Manufacturing Process Optimization

Application to Machining
Selection of Cutting Conditions

• Selection of cutting speed, or cutting speed and feed can be made using unconstrained or constrained mathematical optimization methods.

• Optimization is based on time, cost, or profit rate criteria.
Optimization Criteria

- Commonly employed optimization criteria (objectives) are:
  - *Max. production rate* or *min. production time*: aims to maximize number of parts produced in a unit time interval or minimizes the time per unit part. Neglects cost and/or profit.

  - *Min. production cost*: aims to determine the least production cost. Coincides with max. profit rate criterion for constant unit revenue. Ignores time constraints.

- Usually one of the two criteria is used. Sometimes both criteria are used simultaneously.
Decision Variables

- Depth of cut \((d)\)
- Feed \((f)\)
- Cutting speed \((V)\)

Depth of cut usually decided based on part geometry and material to be removed and can be assumed constant.
Definitions (1)

- **Unit production time**, \( t \) (min/pt): time to manufacture a unit of the product.

\[
 t = t_p + t_m + t_r
\]

Where,
- \( t_p \) = setup time (min/pt); includes time needed to load/unload parts into machine, tool setup time, etc.
- \( t_m \) = machining time (min/pt)
- \( t_r \) = total tool replacement time (min/pt) = \( t_c(t_m/T) \)
- \( t_c \) = time needed to replace a worn cutting edge with a new one (min/edge)
- \( T \) = tool life (min)

- **Production rate** = \( (1/t) \)
Definitions (2)

- **Unit production cost,** $u$ ($$/pt$): cost to manufacture a unit of the product.

\[
U = U_o + U_m + U_t = k_o t_p + (k_o + k_m) t_m + [k_t + k_o t_c] \left( \frac{t_m}{T} \right)
\]

Where,

- $U_o =$ capacity utilization cost ($$/pt$); includes machine cost, labor cost, overhead etc. = \( k_o t = k_o [t_p + t_m + t_c(t_m/T)] \)
- $k_o =$ machine utilization rate ($$/min$$)
- $U_m =$ machining cost (associated with actual machining time); includes cost of electricity, cutting fluids etc. = \( k_m t_m \)
- $k_m =$ machining overhead ($$/min$$)
- $U_t =$ tool utilization cost; includes cost of cutting tool, tool resharpening, etc. = \( k_t \left( \frac{t_m}{T} \right) \)
- $k_t =$ cost per cutting edge ($$/edge$$)
Unconstrained Optimization (1)

• Basic problem formulation: Find cutting speed $V$ that optimizes (Minimizes or Maximizes) $Z$, where $Z =$ appropriate optimization criterion.

• In order to solve this problem, the objective function $Z$ should be expressed in terms of $V$. Feed and depth of cut are assumed to be fixed.

• Note that only tool life $T$ and the machining time $t_m$ are functions of $V$. 
Unconstrained Optimization (2)

• In general,

\[ t_m = \frac{\lambda}{V} \]

Where \( \lambda \) is a constant based on the machining process and \( V \) is in m/min units.

• For turning, boring, drilling, and reaming operations:

\[ t_m = \frac{L}{(Nf)} \quad \text{and} \quad V = \frac{\pi DN}{1000} \]

\[ \lambda = \frac{\pi DL}{(1000f)} \]

where \( D \) = diameter of part/tool (mm), \( L \) = length of cut (mm), \( f \) = feed (mm/rev), \( N \) = spindle speed (rpm)
Unconstrained Optimization (3)

• Taylor’s tool life equation:

\[ VT^n = C \]

Where \( T = \) tool life (min), \( C = 1 \) min. tool life cutting speed, \( n = \) function of tool-workpiece material.
Unconstrained Optimization (4)

• Expressing $t, u$ in terms of $V$, we get:

$$t = t_p + \left( \frac{\lambda}{V} \right) + t_c \left( \lambda V^{\frac{1-n}{n}} / C^{1/n} \right)$$

$$u = k_o t_p + (k_o + k_m) \left( \frac{\lambda}{V} \right) + (k_t + k_o t_c) \left( \lambda V^{\frac{1-n}{n}} / C^{1/n} \right)$$
Unconstrained Optimization (5)

- Optimal cutting speed (and corresponding tool life) can be obtained by standard calculus as:

- Min. time or max. production rate criterion:

\[ V_{opt} = C \sqrt[n]{\left( \frac{1-n}{n} \right)^{n} t_c} \]

\[ T_{opt} = \left( \frac{1-n}{n} \right)^{n} t_c \]
Unconstrained Optimization (6)

- Min. cost criterion:

\[
V_{opt} = C \left[ \frac{(k_o + k_m)}{\left(\frac{1-n}{n}\right)\left(k_t + k_o t_c\right)} \right]^n
\]

\[
T_{opt} = \left(\frac{1-n}{n}\right) \left[ \frac{(k_t + k_o t_c)}{(k_o + k_m)} \right]
\]
Unconstrained Optimization Example (1)

- Given:
  - Taylor tool life equation $VT^{0.23} = 430$, $V$ (m/min)
  - Machining (turning) parameters
    - Depth of cut, $d = 1.00$ mm
    - Feed, $f = 0.2$ mm/rev
  - Workpiece geometry
    - Workpiece diameter, $D = 50$ mm
    - Workpiece length, $L = 200$ mm
  - Time parameters
    - Setup time, $t_p = 0.75$ min/pt
    - Tool replacement time, $t_c = 1.5$ min/edge
Unconstrained Optimization Example (2)

- **Given:**
  - Cost parameters
    - Machine utilization rate, $k_o = 0.50 \$/min
    - Machining overhead, $k_m = 0.05 \$/min
    - Tool cost, $k_t = 2.50 \$/edge
Unconstrained Optimization Example (3)

• Min. time or max. production rate cutting speed and tool life:

\[ V_{opt} = C \sqrt{\left( \frac{1-n}{n} \right) t_c} \]

\[ = 430 \sqrt{\left( \frac{1-0.23}{0.23} \right)^{1.5}}^{0.23} = 296 \text{ m/min} \]

\[ T_{opt} = \left( \frac{1-n}{n} \right) t_c = \left( \frac{1-0.23}{0.23} \right)^{1.5} = 5 \text{ min} \]
Unconstrained Optimization Example (4)

- Min. cost cutting speed and tool life:

\[
V_{opt} = C \left[ \frac{(k_o + k_m)}{(1-n)(k_t + k_o t_c)} \right]^n = 430 \left[ \frac{(0.5 + 0.05)}{(1-0.23)(2.5 + 0.5 \times 1.5)} \right]^{0.23} = 216 \text{ m/min}
\]

\[
T_{opt} = \left( \frac{1-n}{n} \right) \left[ \frac{(k_t + k_o t_c)}{(k_o + k_m)} \right] = \left( \frac{1-0.23}{0.23} \right) \left[ \frac{(2.5 + 0.5 \times 1.5)}{(0.5 + 0.05)} \right] = 19.8 \text{ min}
\]
Summary

• Manufacturing process optimization
  – Application to machining

• Unconstrained optimization
  – Optimum cutting speeds
  – Production time and cost criteria