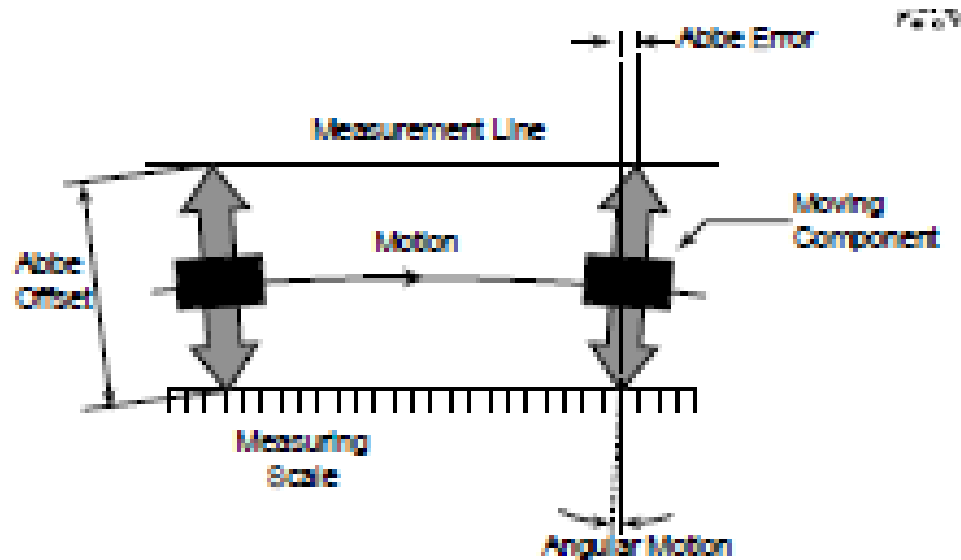
Controllable Errors – Abbe (Alignment)

Ernst Abbe (1840 – 1905)

- A co-founder of Zeiss, Inc.
- Provided numerous technical innovations in optical theory
- Proposed the Principle of Alignment

Controllable Errors – Abbe (Alignment)

Controllable Errors – Abbe (Alignment)

❖ Abbe's Principle of Alignment:

   The scale of a linear measuring system should be collinear with the spatial dimension or displacement to be measured If this is not the case, the measurement must be corrected for the associated Abbe Error.

❖ Abbe Error:

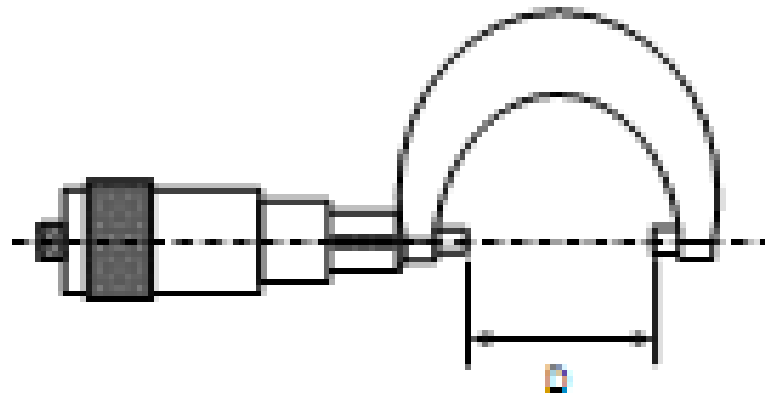   Abbe Error = (Abbe offset)*(sine of angular misorientation)

❖ Abbe Offset:

   The distance between the desired point of measurement and the reference line of the measuring system.

Controllable Errors – Abbe (Alignment)

❖ Abbe Principle – Micrometer a Good Result

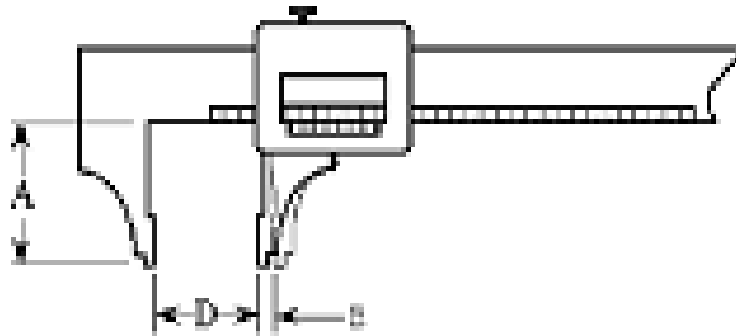− Graduations are located along the same axis as the measurement

Controllable Errors – Abbe (Alignment)

Abbe Principle – Caliper a Bad Result
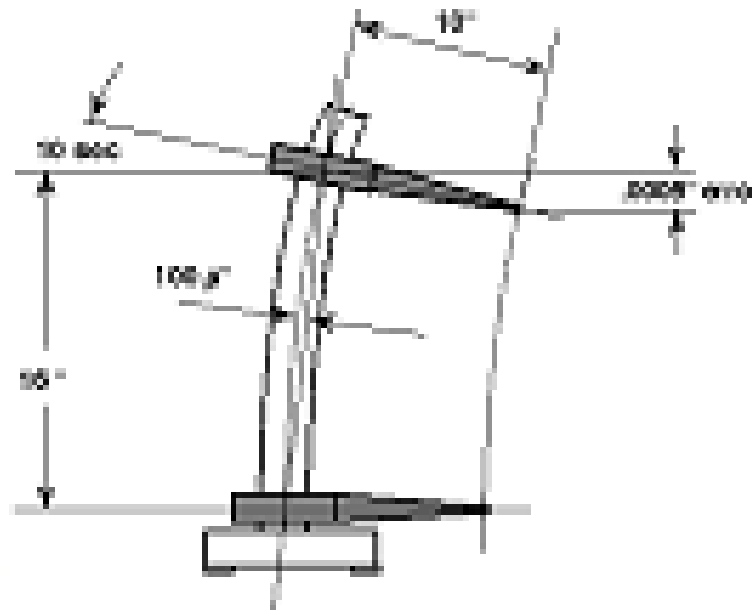
– Graduations are not on the same axis as the measurement

– Abbe Error (E) = A*sin(θ)

Abbe Error-Height Gage

❖ Abbe Error for a Height Gage

- Abbe Error (E) = (10 in) * sin(10 sec)=(10 in) * (10 sec) * (4.8 μin/in.sec) = 480 μin

Fixed and variable Ebbe Offset

- Fixed and variable Ebbe Offset
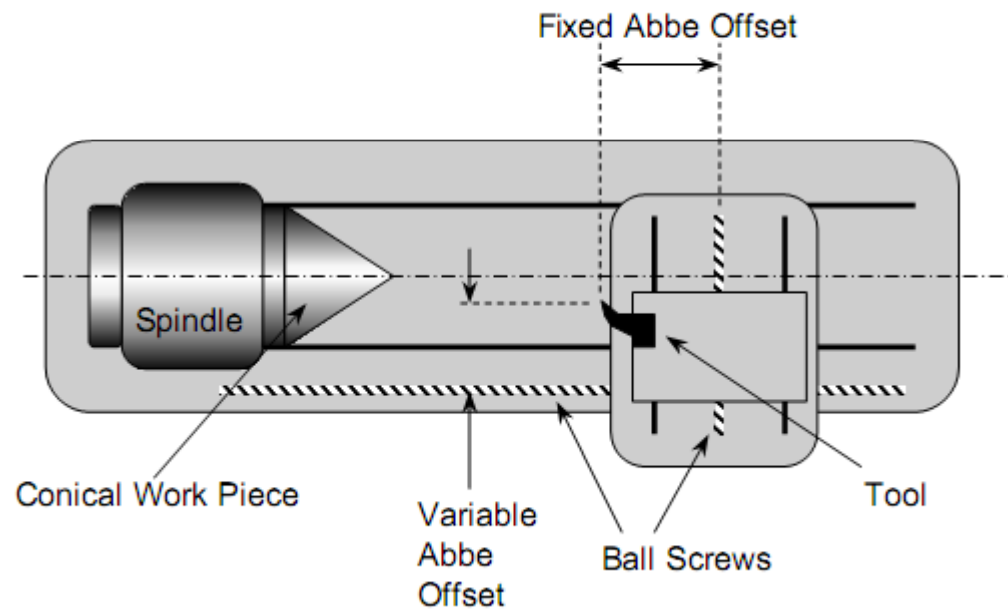
Byran Principle for Straightness Measurement

❖ Corollary to Abbe Principle in straightness

measurement

❖ Minimize M

Controllable Errors - Parallax

❖ Parallax Error-

The error that occurs when the pointer on a scale is not observed along a line normal to the scale
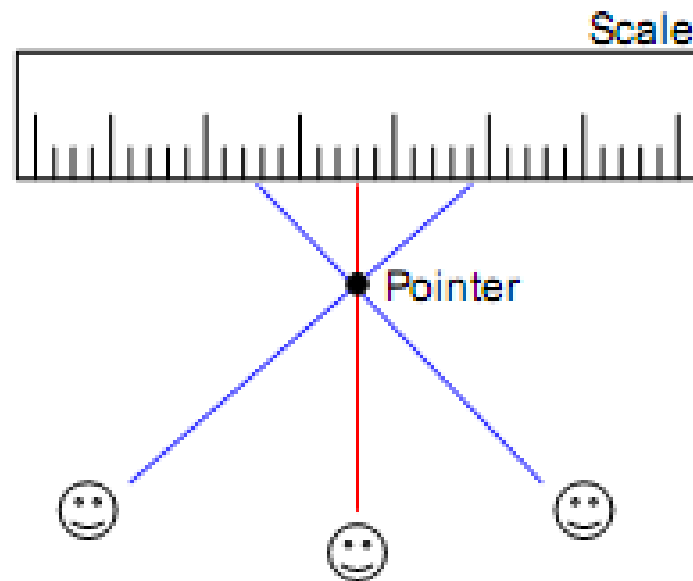
Non-Controllable Errors – Scale

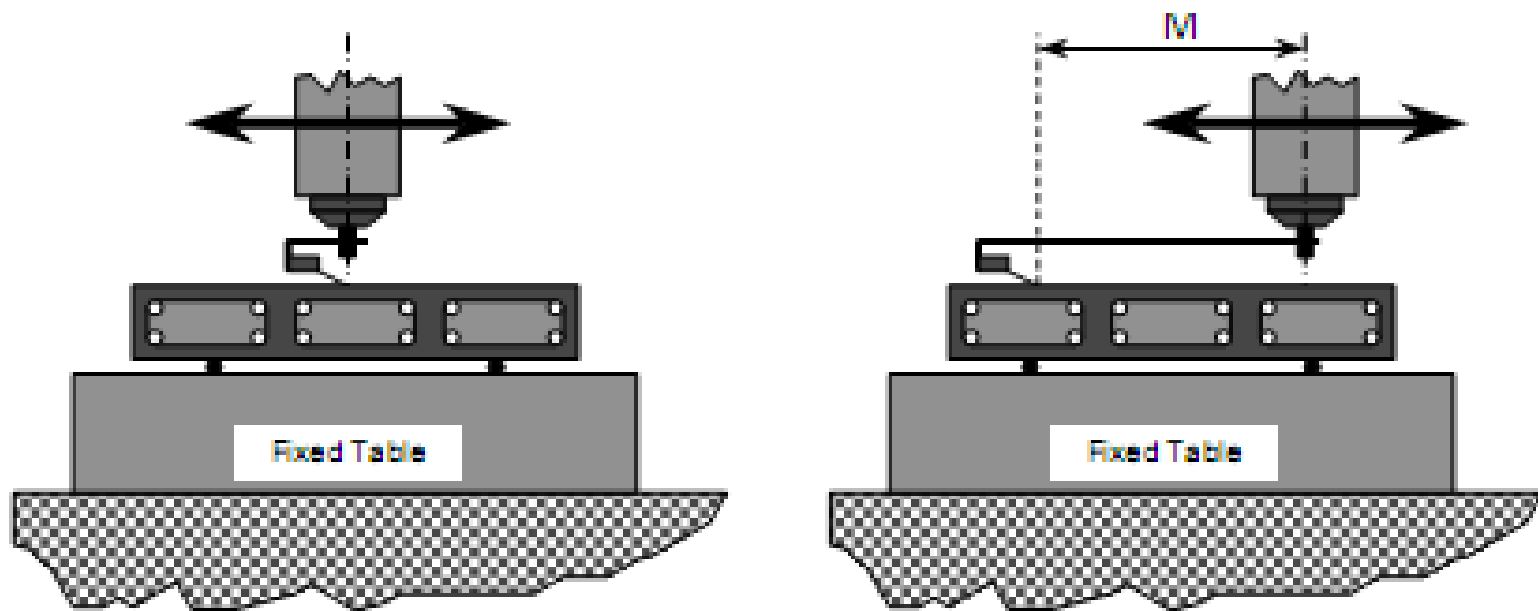❖ Scale Errors – Differences between the position of the readout device (scale) and that of a known reference linear scale.

❖ Scale errors can be reduced or eliminated by:

- Calibrating the scale against known standards of length.
- Using as short a length scale as possible.
- Choosing a setting master whose size is as close to that of the gauge being checked as possible.

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Non-Controllable Errors – Reading

❖ **The accuracy of a scale depends on:**

– The thickness of the rulings

– The spacing of the scale divisions

– The thickness of the datum (pointer) being used

❖ **Reading accuracies:**

– A reading of a pointer or datum line against a scale division can be taken as having an accuracy of ±10% of a scale division.

– A reading of a pointer or datum line between rulings should be taken as having an accuracy of ±20% of a scale division.

Non-Controllable Errors – Reading

❖ Example 1:

– A reading of –3 units is taken on a scale whose divisions represent 0.001 mm.

– The resulting measurement is -0.003 ± 0.0001 mm.

❖ Example 2:

– A reading of –3.4 units is taken on a scale whose divisions represent 0.001 mm.

– The resulting measurement is -0.0034 ±0.0002 mm.

Non-Controllable Errors - Linearity

❖ Output from a measuring system can be linear or non-linear

❖ Calibration of a linear system is easy.

– Requires 2 or 3 points

❖ Calibration of a non-linear system is very cumbersome and time-consuming

❖ Therefore, it is desirable to have a system with linear output

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❖ Linearity Error – For a measuring system with linear output, the maximum deviation of the output of the measuring system from a specified straight line

❖ 4 types of reference lines:

- Terminal Line – A line drawn from the origin to the farthest data point.

- End Point Line – A line drawn between the end points of a data plot.

- Best Fit Line – The line midway between the two closest parallel straight lines that enclose all of the data points.

- Least Square Line – The line for which the sum of the squares of the deviations of the data points from the line being fit is minimized.

Non-Controllable Errors - Linearity

❖ The slope m and intercept b of the least square best-fit straight line

$$\hat{m} = \frac{n(\sum xy) - (\sum y)(\sum x)}{n(\sum x^2) - (\sum x)^2}$$

$$\hat{b} = (\frac{1}{n}\sum y) - \hat{m}(\frac{1}{n}\sum x) = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

Non-Controllable Errors - Linearity

❖ Calculation of linearity errors:

$$\% \text{ linearity} = \frac{y_{i_{max}} - mx_i}{mx_{\text{full range}}} * 100\%$$

$y_{i_{max}}$ = output value at point of maximum deviation from the best-fit straight line

mxi = point on best-fit line corresponding to the maximum deviation point

m = slope of best-fit line

Non-Controllable Errors - Hysteresis

❖ Hysteresis - The difference between the indications of a measuring instrument when the same value of the measured quantity is reached by increasing or by decreasing that quantity.
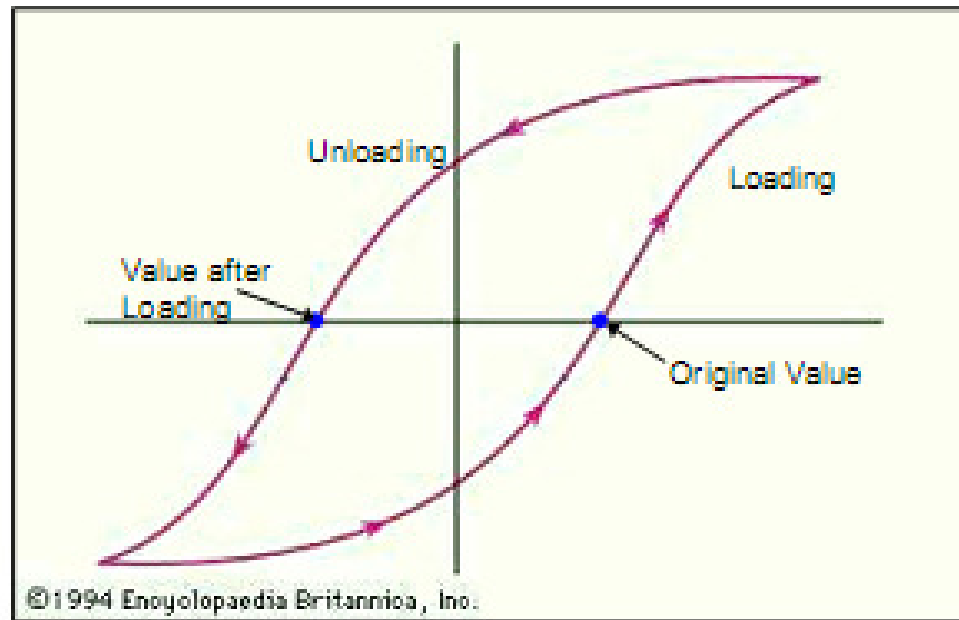
Non-Controllable Errors - Hysteresis

❖ Hysteresis:

- – Hysteresis Error:
  - • The difference in position between the loading and unloading curves
- – Symptoms:
  - • The pointer does not return to zero when the load has been removed
- – Causes:
- – The presence of dry friction
- – The properties of elastic elements (the presence of internal stresses)
- – Remedies:
- – Can be reduced considerably by proper heat treatment.

Non-Controllable Errors - Repeatability

❖ Repeatability – The ability of the measuring instrument to give the same value every time the measurement of a given quantity is repeated.

❖ Random Errors – Errors that cannot be predicted from knowledge of the measuring system and conditions of measurement.

❖ Repeatability is affected by:

– Environmental changes

– Variability in operator performance

– Small variations in the position of the workpiece

– Transient fluctuation in the friction of the measuring instrument.

Non-Controllable Errors - Repeatability

❖ Characterized by the standard deviation of the values
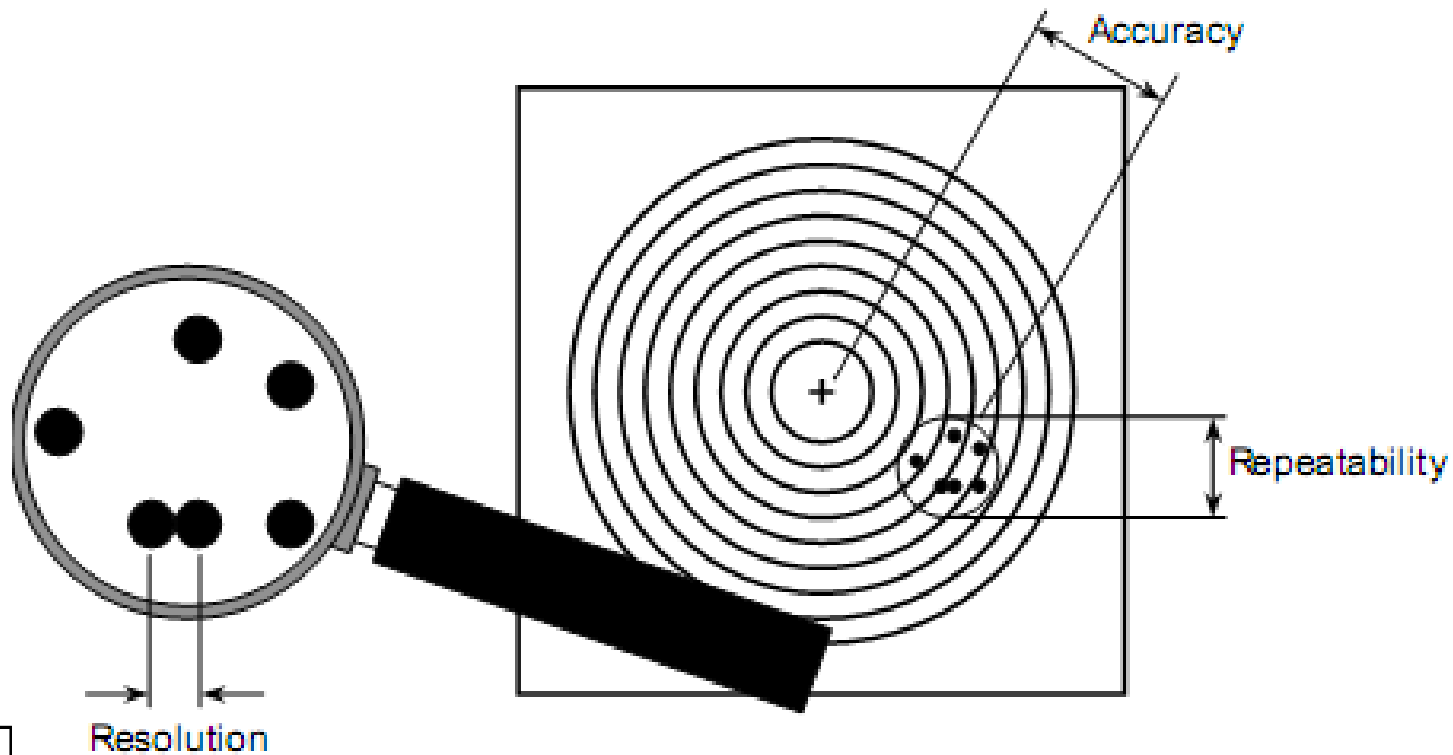
Error Compounding – Accuracy of Determination

❖ No measurement is exact.

❖ Therefore, it is necessary to state the accuracy of determination as well as the measured dimension

❖ Example.:

– 1.445 ±0.002

Compounding Errors

❖ All of the previous errors are cumulative

❖ In order to determine the maximum possible Given individual measurements (a, b, c, …)

❖ Having accuracies of determination (δa, δb, δc, …)

❖ The accuracy of determination for M, dM, as a function of (δa, δb, δc, …) is given by

$$dM = \frac{\partial M}{\partial a}\delta_a + \frac{\partial M}{\partial b}\delta_b + \frac{\partial M}{\partial c}\delta_c$$

Statistical Error Analysis

❖ In the previous slide we saw the worst case scenario

❖ Nominally, we will not see all of the measurements at their limits combining to give us the worst case

❖ Using statistics it can be shown that

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

σ_m = the standard deviation of the mean

σ = the standard deviation of the individual observations

n = the sample or sub-group size

Statistical Error Analysis

❖ If ±δ is the estimated accuracy of determination for a single measurement

❖ Then we can state that we are 99% confident that all observations lie within ±δ. This is also known as the 3σ limit.

❖ Roughly speaking

$$99\% \text{ Confidence Limits} = \pm \frac{3\sigma}{\sqrt{n}} = \pm \frac{\delta}{\sqrt{n}}$$

$$95\% \text{ Conf. Limits} = \pm \frac{2\sigma}{\sqrt{n}} = \pm \frac{\frac{2}{3}\delta}{\sqrt{n}} \qquad 65\% \text{ Conf. Limits} = \pm \frac{\sigma}{\sqrt{n}} = \pm \frac{\frac{1}{3}\delta}{\sqrt{n}}$$

Statistics and Multiple Accuracies

❖ The system variances can be analyzed as well

❖ Given

  M = a+b+c

❖ with variances $\sigma_a^2, \sigma_b^2, \sigma_c^2$

$$\sigma_M^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2$$

$$\sigma_M = \sqrt{\sigma_a^2 + \sigma_b^2 + \sigma_c^2}$$

❖ Remember, variances add. Standard deviations do not.

Summary

❖ All measurements are subject to error

❖ The possible deviation from the stated measurement should be estimated and given as an accuracy of determination

❖ Accuracy of determination can be improved by repeating the measurement a number of times and stating the mean value

❖ Statistical methods can be used to express the confidence in a particular measurement.