Statistical Quality Control

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Outline

- What is *SQC*?
- Basics of Statistics
- Design of Control Charts
  - Inspection by Variables
  - Inspection by Attributes
- Process Capability Index

What is *Statistical Quality Control* ?

Set of Statistical Tools to identify Quality Problems in the Product and the Production Process.

SQC Tools

Descriptive Statistics

- mean, standard deviation, range, distribution

Statistical Process Control – SPC

- Process inspection from random samples
- Assessing Process Control Causes

Acceptance Sampling

- Random Inspection to *Accept / Reject* the lot

Product Quality Variation

Assignable Causes

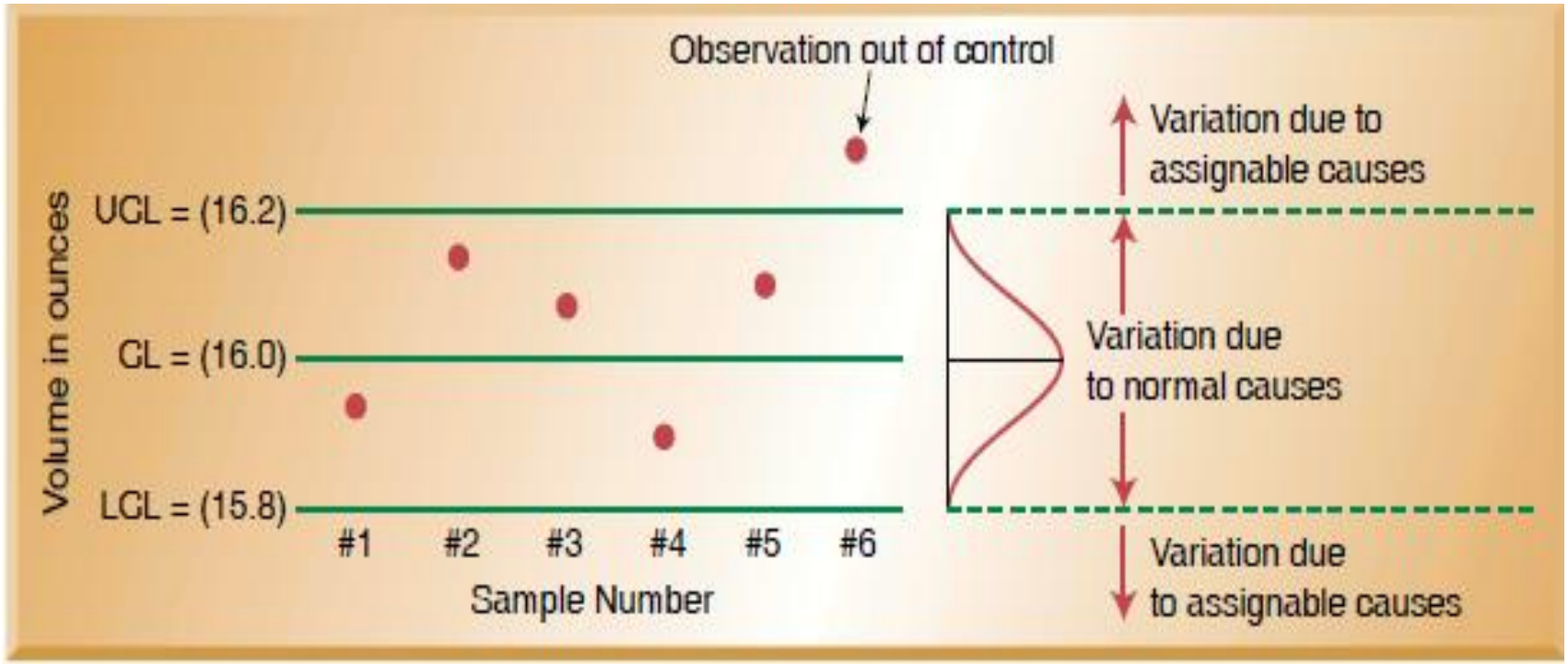
- Setup error

Random Causes

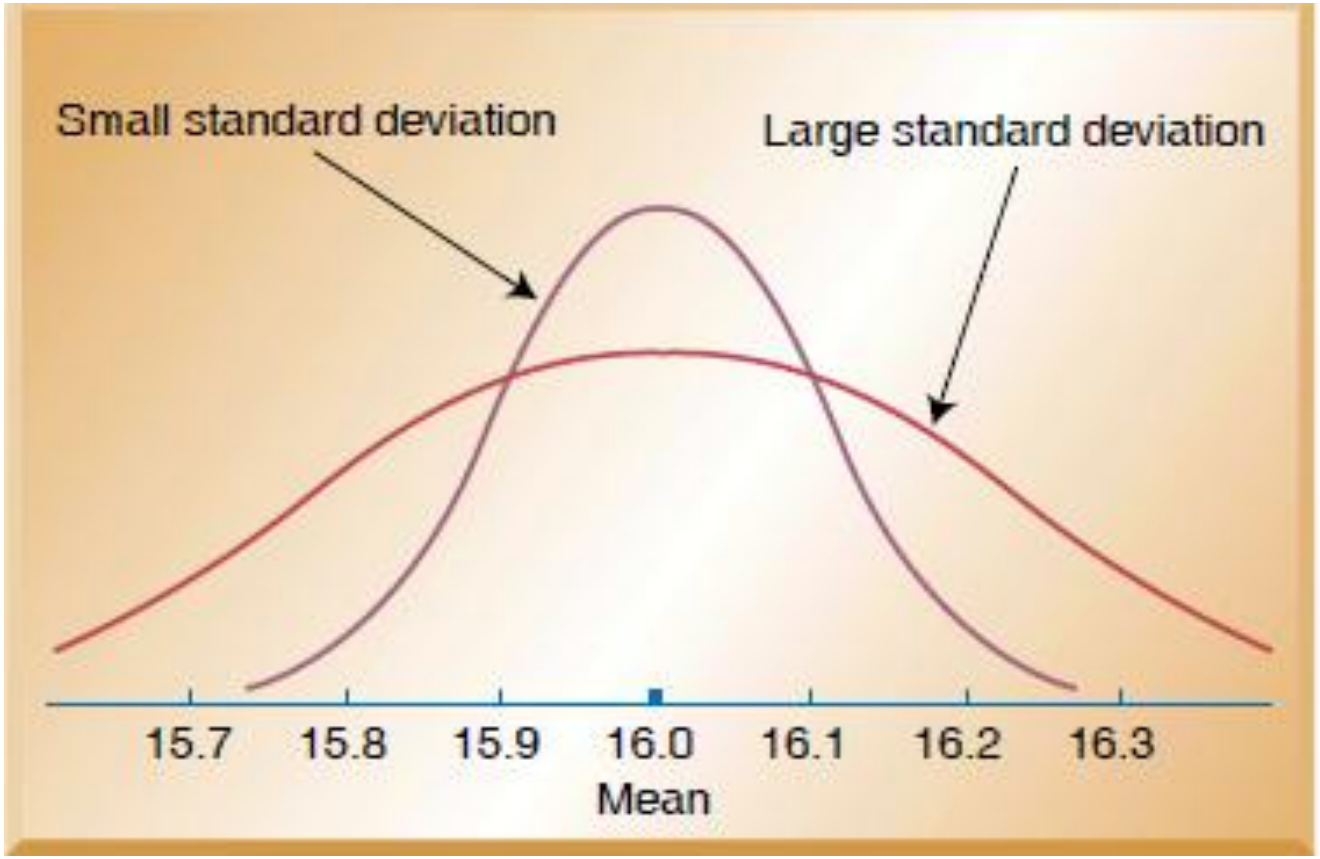
- Natural random errors
- Tool, Work, Operator

Quality control aims at *Natural Random Variations* in a Process

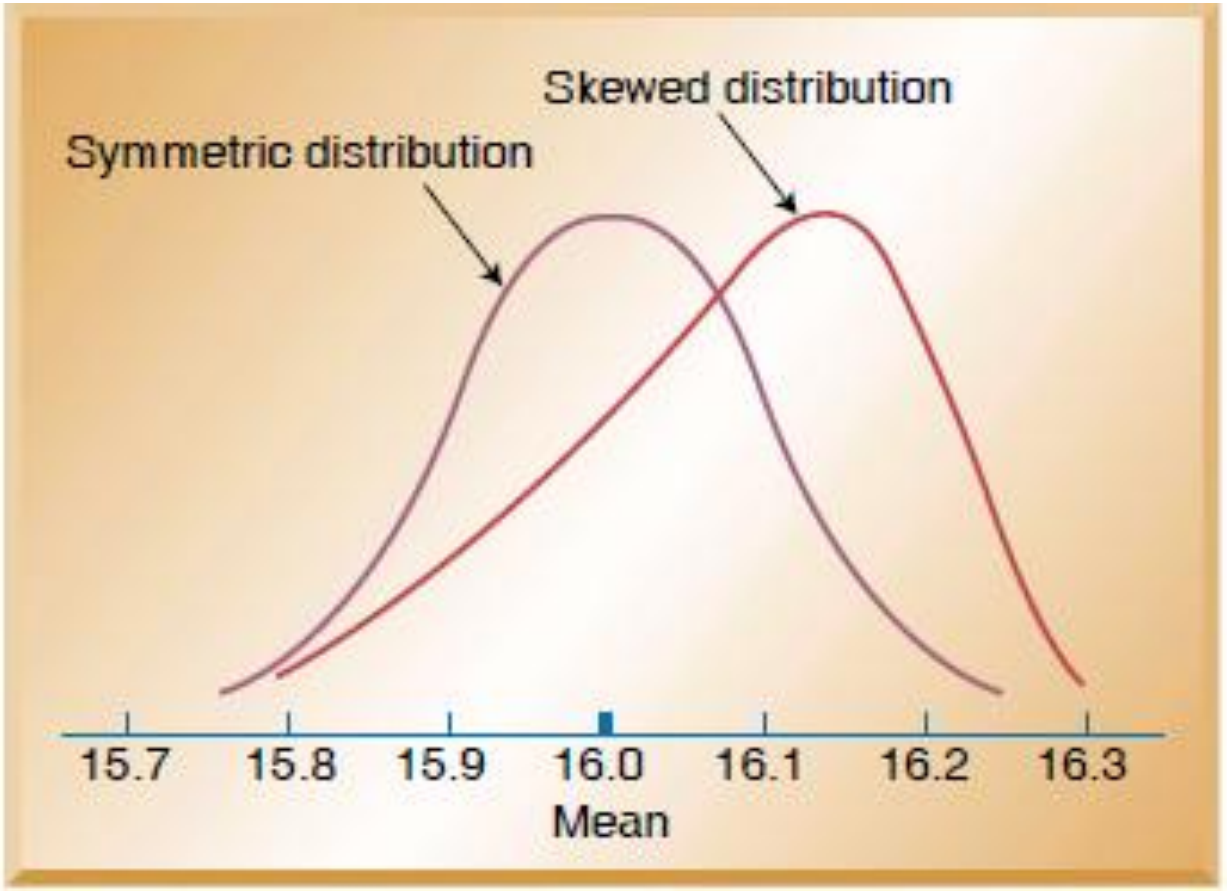
Causes of Data Variability



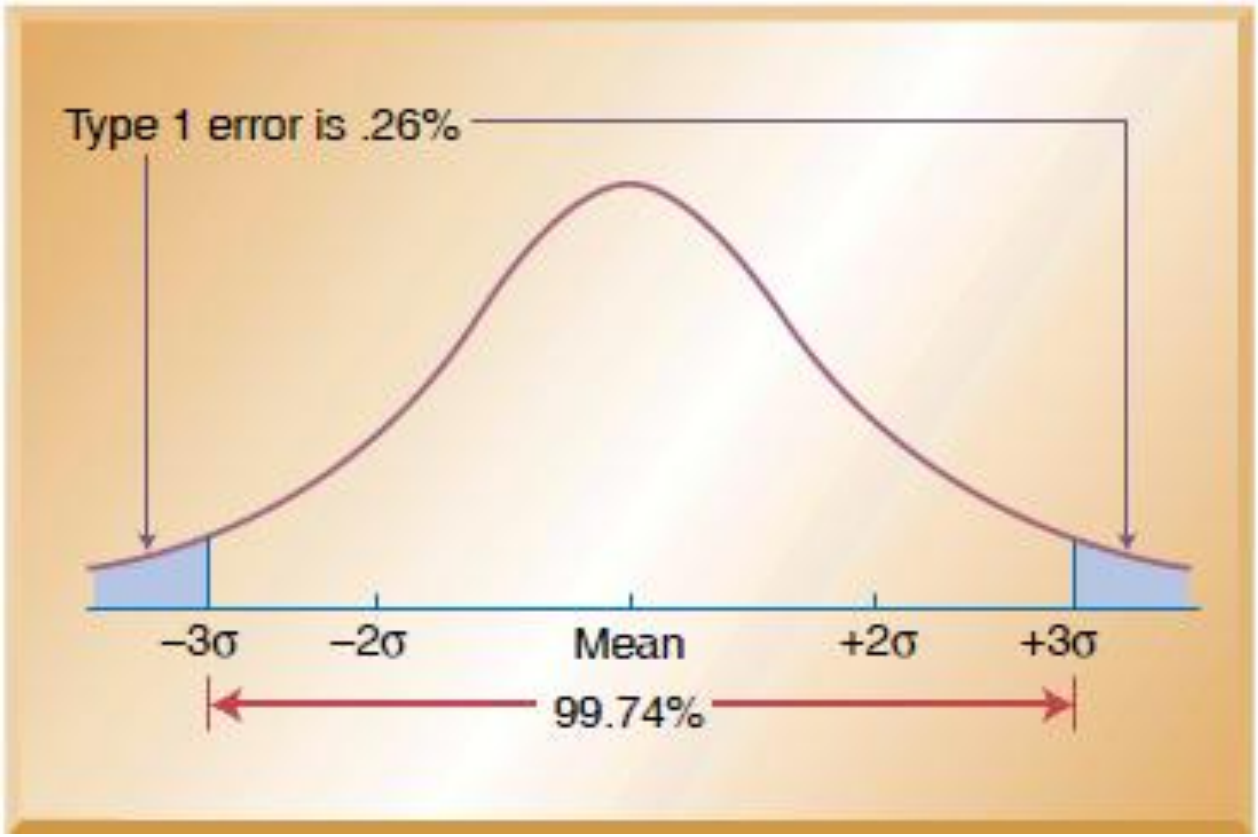
Normal Distribution



Skewed Distribution



Acceptance Error



Basics of Statistics

Measures of Central Tendency of Data

- Mean $\overline{X}$
- Variability
  - *Range R*
  - *Standard Deviation* σ

Arithmetic mean

Measures central tendency of data

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

x_i : data

n : no. of observations

Range and Standard Deviation

Measures of variability of data around the mean.

Range

$R = x_{max} - x_{min}$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Statistical Process Control - SPC

Objectives

- *Evaluate* Quality of products/ services
- *Determine* Common/ Normal variation
- *Identify* Quality Problems during Process
- *Provide* direction to correct Production Process *in run*

Control Charts

Using Variables

- Length, diameter, weight, volume.
- Use *X- Bar and R charts*

Using Attributes

- No of broken cookies in a pack, rotten apples in a carton, defects in a lot
- Use *P and C charts*

X-Bar and R charts

Samples are drawn from a Production Process and Inspected

Let

K = No. of samples

n = No. of observations per sample

X- Bar Chart

Monitors mean of the process.

Sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Mean of Samples

$$\bar{\bar{x}} = \frac{1}{k} \sum_{i=1}^{k} \bar{x}_i$$

X – Bar Control Limits

Upper Control Limit

$$UCL = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}}$$

Lower Control Limit

$$LCL = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}}$$

R Control Limits

Monitors *Dispersion* or *Variability* of Process

Centerline = $\overline{R}$

UCL = $D_4 \cdot \overline{R}$

LCL = $D_3 \cdot \overline{R}$

Factors : D_4, D_3 (From Table)

Factors for X Bar and R Charts

Sample Size n	Factor for $\bar{x}$-Chart	Factors for R-Chart	
	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72

Example

A quality control inspector at a soft drink company has taken 25 samples each with 4 observations on the volume of bottle filled. Standard deviation of bottling operation is 0.14 oz.

Develop Control Charts for the inspection of the bottling operation.

Inspection Data

Sample Number	Observations (bottle volume in ounces) 1	2	3	4	Average $\bar{x}$	Range R
1	15.85	16.02	15.83	15.93	15.91	0.19
2	16.12	16.00	15.85	16.01	15.99	0.27
3	16.00	15.91	15.94	15.83	15.92	0.17
4	16.20	15.85	15.74	15.93	15.93	0.46
5	15.74	15.86	16.21	16.10	15.98	0.47
6	15.94	16.01	16.14	16.03	16.03	0.20
7	15.75	16.21	16.01	15.86	15.96	0.46
8	15.82	15.94	16.02	15.94	15.93	0.20
9	16.04	15.98	15.83	15.98	15.96	0.21
10	15.64	15.86	15.94	15.89	15.83	0.30
11	16.11	16.00	16.01	15.82	15.99	0.29
12	15.72	15.85	16.12	16.15	15.96	0.43
13	15.85	15.76	15.74	15.98	15.83	0.24
14	15.73	15.84	15.96	16.10	15.91	0.37
15	16.20	16.01	16.10	15.89	16.05	0.31
16	16.12	16.08	15.83	15.94	15.99	0.29
17	16.01	15.93	15.81	15.68	15.86	0.33
18	15.78	16.04	16.11	16.12	16.01	0.34
19	15.84	15.92	16.05	16.12	15.98	0.28
20	15.92	16.09	16.12	15.93	16.02	0.20
21	16.11	16.02	16.00	15.88	16.00	0.23
22	15.98	15.82	15.89	15.89	15.90	0.16
23	16.05	15.73	15.73	15.93	15.86	0.32
24	16.01	16.01	15.89	15.86	15.94	0.15
25	16.08	15.78	15.92	15.98	15.94	0.30
Total					398.75	7.17

X – Bar and R Control Chart Limits

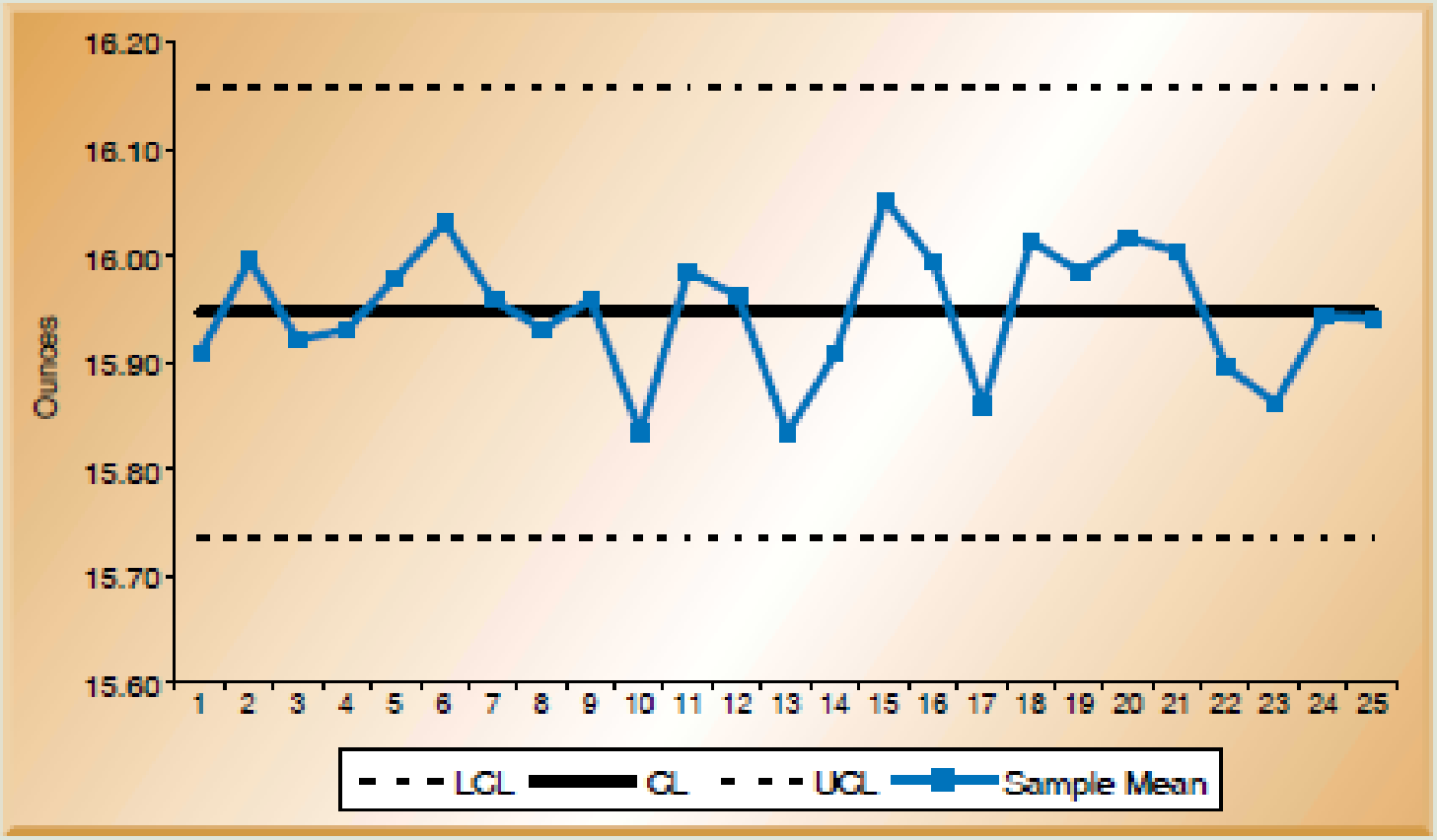
X – Bar Chart

CL $\bar{\bar{x}}$ = 15.95
UCL = 16.16
LCL = 15.74

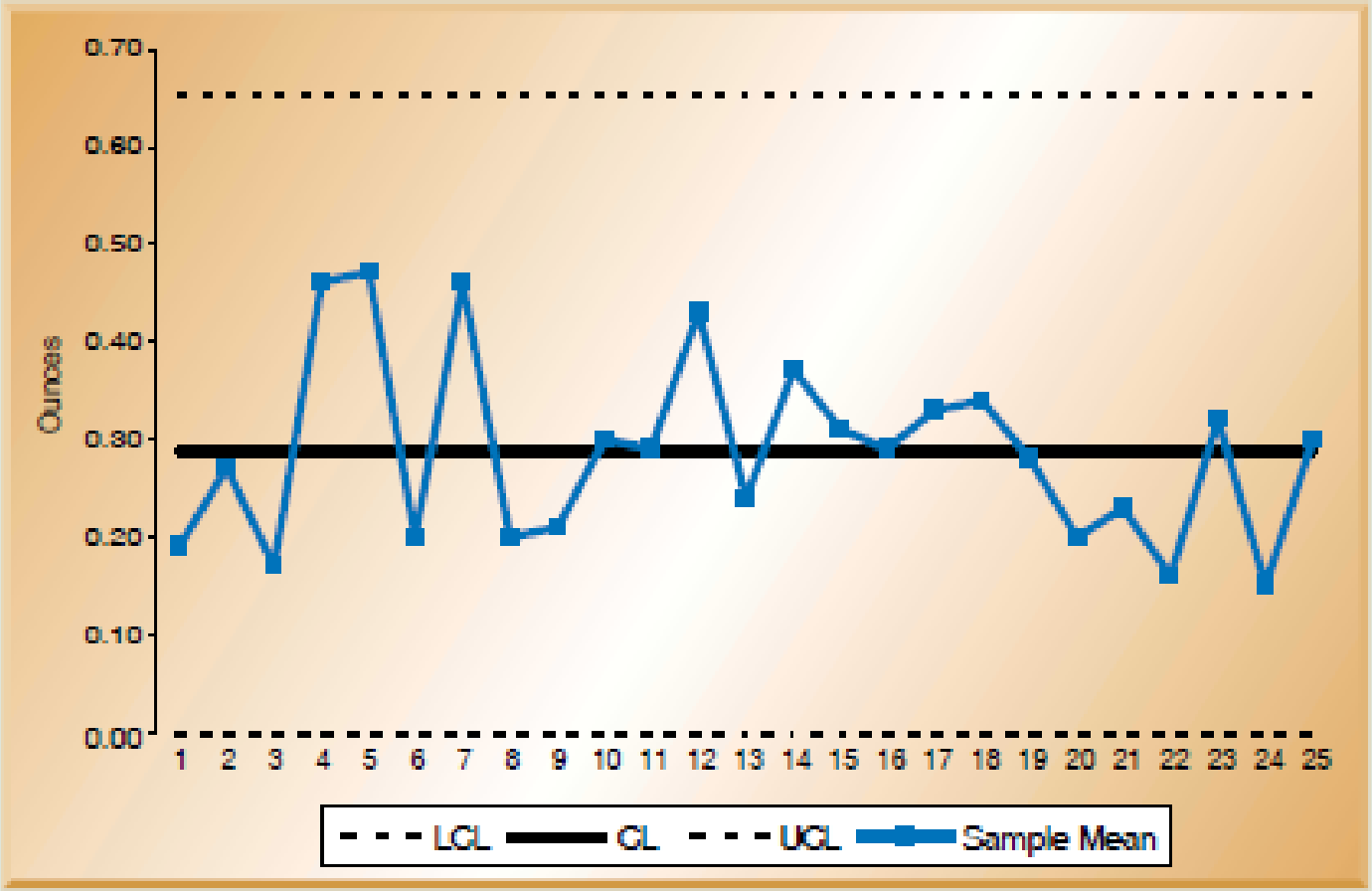
R – Chart

CL $\bar{R}$ = 0.29
UCL = 0.6612
LCL = 0

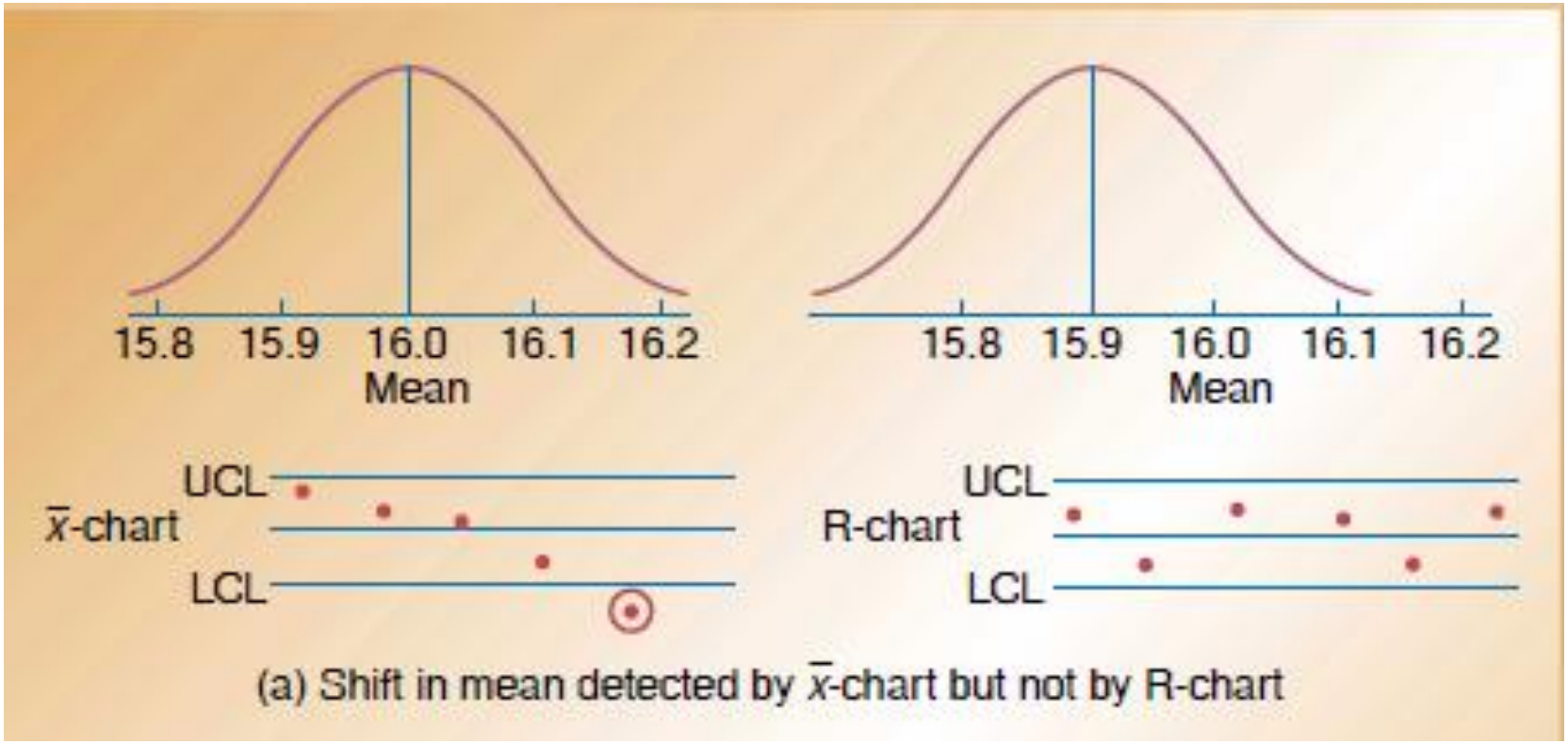
X Bar Chart


16.20
16.10
16.00
15.90
15.80
15.70
15.60
Ounces
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
LCL
CL
UCL
Sample Mean

R Chart

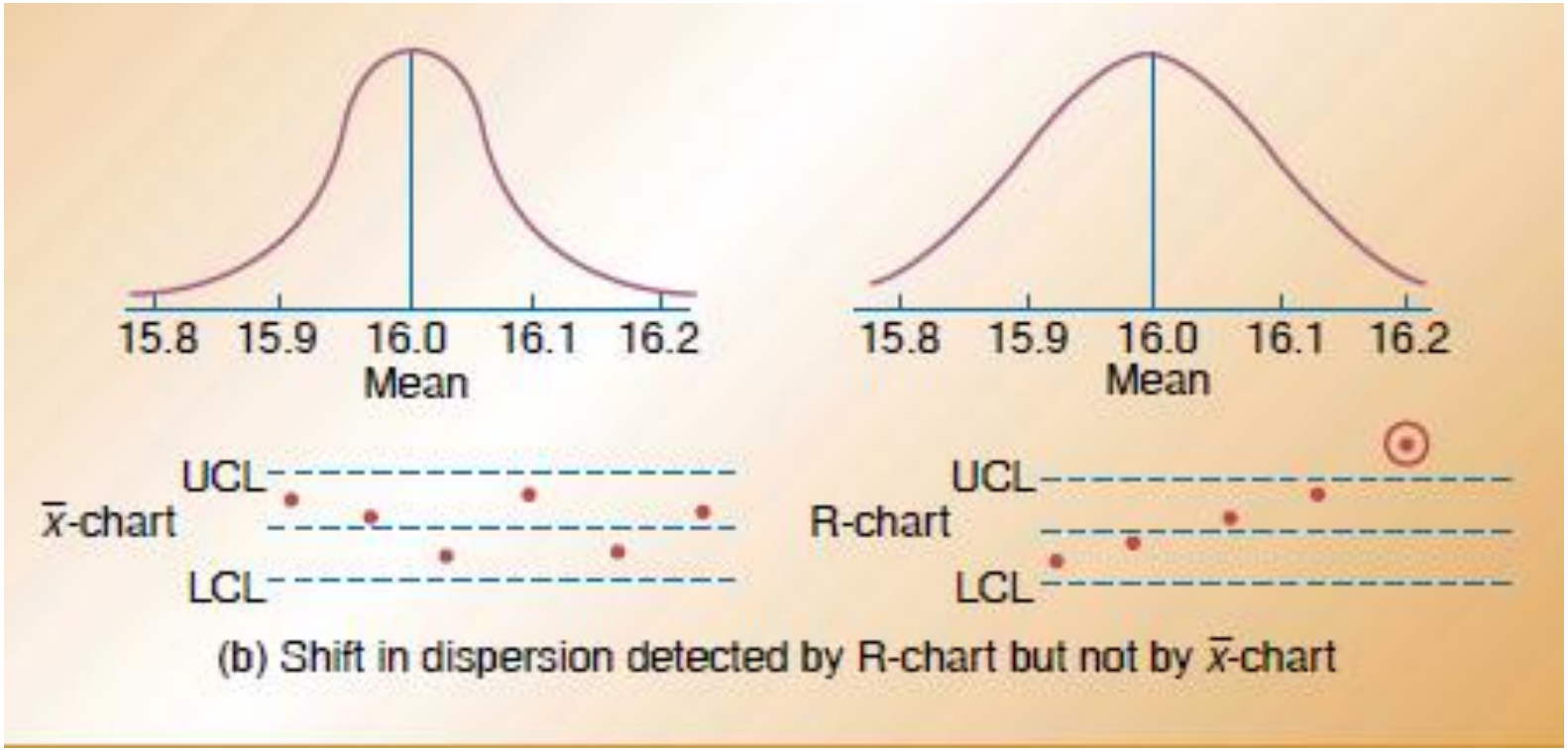

0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
Ounces
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
LCL
CL
UCL
Sample Mean

Interpreting X Bar and R Charts



(a) Shift in mean detected by $\bar{x}$-chart but not by R-chart

Interpreting X Bar and R Charts



Inspection by Attributes

Measures Quality characteristics for parts with Attributes which are *Discrete* in nature.

- Number of Defects in a sample, Broken cookies in a pack, Rotten apples in a box
- Both Sample size and number of defects are Countable.

P Control Chart

Chart constructed in terms of

$\overline{p}$ = Average Proportion Defective in a sample

P Control Chart limits

$\bar{p}$ = Average Proportion Defective in a sample

Chart Limits

Centerline : $\bar{p}$

UCL = $\bar{p} + 3 * \sigma_p$

LCL = $\bar{p} - 3 * \sigma_p$

$$\sigma_p = \sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

where n = no of observations in a sample

Example

In a Production Process, data on number of defective parts has been collected. 20 samples have been drawn each with 20 observations.

Construct P chart for the inspection process.

Inspection Data – No of Defects

Sample Number	Number of Defective Tires	Number of Observations Sampled	Fraction Defective
1	3	20	.15
2	2	20	.10
3	1	20	.05
4	2	20	.10
5	1	20	.05
6	3	20	.15
7	3	20	.15
8	2	20	.10
9	1	20	.05
10	2	20	.10
11	3	20	.15
12	2	20	.10
13	2	20	.10
14	1	20	.05
15	1	20	.05
16	2	20	.10
17	4	20	.20
18	3	20	.15
19	1	20	.05
20	1	20	.05
Total	40	400	

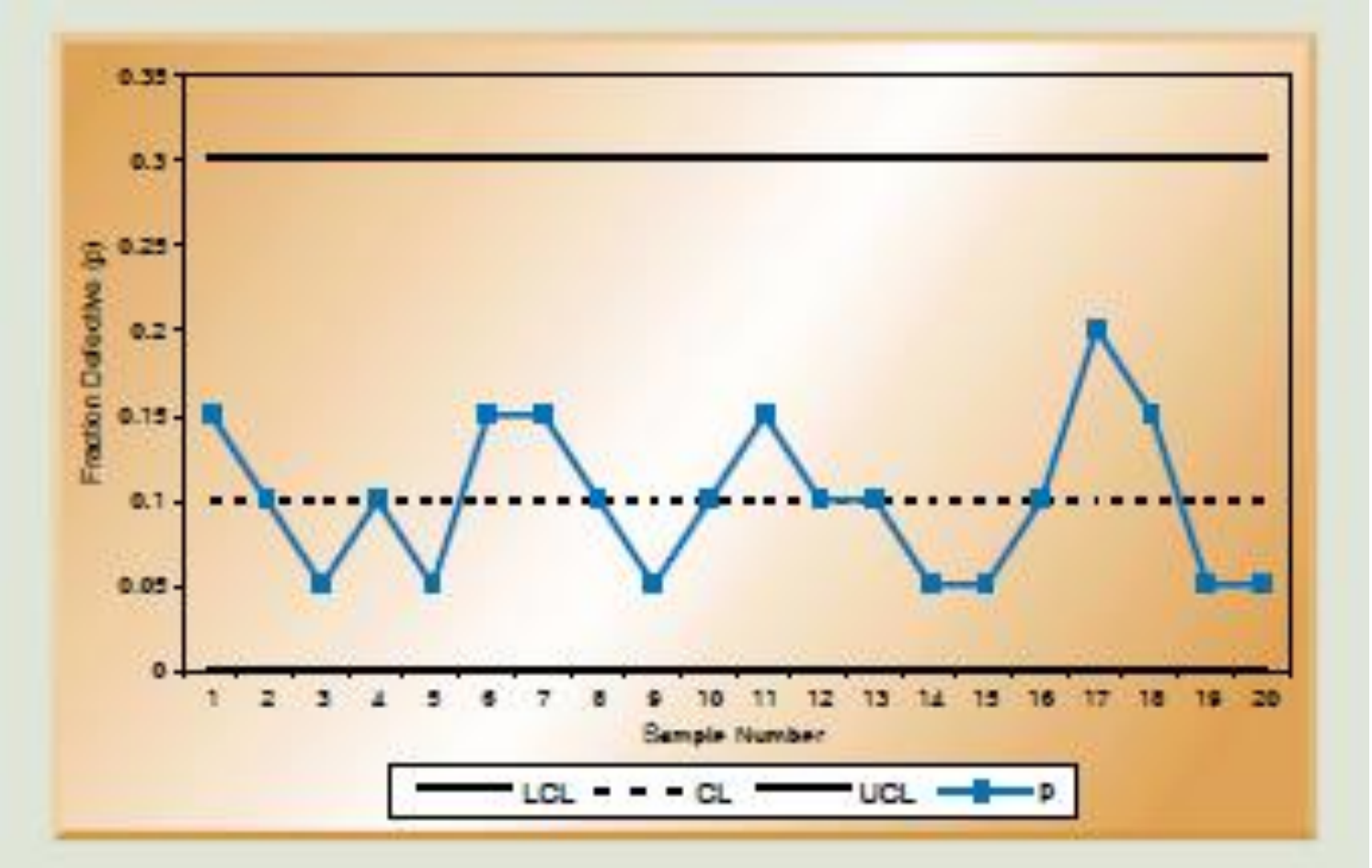
P Chart Control Limits

Sample size (k) : 20
Observations per sample (n) : 20
Total Observations : 400
Total number of defects observed : 40
Fraction defective $\bar{p}$ = 0.1
σ_p = 0.067

<u>Chart limits</u>

UCL = 0.301
LCL = -0.1 -- > 0.0

P Chart


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Fraction Defective (p)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Sample Number
LCL
CL
UCL
p

Process Capability

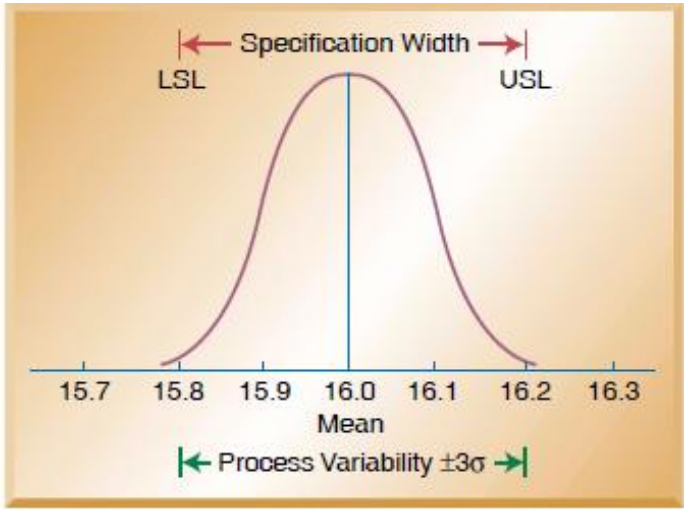
Evaluating ability of Process to meet Product Tolerances

Process Capability Index – C_p

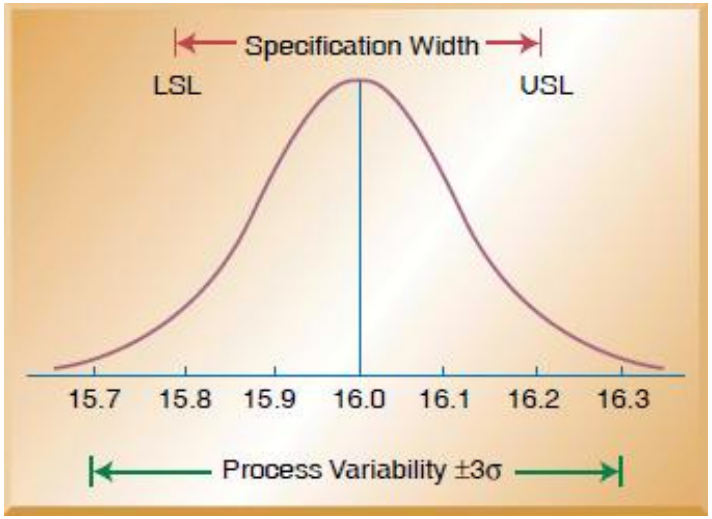
$$C_p = \frac{USL - LSL}{6\sigma}$$

Upper /Lower Specification limits

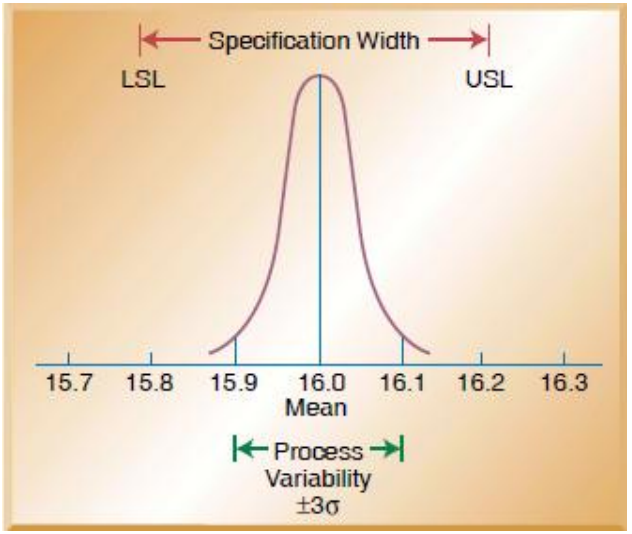
Process Capability Cp



Cp = 1



Cp < 1



Cp > 1

SQC – Implications for Managers

How much and How often to inspect?

- Product Cost and Product Volume
- Process Stability History
- Frequency of Inspection

Where to Inspect?

- Inbound/Outbound/Prior to Costly Processing

SQC Tools used

- X-Bar and R charts
- Acceptance Sampling for Inbound/ Outbound