Determination of the Flatness of a Plane Surface

In Chapter 6 it is demonstrated how an auto-collimator, or a spirit level, may be used to measure the deviation from straightness of a machine tool guide-way. The same principles may be used to determine the deviation from a true plane of a large surface such as a surface table or machine table.

A flat surface is composed of an infinitely large number of lines, or generators, and for it to be truly flat the following conditions must be satisfied:

(*a*) All generators must be straight.

(*b*) All generators must lie in the same plane.

It should be noted that provided condition (*a*) is *completely* realized then condition (*b*) must also hold good. The two conditions are emphasized as it is the verification of condition (*b*) which is the main problem. Also it must be realized that it is not a sufficient test, in the case of a rectangular surface, to measure the straightness of generators parallel to the edges. These may all be straight but the surface need not be flat.

Consider a sheet metal box having a pair of diagonally opposite corners reduced in height, but whose sides are straight. If the box is filled with plaster of paris which is then levelled off with a straight edge which is kept parallel to one end, then all lines across the surface must be straight (they were produced by a straight edge). Similarly all lines at 90° to these generators must be straight, as the straight edge was controlled by two other straight lines, these being the edges of the box. Thus if such a surface is tested for flatness along lines parallel to its sides it will appear to be flat. That it is not is clearly seen from Fig. 12.1, it being concave across one diagonal and convex across another.

It is immediately seen that if the surface is to be verified as being truly flat then it is necessary to measure the straightness of the diagonals, in addition to the generators parallel to the sides.

The measurement of straightness of all of these lines of test may be carried out with an auto-collimator as is described in Chapter 6, but having made these measurements it is necessary to relate each line of test to all of the others, i.e. verifying conditions (*b*) with which this appendix is concerned.

Consider the surface shown in plan view of Fig. 12.2 on which the eight main generators are set out. These should be chosen just inside the edges of the table so that the edge area, which is prone to damage, is avoided. The length of the lines should be whole multiples of the length of the base of the spirit level or reflector stand, whichever instrument is used, and it is advisable to select side and diagonal lengths in the ratio of 3 : 4 : 5.

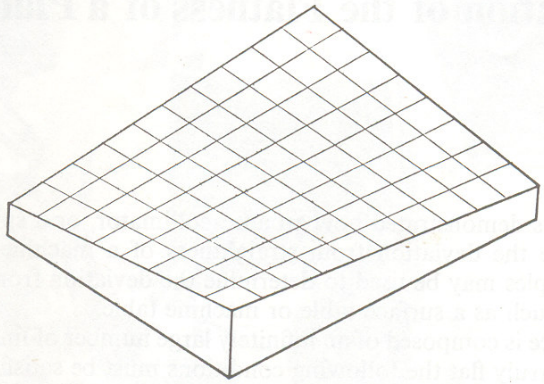


Fig. 12.1. A surface, all of whose generators parallel to the sides are straight, but which is not flat.

The procedure is as follows:

(*a*) Carry out a normal straightness test on each generator.

(*b*) Tabulate each set of results only as far as the cumulative error column.

(*c*) Correct the ends of AC; AG; and CG; to zero. This gives the heights of points A, C, and G as zero and these three points then constitute an arbitrary plane relative to which the heights of all other points may be determined.

(*d*) From (3) the height of O is known relative to the arbitrary plane ACG=OOO. As O is the common mid-point of AE, CG, BF, and HD, all points on AE are now fixed. This is done by leaving A=O and correcting O on AE to coincide with the mid-point O on CG.

(*e*) Correct all other points on AE by amounts proportionate to the movement of its mid-point. Note that as E is twice as far from A as the mid-point, its correction is double that of O, the mid-point.

(*f*) As E is now fixed and C and G are set at zero, it is possible to put in CE and GE, proportionally correcting all intermediate points on these generators.

(*g*) The positions of H and D, and B and F, are known so it is now possible to fit in lines HD and BF. This provides a check on previous evaluation since the mid-point of these lines should coincide with the known position of O, the mid-point of the surface.

Thus the height of all points on the surface are known, relative to an arbitrary plane ACG; but this may not be the best plane and correction must be made for this.

However, consider now an example illustrating the method outlined to relate a series of test lines to each other.

The table below is a set of *cumulative* errors for the lines of test designated in Fig. 12.2 on a surface table.

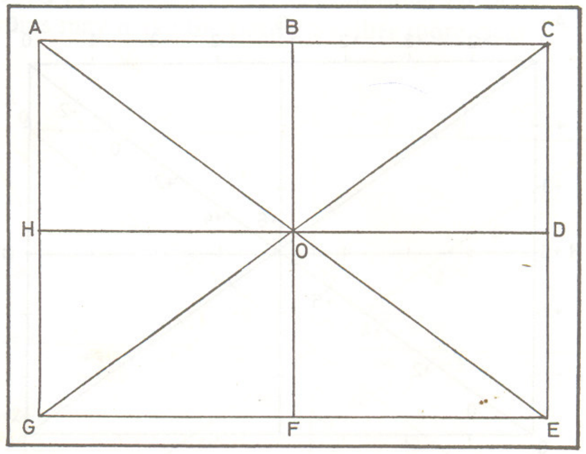


Fig. 12.2. Surface table marked out with the minimum number of lines for a flatness test.

Cumulative Errors of Individual Lines of Test

Lines of Test							
A–C	*A–E*	*A–G*	*G–C*	*G–E*	*C–E*	*B–F*	*H–D*
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
−1	0	+1	+2	+1	−1	+1	+3
−4	−1	+2	+4	−3	+2	+2	+7
−7	−2	−2	+5	−6	+5	−2	+9
−12	−4	−6	+6	−8	+3	−5	+9
−15	−8	−6	+4	−9	+2	−7	+6
−15	−12		+2	−11			+9
−18	−17		0	−12			+10
	−21		−2				
	−24		0				

It is convenient now to consider these lines of test on a plan view of the surface as in Fig. 12.3 in which lines AC, AG and CG have been corrected to zero at each end. Thus the plane ACG is fixed with the points A, C, and G at zero, and points on these three lines are all known relative to this plane.

It is seen that the mid-point is positioned at +6 units above the plane, and the mid-point of line AE must coincide with this position, while point A is known to be O.

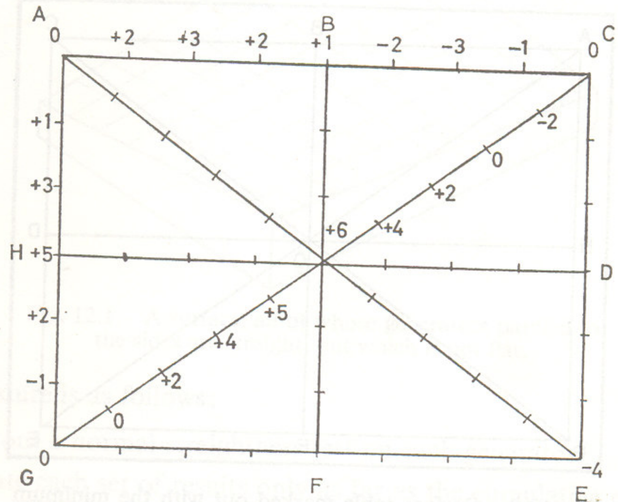


Fig. 12.3. Three corners of a surface adjusted to zero enable the height of the mid-point to be fixed relative to a plane through the corners. This enables the height of the other corner to be determined.

Correction for Line A E

Cumulative Error	*Correction*	*Height Relative to Plane ACG*
0	0	0
0	+2	+2
0	+4	+4
−1	+6	+5
−2	+8	+6
−4	+10	+6
−8	+12	+4
−12	+14	+2
−17	+16	−1
−21	+18	−3
−24	+20	−4

From the table of cumulative errors the value of the mid-point of AE is seen to have a value of -4 units. For this to become $+6$ units it must be raised by $+10$ units and thus point E, which is twice as far from A, must be raised by $+20$ units, giving E a final value of $(-24+20)=-4$ units. All other points on AE are corrected by proportionate amounts, so that a table for AE may be drawn up as shown below.

These values may be inserted on the diagram of the surface as in Fig. 12.3. They are included in Fig. 12.4 along with all other corrected figures, as the two separate diagrams may make the position rather more clear.

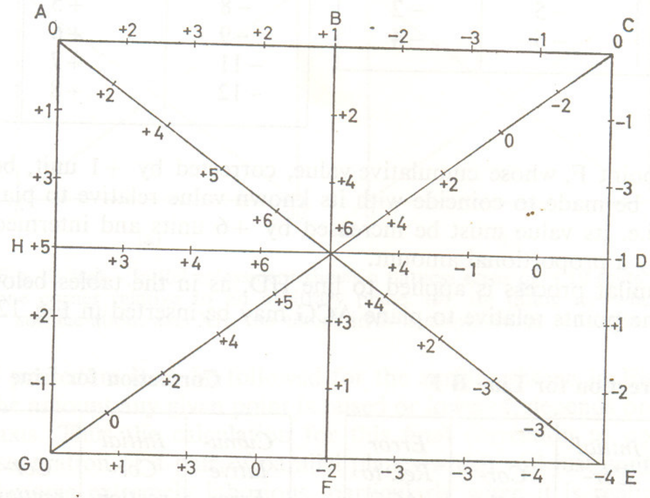


Fig. 12.4. Height of all measured points related to an arbitrary plane ACG.

The height of point E being known as -4 units, relative to plane ACG, enables the relative heights of all points on lines CE and GE to be fixed relative to this plane. Considering line CE it is seen the value of E in the table of cumulative errors is $+2$ units. Hence to make it -4 units it must be corrected by the amount -6 units and all other points corrected by proportional amounts.

Similarly on line GE, point E has a value of -12 units in the table of cumulative errors and it must therefore be corrected by $+8$ units, and by proportional amounts on intermediate points. These tables of corrected values are shown below.

It remains now only to fix all points on lines BF and HD relative to the plane ACG.

Considering line BF it is seen that relative to plane ACG, point B has a value of $+1$ unit, but the value of point B in the table of cumulative errors is O, so that initially all points on BF must be increased by $+1$ unit.

states that the departure from flatness is the *minimum* separation of a pair of parallel planes which will just contain all points on the surface. Consider a surface as shown in Fig. 12.5 (*a*) in which three corners have heights of zero, relative to same arbitrary plane, and the fourth corner has a value of +10 units relative to this plane. It might be thought that the departure from flatness is +10 units, but if the plane is allowed to tilt about the axis XX and the two opposed free corners allowed to become equal as in Fig. 12.5 (*b*) it is seen that the departure from flatness is only +5 units.

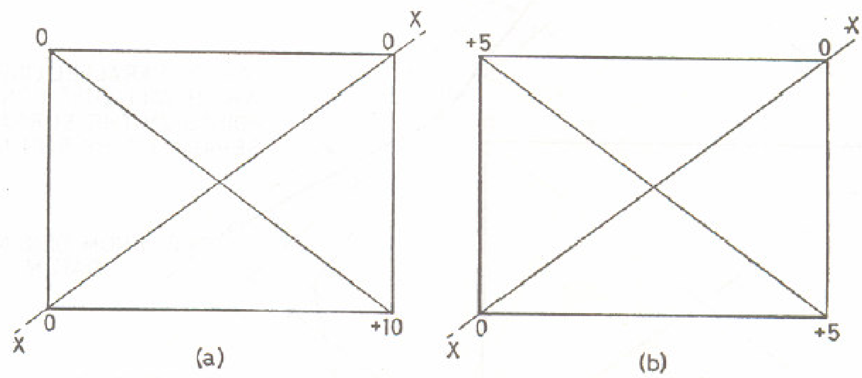


Fig. 12.5(*a*). Initial assessment shows a flatness error of +10 units at one corner relative to an arbitrary plane. (*b*). By tilting the whole surface about axis XX, the actual error is shown to be +5 units.

If this procedure is to be followed for the surface shown in Fig. 12.4, it is seen that the amount any given point is raised or lowered, depends on its distance from the axis. Thus the calculation for this final correction to determine the minimum separation of a pair of parallel planes which will just contain the surface, can become extremely laborious, particularly when it is realized that the process must be carried out at least twice, on axes at right angles to each other.

A possible simplification of this process has been suggested, using a graphical method as outlined below. If we consider again Fig. 12.5 (*a*) and make a projection of the surface along the line of tilt we see the surface as in Fig. 12.6.

It is seen that a pair of parallel lines may be drawn, which just enclose all points on the surface, whose separation is much less than +10 units. In fact, if the scale is considered, it is 5 units as was found by tilting.

To apply this technique to the points on a surface such as that in Fig. 12.4 the procedure is as follows.

(*a*) Arrive at the condition shown in Fig. 12.4 and select two points, preferably on opposite sides, whose values are the maximum positive and maximum negative relative to the arbitrary plane, in this case ACG. Connect these points and project at right angles to the line XX connecting them.

(*b*) Set off to scale the height of all points relative to a line YY, parallel to XX, which represents plane ACG.

Correction for Line C E

Cumulative Error	*Correction Rel. to ACG*	*Error Rel. to ACG*
0	0	0
0	−1	−1
−1	−2	−3
+2	−3	−1
+5	−4	+1
+3	−5	−2
+2	−6	−4

Correction for Line G E

Cumulative Error	*Correction Rel. to ACG*	*Error Rel. to ACG*
0	0	0
0	+1	+1
+1	+2	+3
−3	+3	0
−6	+4	−2
−8	+5	−3
−9	+6	−3
−11	+7	−4
−12	+8	−4

Then point F, whose cumulative value, corrected by +1 unit, becomes −8 units, must be made to coincide with its known value relative to plane ACG of −2 units, i.e. its value must be increased by +6 units and intermediate values corrected by a proportional amount.

If a similar process is applied to line HD, as in the tables below, then the values of the points relative to plane ACG may be inserted in Fig. 12.4.

Correction for Line B F

Cumulative Error	*Initial Correction*	*Correction*	*Error Rel. to ACG*
0	+1	0	+1
0	+1	+1	+2
+1	+2	+2	+4
+2	+3	+3	+6
−2	−1	+4	+3
−5	−4	+5	+1
−9	−8	+6	−2

Correction for Line H D

Cumulative Error	*Initial Correction*	*Correction*	*Error Rel. to ACG*
0	+5	0	+5
0	+5	−2	+3
+3	+8	−4	+4
+7	+12	−6	+6
+9	+14	−8	+6
+9	+14	−10	+4
+6	+11	−12	−1
+9	+14	−14	0
+10	+15	−16	−1

It should be noted that the mid-points of both of these lines of test coincide correctly with the value of +6 units for the mid-point of the surface. This provides a useful check on the calculations up to this point.

It may be thought that this is the end of the matter, but this is not so, because the plane ACG was chosen entirely arbitrarily, and the definition of flatness error

(c) By inspection select the closest pair of parallel lines which will contain all of the points. It should be noted that one line will have two points on it, and the other line one point.

(d) Draw a centre line ZZ between these two and refer all points to this line.

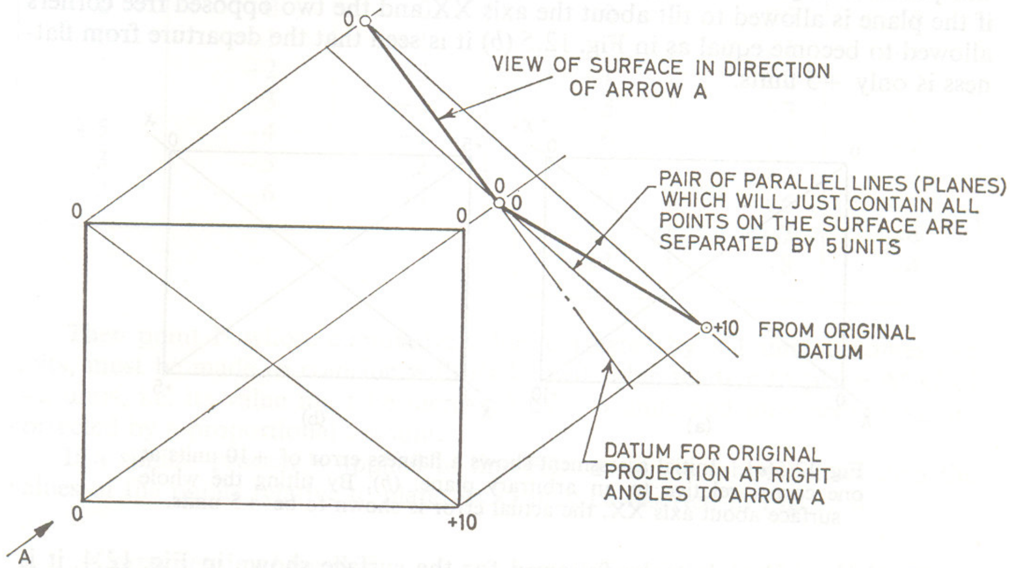


Fig. 12.6. The true flatness error of +5 units, obtained by tilting in Fig. 12.5 (*b*), can also be obtained by projection.

It is important to realize that the two parallel lines represent planes at right angles to the plane of the paper. It may be possible to bring them still closer by inclining them, as a pair, to one side or the other. This can be done by repeating the above process, i.e. draw another plan view of the surface inserting the results from (*d*) above, and project again at right angles to the line of the original projection.

This procedure has been carried out for the surface referred to previously, the results being shown in Fig. 12.7.

It must be emphasized that this is not an exact method. It contains an error due to the differences in scales for lengths and heights of the surface. Also more than two projections may be required but in practice it has been found that the percentage reduction in the separation of the parallel planes containing the surface, by continued projection, is not significant unless the line of the original projection is particularly badly chosen.

Another method of carrying out this process is to refer all points to x, y and z axes, thus fixing them in space. It is then possible to determine the minimum separation of the parallel planes containing the surface by finding the best plane

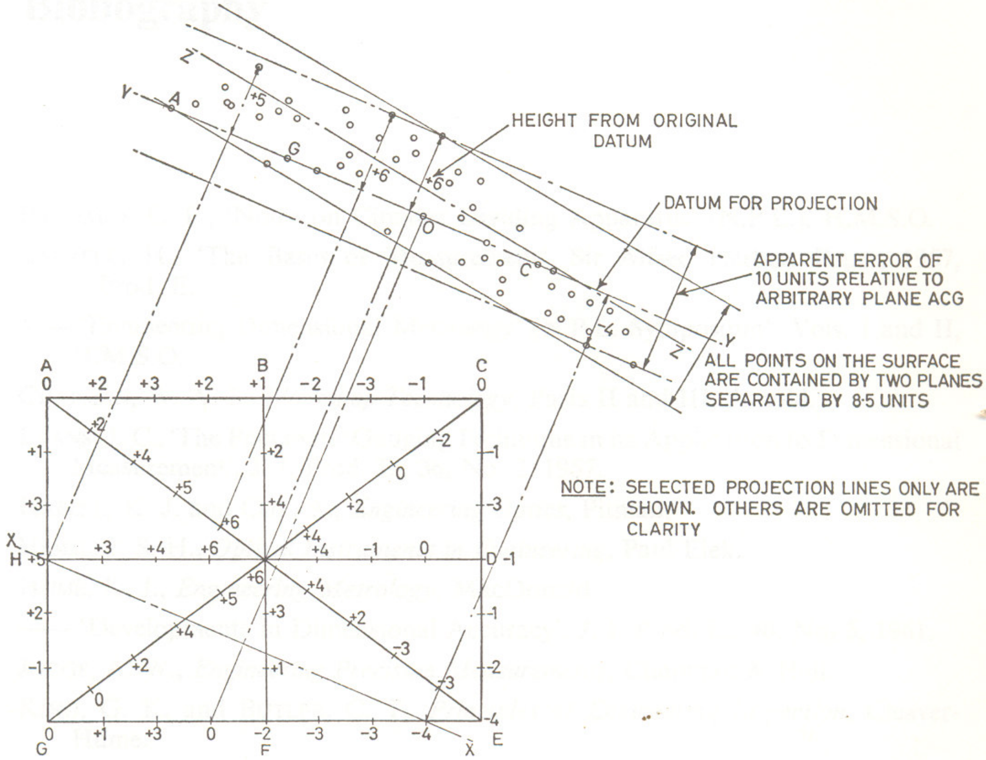


Fig. 12.7. Determination of flatness error by graphical methods.

The first projection only is shown. It may be necessary to refer all points to line ZZ and project again at 90° to the original projection, but in this case it is unlikely as some of the new maxima are widely spaced.

so that the sum of the squares of all points from it is a minimum. This is an extension of the method of least squares (1·51) operating in three dimensions. With a large number of points to be considered, a computer is necessary for this calculation.

Finally it must be realized that whatever method is used it is a laborious process. Many more points would be taken than have been used in the example, a complete grid of the surface being tested, all lines related, and cross-checked in the calculations. The authors feel that without the aid of a computer the effort involved by the graphical method increases least as the number of points surveyed increases.

10.1. Introduction

Gauges are used to check the products. The accuracy of gauges plays very important role in inspection of the products. If a gauge is not accurate, the reading on the job indicated by it will also not be accurate. Hence more accurate methods are required for testing the gauges.

Micro-comparator is one of the instruments used for checking various gauges in the industry. It ensures that the wear effects do not allow the gauge dimensions to fall outside the prescribed limits.

The absolute accuracy of a comparator is guaranteed by use of standard slip gauges, and slip gauges employed for setting working gauges are themselves checked by micro-comparator for wear effects. For very accurate and for absolute results light wave interference method is used.

The indirect method of testing gauges consists in using two comparators where relative difference from standard size is obtained. Comparators are available upto the accuracy of 75 millionth of a mm.

In addition to gauges, all linear and angular measuring instruments must be calibrated from time to time for best results. So we will first consider the calibration of linear and angular measuring instruments.

10.2. Calibration of Linear and Angular Measuring Instruments

Every measuring instrument must be provable, *i.e.* it must be caused to prove its ability reliably. The procedure for this is calibration. The variation in any observation on a product depends upon the variation in the product due to process of manufacture and variation due to measuring process, *i.e.* it can be expressed as : $\sigma_{observation} = \sigma_{process} + \sigma_{measurement}$.

In order to keep $\sigma_{observation}$ minimum so that product as a whole is reliable, $\sigma_{measurement}$ should be kept minimum which in other words means that measurement process or the instruments used should be precise with minimum of variation in the measured values. In order to maintain the precision accuracy of measuring device its periodical calibration is essential as from the moment an instrument is put into use it begins to deteriorate in accuracy and its precision. To a degree, this takes place even if the instrument is not being used. Regarding calibration it is said that in a plant where accuracy is not properly organised, 30% to 50% of the measuring equipment used do not give true results.

In order to maintain accuracy of measuring instruments, following procedure should be followed :

(*i*) Each instrument should be numbered. It serves the easy location of the instrument.

(*ii*) A card record should be established for each instrument.

Table 10.1
Calibration Card

Defects Found (if any)	*Errors/ Defects After repairs (if any)*	*Details of repairs carried out*	*Remarks*	*Calibrated by*	*Place of use*	*Next calibration due on*	*Initial Lab. I/C*

Instrument .... Type and Class .... Inventroy No. ....

Table 10.2
Annual Calibration Programme For General Measurement Instruments

Empty—Indicates that calibration is due.

O—Indicates that calibration has been completed for all the instruments due for calibration in the month.

Months	*SHOPS*								
	I							*II*	
	Calipers, Micrometers, Protractors, dial gauges etc.	*Sine bars*	*Limit gauges*	*Slip gauges*	*Levels, surface plates etc.*	*Others*			
January	O		O						
February	O	O	O	O	O				
March	O		O						
April	O		O			O			
May	O		O	O					
June	O	O	O						
July	O		O	O					
August	O		O		O				
September	O		O	O					
October	O	O	O						
November	O		O	O					
December	O		O			O			

(*iii*) Checking interval should be established.

(*iv*) Some system should be adopted for providing adherence to the checking schedule.

(*v*) The record of the findings of the check should be maintained.

(*vi*) Record of checks should be further studied and analysed so as to improve upon the system.

It shall be preferable to have individual history card for each instrument. History card can be of type given in Table 10.1. These types of history cards can be prescribed using Kardex cabinets.

As regards checking interval, this mainly depends upon the frequency of use of the instruments, and the precision requirement of the measurements. For example in machine shop and tool room the measurements made are of more precise nature as compared to those made in casting and fabrication shops, so instruments used in these shops require more frequent calibration. It is always better to prepare annual calibration programme for instruments used in various shops, so as per the programme the shops send the instruments for calibration. The annual programme can be of the type as suggested in Table 10.2 which is for general measuring instruments such as vernier calipers, micrometers, protectors, limit gauges, slip gauges, dial gauges etc. For optical measuring instruments like universal microscopes, tool maker's microscopes etc. annual programme can be made for periodical calibration as well as for general preventive maintenance which besides general cleaning and lubrication will also consist of carrying visual checks like relative movements of moving parts, presence of corrosion, scratch marks, visibility and correct working of optical system etc. Such visual checks should be carried out more frequently. In Table 10.3 is shown such chart wherein the annual schedule for calibration and preventive maintenance of optical instruments is drawn. Table 10.4 also shows the history card to be made for limit gauges. For every gauge or gauge

Table 10.3

Annual Schedule for Preventive Maintenance and Calibration of Optical Metrological Instruments

Δ Indicates about preventive maintenance being due.

□ Indicates about calibration being due.

Instruments	*Months*											
	Jan.	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
Universal Microscope Sl. No. ...	Δ □	Δ	Δ	Δ □	Δ	Δ	Δ □	Δ	Δ	Δ □	Δ	Δ
Tool Makers Microscope Sl. No. ...	Δ □	Δ	Δ	Δ	Δ □	Δ	Δ	Δ	Δ □	Δ	Δ	Δ
Interferometer Sl. No. ...	Δ	□	Δ		Δ		Δ	□	Δ		Δ	
Shadow graph Sl. No. ...	Δ	Δ □	Δ	Δ	Δ	Δ □	Δ	Δ	Δ	Δ □	Δ	Δ

Table 10.4

GAUGE CARD SPECIFICATIONS

Gauge	Major Dia and Tolerance	Effective Dia and Tolerance			Tolerance		Minor Dia and Tolerance	
		For New Gauge	Transfer to Insp.	Wear Limit	Pitch	Half Profile Angle		Wire size ________ Factor X ________
Go								
Not Go								

RESULTS OF PERIODIC CHECKING

Date	DIMENSIONS OBTAINED												Remarks About Stability		Place of Use	Next Checking Due	Checked By
	Go						No Go										
	Major Dia	Effective Dia	Half Profile Angle		Pitch	Minor Dia	Major Dia	Effective Dia	Half Profile Angle		Pitch	Minor Dia	Go	Not Go			
			Left	Right					Left	Right							

Type .... Size .... Card .... Instt. No ....

set an individual history card should be maintained. For gauges of any particular size, sometimes it becomes difficult for shop or inspection personnel to know whether the gauges are available or not.

For this purpose the charts could be maintained which indicate the maximum/minimum inventory stock quantity level for these gauges so that at any time when the quantity available in the stores goes down the minimum value, action should be taken for the procurement.

Here are described the methods of calibration of some of the important metrological instruments.

10.2.1. General Metrological Instruments. (*a*) *Vernier Calipers and other Vernier Instruments.*

The following table gives the allowable deviation in the parameters.

Parameters	*Permissible Error*	
	Least count 0.02 mm	*Least count 0.05 mm*
Zero error	0.02	0.05
Flatness of measuring jaws	0.003	0.004
Parallelness of measuring jaws	0.010	0.015
Error in readings	0.02	0.05
Spherical portion size of inside measuring jaws	± 0.02	± 0.03

The zero error is checked by bringing in contact the jaws, and the shift of zero of main scale is observed with respect to zero of vernier scale. The flatness of the measuring jaws is checked using a straight edge having sharp edge of class 1 accuracy. The straight edge is put over the surface and the light gap observed between the straight edge and the surface and compared with standard light gap formed between another straight edge and an optical flat. The parallelness is checked by inserting a slip gauge of any value between the jaws at various positions and determining the out of parallelness using slip gauges.

Error in readings along the entire range is also found out using slip gauges. In case of vernier calipers having spherical inside measuring jaws, the width of spherical portion is checked using a passmeter or dial type micrometric comparator.

(*b*) *Dial Gauges.* The dial indicators of least count 0.01 mm can be conveniently calibrated using passmeter or micrometer dial comparator of least count (L.C.) 0.002 mm. Here dial indicator is inserted in place of fixed right hand side flat jaw and the dial tip rests on the movable jaw. The dial indicators can also be calibrated using slip gauges and by fixing the dial gauge in a comparator stand.

The following table gives the permissible errors in the dial indicators :

Parameter	*Permissible Error*		
	L.C. 0.01 mm	*L.C. 0.001 mm*	*L.C. 0.002 mm*
Maximum error along entire range and in any one turn	0.02 mm along entire range	0.003 mm	0.004 mm
	0.006 if any one turn.		
Variation in readings along entire range.	0.02 mm.	0.003 mm	0.004 mm

(c) *Micrometers*. In case of micrometers the following are the main points to be checked :

(i) General appearance and relative movement of moving parts.

(ii) Checking initial zero setting for micrometers of size 25—50 mm or more.

(iii) Flatness of measuring surfaces.

(iv) Parallelness of measuring surfaces.

(v) Error in readings.

In general appearance, the micrometer is thoroughly checked for presence of scratches, dents etc. on measuring jaws, as well as, for corrosion marks, scratches, dents etc. on the surfaces of measuring drums, for proper working of ratchet system. The relative movement of moving parts is also checked which should be smooth. The working of lock system is also checked. The zero error of micrometer is checked and if it is found wrong it is adjusted easily for micrometers of size 25—50 mm and more. The size of the setting piece is checked on interferometer or any other comparator set to read upto 0.0001 mm. The permissible error allowed in its size 0.001 mm for micrometers upto size of 100 mm. The flatness error is checked by keeping optical flat on each jaw. The maximum permissible error is 0.0009 mm. The parallelness error is also checked using four optical flats of different width so that one complete turn of micrometer drum is made. The optical flat is set in such a way that total fringes on both sides are minimum. The permissible error in parallelity is 0.002 mm for micrometers upto 100 mm size and 0.004 mm for micrometers above 100 mm and upto 200 mm size.

The error in readings is checked using slip gauges so as to cover the entire range. The maximum permissible error is 0.004 mm for class I micrometers and 0.008 mm for class II micrometers.

Study Material for Expt No. 3 : Geometric characteristics
Part A

CIRCULARITY ○

Definition. Circularity is the condition on a surface of revolution where:

1. in the case of a cylinder or cone, all points of the surface intersected by any plane perpendicular to a common axis are equidistant from their axis;
2. in the case of a sphere, all points of the surface intersected by any plane passing through a common center are equidistant from that center.

CIRCULARITY TOLERANCE

A circularity tolerance specifies a tolerance zone bounded by two concentric circles within which each circular element of the surface must lie and applies independently at any plane as described above.

CIRCULARITY TOLERANCE APPLICATION

Limits of size exercise control of circularity within the size tolerance. Often this provides adequate control. However, where necessary to further refine form control, circularity tolerancing can be used on any figure of revolution or circular cross section.

The example illustrates a part with a circularity tolerance of .002 specified on a cylindrical part.

The interpretation shows how one establishes the .002 tolerance zone. Note that the tolerance zone is the width of the annular zone between the two concentric circles.

A circularity tolerance zone is established relative to the actual size of the part when measured at the surface periphery at any cross section perpendicular to the part axis. It should be noted that the circularity tolerance applies only at the cross-sectional point of measurement, and is relative to the *size* at that point. Therefore, a cylindrical part with circularity tolerance control could taper or otherwise vary in its surface contour within its size tolerance range, yet still meet circularity requirements if it is within the circularity tolerance at that point.

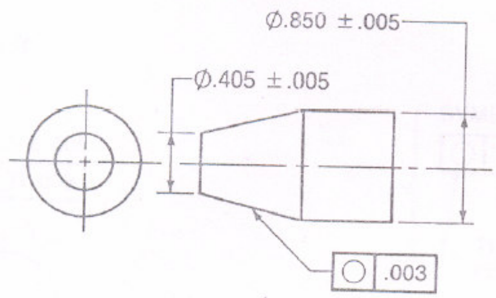
The part size in this example has been assumed to measure .503 at its largest point at the cross section selected for measurement. The .002 circularity tolerance zone is then established by two theoretically perfect concentric circles, one at the .503 diameter and the other .004 *smaller* at the .499 diameter. This establishes the tolerance zone of .002 *width* between the concentric circles. To be acceptable, the part surface at that cross section must fall within the .002 wide tolerance zone.

As is seen, the tolerance zone is established relative to the part size wherever it may fall in its size tolerance range. That is, the part *size* is first determined and its circularity is then defined as a refinement of the part *form* relative to that *size*. Unless otherwise specified, any established size at any point along the surface can be used to determine the circularity tolerance zone. It is therefore seen that the circularity tolerance may be based on *different* sizes on the same part. The circularity tolerance zone, however, remains constant.

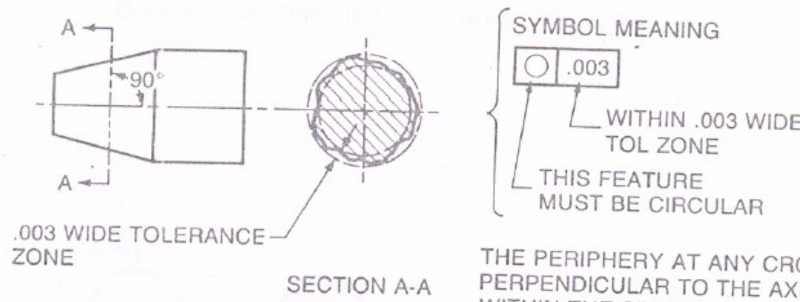
CIRCULARITY OF A CONE

Example 1 below illustrates a cone-shaped part for which a circularity tolerance of .003 is specified. As previously discussed, the periphery at any cross section perpendicular to the axis must be within the specified tolerance of size and must lie between the two concentric circles (one having a radius .003 larger than the other).

EXAMPLE 1



MEANING

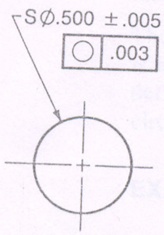


THE PERIPHERY AT ANY CROSS SECTION PERPENDICULAR TO THE AXIS MUST BE WITHIN THE SPECIFIED TOLERANCE OF SIZE AND MUST LIE BETWEEN TWO CONCENTRIC CIRCLES (ONE HAVING A RADIUS .003 LARGER THAN THE OTHER).

CIRCULARITY OF A SPHERE

Circularity of a spherical part is given the same basic interpretation (see Example 2 below) except that the tolerance control reference is to *any cross section passing through a common center* rather than to *any cross section perpendicular to the axis,* as in the conventional application of circularity tolerancing.

EXAMPLE 2



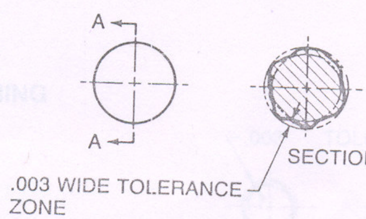
SYMBOL MEANING

○	.003

WITHIN .003 WIDE TOL ZONE
THIS FEATURE MUST BE CIRCULAR

THE PERIPHERY AT ANY CROSS SECTION PASSING THROUGH A COMMON CENTER MUST BE WITHIN THE SPECIFIED TOLERANCE OF SIZE AND MUST BE BETWEEN TWO CONCENTRIC CIRCLES (ONE HAVING A RADIUS .003 LARGER THAN THE OTHER). HENCE, THE SURFACE MUST LIE BETWEEN TWO CONCENTRIC SPHERES SEPARATED .003 APART.

MEANING



PERPENDICULARITY ⊥
(SQUARENESS, NORMALITY)

Definition. Perpendicularity is the condition of a surface, median plane, or axis which is at exactly 90° to a datum plane or axis.

PERPENDICULARITY TOLERANCE

A perpendicularity tolerance specifies:

1. a tolerance zone defined by two parallel planes perpendicular to a datum plane within which
   a. the surface of a feature must lie (see Fig. 1);
   b. the median plane of a feature must lie (see Fig. 2);
2. a tolerance zone defined by two parallel planes perpendicular to a datum axis within which the axis of a feature must lie (see Fig. 3);
3. a cylindrical tolerance zone perpendicular to a datum plane within which the axis of a feature must lie (see Fig. 4);
4. a tolerance zone defined by two parallel, straight lines perpendicular to a datum plane or datum axis within which an element of the surface must lie (see Fig. 5—radial perpendicularity).

PARALLELISM //

Definition. Parallelism is the condition of a surface or axis which is equidistant at all points from a datum plane or axis.

PARALLELISM TOLERANCE

A parallelism tolerance specifies:

1. a tolerance zone defined by two planes or lines parallel to a datum plane (or axis) within which the considered feature (axis or surface) must lie (see Figs. 1 and 2);
2. a cylindrical tolerance zone parallel to a datum axis within which the axis of the feature under consideration must lie (see Fig. 3).

FIGURE 1

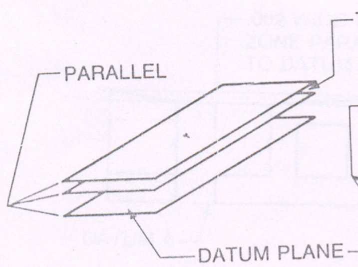


FIGURE 2

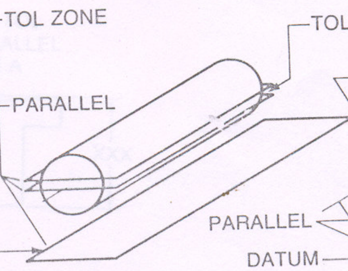
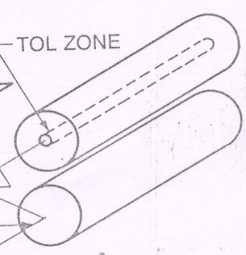


FIGURE 3



PARALLELISM APPLICATION

Note in the following example that the bottom surface has been selected as the datum and the top surface is to be parallel to datum plane A within .002.

The Meaning beneath the example clarifies the symbol: It reads, "This feature must be parallel within .002 to datum plane A."

The lower example illustrates the tolerance zone and the manner in which the surface must fall within the tolerance zone to be acceptable. Note that the tolerance zone is established parallel to the datum plane A. Note also that the parallelism tolerance, when applied to a plane surface, controls flatness if a flatness tolerance is not specified (that is, the implied flatness will be *at least* as good as the parallelism).

Part B

6.23 Tests for Straightness and Flatness

It will be appreciated that for a carriage to move along a straight line in both vertical and horizontal planes, the controlling guide-ways must themselves be straight. Tests for this condition may be carried out in several ways, the most convenient of which are by precision level and by the auto-collimator. It is the latter method which will be discussed here, but the method of tabulating and using the results of individual measurements is similar in each method.

The principle of measurement by the auto-collimator has been dealt with in Chapter 4, but the method of determination of straightness and flatness is dealt with now.

Assume that the straightness of a lathe bed 2 m in length is to be measured. The general arrangement of measurement would be as in Fig. 6.4, the auto-collimator being set up independently of the lathe bed, about $\frac{1}{2}$ m from one end, the parallel beam from the instrument being projected along the length of the bed. A particularly rigid support, preferably of the tripod type, is required for this. Assuming the bed to have flat-ways, the plane reflector is set on to the end of the bed nearer the instrument and a reflection obtained from it such that the image of the cross-wires of the collimator appear nearer the centre of the field. The reflector is then moved to the other end of the bed, and provided the general line of movement of the reflector has been reasonably parallel to the optical axis of the instrument, then the image of the cross-wires will appear in the field of the eyepiece at this position of the reflector also. This procedure ensures that reflections at intermediate positions will be within the field, and is thus an approximate check on the level of the bed in the horizontal plane.

A straight-edge should now be set down on the bed, to ensure that the reflector is stepped along it in a straight line.

Assume that the distance between the support feet of the reflector is 103·5 mm, and that the interval length at which measurements are taken is also 103·5 mm.

$$\text{Now, since 1 min of arc} = \frac{2\pi}{360 \times 60} \text{ radians}$$

$$\text{then, on a base length of 103·5 mm, 1 min of arc} = \frac{2\pi}{360 \times 60} \times 103{\cdot}5 \text{ mm}$$

$$= 0{\cdot}03 \text{ mm}$$

That is, each tilt of 1 min of arc of the reflector as it is stepped along the bed-way corresponds very closely to a rise or fall of the guide-way surface of 0·03 mm.

Having ensured that an image of the cross-wires will be received by the auto-collimator when the reflector is set at the end positions of the bed, the reflector is now set at the forward end of the bed, nearest the instrument, to begin the series of readings. This condition, and those for subsequent readings, is shown in Fig. 6.4 in which the rise and fall of the bed surface is greatly exaggerated.

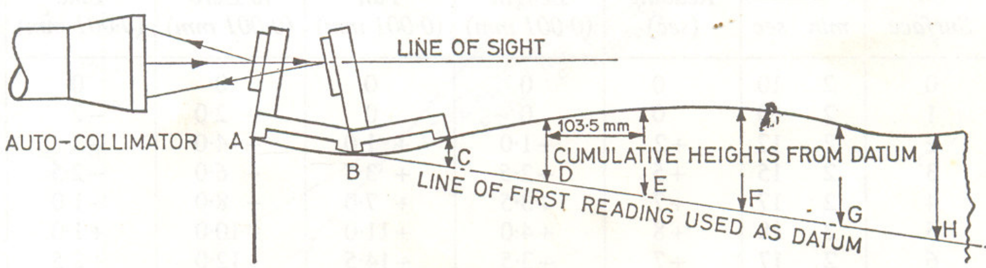


Fig. 6.4. Auto-collimator used for checking the straightness of lathe-bed guide-ways.

With the reflector set at A–B, the setting wires in the auto-collimator eyepiece are moved to straddle symmetrically the image of the horizontal cross-wire, by the suitable rotation of the micrometer drum, and the micrometer reading is noted. The reflector is then moved 103·5 mm to the position B–C and a second reading is taken on the micrometer drum. Successive readings at C–D, D–E, E–F, etc., are taken until the length of the bed has been stepped along. A second set of readings should now be obtained by stepping the reflector in the reverse direction along the bed, to reveal any serious errors in the first set of readings. Assuming none have occurred, the mean values of each set of readings may now be recorded, and these represent the angular positions of the reflector, in seconds relative to the optical axis of the auto-collimator at each of its positions along the bed.

The method of tabulation of the results of measurement are shown on p. 121.

Column 1 gives the position of the plane reflector at 103·5 mm intervals along the bed. Column 2 gives the mean reading of the auto-collimator to the nearest second. In practice it is possible to observe sub-divisions of seconds, and this should be done. Column 3 gives the differences of each reading from the first. In column 4 these differences are converted to the corresponding linear rise or fall, on the basis of 1 sec of arc = 0·0005 mm per 103·5 mm. The second zero introduced at the head of column 4, when associated with the previous zero in this column, represents the heights of the two feet of the reflector support mounting when in its original position. Column 5 gives the heights of the support feet of the reflector above the datum line drawn through their first position. That is, the values in column 5 are obtained by successively adding, algebraically, the values in column 4. This is necessary because the individual heights obtained in column 4 are the heights of the back feet of the support relative to the front feet in a given position and not relative to the datum.

1	2		3	4	5	6	7
Position on Surface	*Reading*		*Difference from 1st Reading (sec)*	*Rise or Fall in Interval Length (0·001 mm)*	*Cumulative Rise or Fall (0·001 mm)*	*Adjustment to bring both ends to Zero (0·001 mm)*	*Errors from Straight Line (0·001 mm)*
	min	*sec*					
0	2	10	0	0	0	0	0
1	2	10	0	0	0	− 2·0	−2
2	2	12	+2	+1·0	+ 1·0	− 4·0	−3
3	2	15	+5	+2·5	+ 3·5	− 6·0	−2·5
4	2	17	+7	+3·5	+ 7·0	− 8·0	−1·0
5	2	18	+8	+4·0	+11·0	−10·0	+1·0
6	2	17	+7	+3·5	+14·5	−12·0	+2·5
7	2	15	+5	+2·5	+17·0	−14·0	+3·0
8	2	13	+3	+1·5	+18·5	−16·0	+2·5
9	2	9	−1	−0·5	+18·0	−18·0	0
10	2	12	+2	+1·0	+19·0	−20·0	−1·0
11	2	14	+4	+2·0	+21·0	−22·0	−1·0
12	2	16	+6	+3·0	+24·0	−24·0	0

The total rise in the surface of the bed over a $1\frac{1}{4}$ m length from a datum along the line of the first reading is 24 μm. In column 6 this total rise is proportioned over the twelve readings taken, i.e. in increments of $24/12 = 2$ μm. These values (column 6) are subtracted from the values in column 5 to give the errors (column 7) in the bed from a straight line joining the end points and within which the series of readings were obtained (i.e. it is as though a straight-edge were laid along the bed profile and touching the end points of the test surface when they are in a horizontal plane). The rise and fall of the surface relative to the straight-edge would be the values given in column 7.

A graphical representation of this is shown in Fig. 6.5 in which the values given in columns 5, 6, and 7 are plotted. In the graph of cumulative errors a straight line has been passed through the end points, and represents the straight line connecting the ends of the bed. In the graph of straightness errors, this line has been used as the axis, and thus the values plotted in the previous graph have the same relationship to it.

It is important to note that the increasing values for the readings given in column 2 of the table indicate the increasing angle of tilt of the top of the reflector towards the optical axis of the auto-collimator. Increasing readings have therefore indicated positive (+) values for the linear rise and fall, and vice versa. The lathe bed is thus both concave and convex along its length relative to the datum line joining its end points.

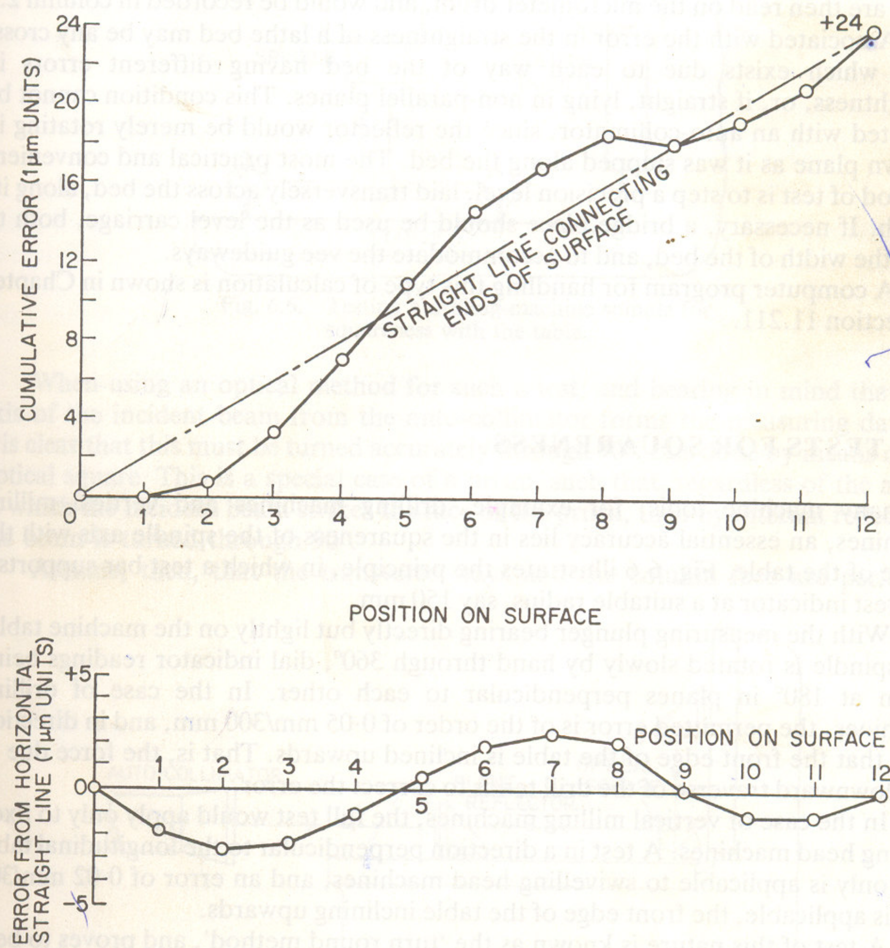


Fig. 6.5. Graphs of cumulative error and actual error in a machine bed, determined using an auto-collimator.

The test described relates to a flat-bed lathe, but the method applies also to a bed with vee guide-ways. In this case, however, the plane reflector mount must be supported on a carriage having a vee accurately ground in its base, to suit the vee of the bed. The apex of the carriage vee should be relieved so that contact is made only at the sides of the vee. With this arrangement, the straightness of the bed in the horizontal plane can be determined, as well as in the vertical (i.e. the tilt about the vertical axis indicates changes in the angle of the reflector in the horizontal plane).

As in the previous test, the reflector is stepped along the bed in interval lengths of 103·5 mm but in this case the auto-collimator tube is rotated 90° in its housing, to a pre-set stop, so that the pair of setting wires in the eyepiece are vertical. Changes of position of the image of the vertical member of the cross-wires are then read on the micrometer drum, and would be recorded in column 2.

Associated with the error in the straightness of a lathe bed may be any cross-wind which exists due to each way of the bed having different errors in straightness, or, if straight, lying in non-parallel planes. This condition cannot be detected with an auto-collimator, since the reflector would be merely rotating in its own plane as it was stepped along the bed. The most practical and convenient method of test is to step a precision level, laid transversely across the bed, along its length. If necessary, a bridge piece should be used as the level carriage, both to span the width of the bed, and to accommodate the vee guideways.

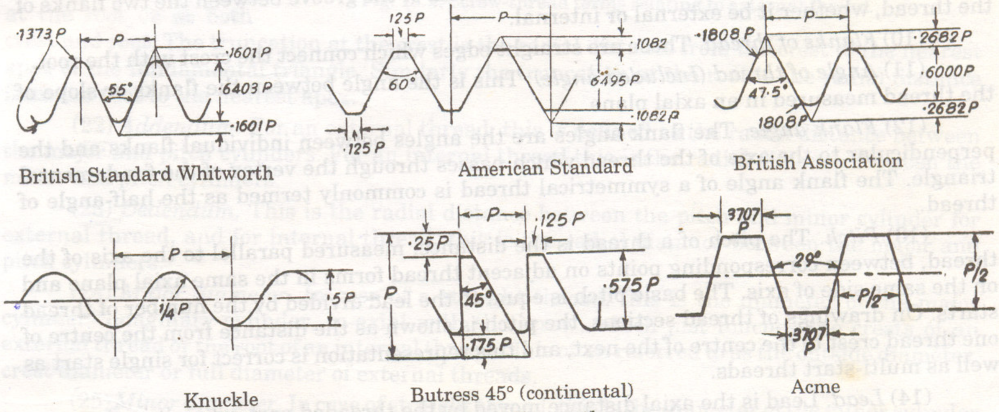
A computer program for handling this type of calculation is shown in Chapter 11, section 11.211.

Study Material for Expt No. 4: Screw Thread Measurement

Metrology of Screw Thread

13.1. Introduction

Screw thread has generally two functions to perform, *viz.*, transmission of power and motion, and to act as fastener. Second function is rather more important, so we will be more concerned with Vee-form of threads. The object of dimensional control in case of plain shaft and hole is to ensure a certain consistency of fit. In the case of threaded work, the object is to ensure mechanical strength which is dependent upon the amount of flank contact and not upon the fit.



The most commonly used thread-forms all over the world are shown in Fig. 13.1. These mainly consist of the British Whitworth and B.A. series, and in the Metric series the American Standard series. Unified thread which is an attempt to achieve interchangeability between threads used in England and America has been added recently. The ISO metric threads recommended for use in Indian Industry by BIS have been dealt with separately in details in Chapter 14.

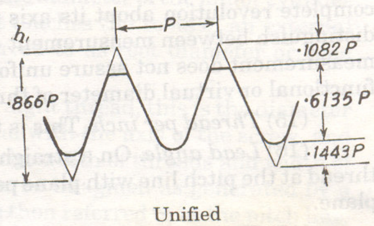


Fig. 13.1. [Not to scale]
Commonly used thread forms.

13.2. Screw Threads Terminology

(1) *Screw thread.* A screw thread is the helical ridge produced by forming a continuous helical groove of uniform section on the external or internal surface of a cylinder or cone. A screw thread formed on a cylinder is known as straight or parallel screw thread, while the one formed on a cone or frustrum of a cone is known as tapered screw thread.

(2) *External thread.* A thread formed on the outside of a workpiece is called external thread *e.g.*, on bolts or studs etc.

(3) *Internal thread.* A thread formed on the inside of a workpiece is called internal thread *e.g.* on a nut or female screw gauge.

(4) *Multiple-start screw thread.* This is produced by forming two or more helical grooves, equally spaced and similarly formed in an axial section on a cylinder. This gives a 'quick traverse' without sacrificing core strength.

(5) *Axis of a thread.* This is imaginary line running longitudinally through the centre of the screw.

(6) *Hand (Right or left hand threads).* Suppose a screw is held such that the observer is looking along the axis. If a point moves along the thread in clockwise direction and thus moves away from the observer, the thread is right hand ; and if it moves towards the observer, the thread is left hand.

(7) *Form of thread.* This is the shape of the contour of one complete thread as seen in axial section.

(8) *Crest of thread.* This is defined as the prominent part of thread, whether it be external or internal.

(9) *Root of thread.* This is defined as the bottom of the groove between the two flanks of the thread, whether it be external or internal.

(10) *Flanks of thread.* These are straight edges which connect the crest with the root.

(11) *Angle of thread (Included angle).* This is the angle between the flanks or slope of the thread measured in an axial plane.

(12) *Flank angle.* The flank angles are the angles between individual flanks and the perpendicular to the axis of the thread which passes through the vertex of the fundamental triangle. The flank angle of a symmetrical thread is commonly termed as the half-angle of thread.

(13) *Pitch.* The pitch of a thread is the distance, measured parallel to the axis of the thread, between corresponding points on adjacent thread forms in the same axial plane and on the same side of axis. The basic pitch is equal to the lead divided by the number of thread starts. On drawings of thread sections, the pitch is shown as the distance from the centre of one thread crest to the centre of the next, and this representation is correct for single start as well as multi-start threads.

(14) *Lead.* Lead is the axial distance moved by the threaded part, when it is given one complete revolution about its axis with respect to a fixed mating thread. It is necessary to distinguish between measurement of lead from measurement of pitch, as uniformity of pitch measurement does not assure uniformity of lead. Variations in either lead or pitch cause the functional or virtual diameter of thread to differ from the pitch diameter.

(15) *Thread per inch.* This is the reciprocal of the pitch in inches.

(16) *Lead angle.* On a straight thread, lead angle is the angle made by the helix of the thread at the pitch line with plane perpendicular to the axis. The angle is measured in an axial plane.

(17) *Helix angle.* On straight thread, the helix angle is the angle made by the helix of the thread at the pitch line with the axis. The angle is measured in an axial plane.

(18) *Depth of thread.* This is the distance from the crest or tip of the thread to the root of the thread measured perpendicular to the longitudinal axis or this could be defined as the distance measured radially between the major and minor cylinders.

(19) *Axial thickness.* This is the distance between the opposite faces of the same thread measured on the pitch cylinder in a direction parallel to the axis of thread.

angle of thread. The virtual diameter being the modified effective diameter by pitch and angle errors, is the most important single dimension of a screw thread gauge.

In the case of taper screw thread, the cone angle of taper for measurement of effective diameter, and whether pitch is measured along the axis or along the pitch cone generator also need to be specified.

13.2.1. Errors in Threads. In the case of plain shafts and holes, there is only one dimension which has to be considered (*i.e.* diameter), and errors on this dimension if exceed the permissible tolerance, will justify the rejection of part. While in the case of screw threads there are at least five important elements which require consideration and error in any one of these can cause rejection of the thread. In routine production all of these five elements (major diameter, minor diameter, effective diameter, pitch and angle of the thread form) must be checked and method of gauging must be able to cover all these elements.

Errors on the major and minor diameters will cause interference with the mating thread. Due to errors in these elements, the root section and wall thickness will be less, also the flank contact will be reduced and ultimately the component will be weak in strength. Errors on the effective diameter will also result in weakening of the assembly due to interference between the flanks.

Similarly pitch and angle errors are also not desirable as they cause a progressive tightening and interference on assembly. These two errors have a special significance as they can be precisely related to the effective diameter.

Now we will consider come errors in detail and define some terms.

13.2.2. Drunken Thread. This is the one having erratic pitch, in which the advance of the helix is irregular in one complete revolution of the thread.

Thread drunkenness is a particular case of a periodic pitch error recurring at intervals of one pitch. In such a thread, the pitch measured parallel to the thread axis will always be correct, the only error being that the thread is not cut to a true helix. If the screw thread be regarded as an inclined plane wound around a cylinder and if the thread be unwound from the cylinder, (*i.e.* development of the thread be taken) then the drunkenness can be visualised. The helix will be a curve in the case of drunken thread and not a straight line as shown in Fig. 13.3.

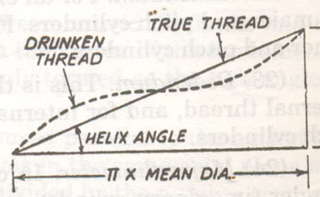


Fig. 13.3. Drunken thread.

It is very difficult to determine such errors and moreover they do not have any great effect on the working unless the thread is of very large size.

13.2.3. Pitch Errors in Screw Threads. Generally the threads are generated by a point cutting tool. In this case, for pitch to be correct, the ratio of the linear velocity of tool and angular velocity of the work must be correct and this ratio must be maintained constant, otherwise pitch errors will occur. If there is some error in pitch, then the total length of thread engaged will be either too great or too small, the total pitch error in overall length of the thread being called the cumulative pitch error. Various pitch errors can be classified as :

13.2.3.1. Progressive Pitch Error. This error occurs when the tool work velocity ratio is incorrect though it may be constant. It can also be caused due to pitch errors in the lead screw of the lathe or other generating machine.

The other possibility is by using an incorrect gear or an approximate gear train between work and lead screw *e.g.,* while metric threads are cut with an inch pitch lead screw and a translatory gear is not available. A graph between the cumulative pitch error and the length of thread is generally a straight line in case of progressive pitch error (Fig. 13.4).

13.2.3.2. Periodic Pitch Error. This repeats itself at regular intervals along the thread. In this case, successive portions of the thread are either longer or shorter than the mean. This type of error occurs when the tool work velocity ratio is not constant. This type of error also results when a thread is cut from a lead screw which lacks squareness in the abutment causing the leadscrew to move backward and forward once in each revolution. Thus the errors due to these cases are cyclic and pitch increases to a maximum, then reduces through normal value to a minimum and so on. The graph between the cumulative pitch error and length of threads for this error will, therefore, be of sinusoidal form.

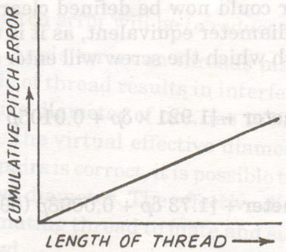


Fig. 13.4. Progressive Error.

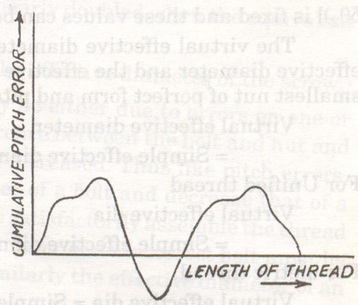


Fig. 13.5. Periodic Error.

13.2.3.3. Irregular Errors. These arise from disturbances in the machining set-up, variations in the cutting properties of material etc. Thus they have no specific causes and correspondingly no specific characteristics also. These errors could be summarised as follows :

Erratic Pitch. This is the irregular error in pitch and varies irregularly in magnitude over different lengths of thread.

Progressive Error. When the pitch of a screw is uniform, but is shorter or longer than its nominal value, it is said to have progressive error.

Periodic Error. If the errors vary in magnitude and recur at regular intervals, when measured from thread to thread along the screw are referred to as periodic errors.

13.4.5.4. Three wire method. This method of measuring the effective diameter is an accurate method. In this three wires or rods of known diameter are used : one on one side and two on the other side [Fig. 13.17 (*a*) and (*b*)]. This method ensures the alignment of micrometer anvil faced parallel to the thread axis. The wires may be either held in hand or hung from a stand so as to ensure freedom to the wires to adjust themselves under micrometer pressure.

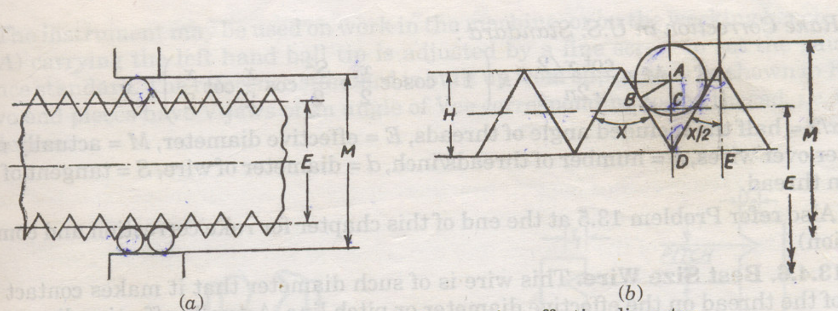


Fig. 13.17. Three wire method of measuring effective diameter.

M = distance over wires, E = effective diameter, r = radius of the wires, d = diameter of wires, h = height of the centre or the wire or rod from the effective diameter, x = angle of thread.

From Fig. 13.17 (b), $AD = AB \text{ cosec } x/2 = r \text{ cosec } x/2$

$$H = DE \cot x/2 = p/2 \cot x/2$$

$$CD = H/2 = p/4 \cot x/2$$

$$h = AD - CD$$

$$h = r \text{ cosec } x/2 - p/4 \cot x/2$$

$$\text{Distance over wires } = M = E + 2h + 2r$$

$$= E + 2\,(r \text{ cosec } x/2 - p/4 \cot x/2) + 2r$$

$$= E + 2r\,(1 + \text{cosec } x/2) - p/2 \cot x/2$$

or $$M = E + d\,(1 + \text{cosec } x/2) - p/2 \cot x/2$$

(i) *In case of Whitworth thread :*

$x = 55°$, depth of thread = $0.64p$, so that, $E = D - 0.64p$ and $\text{cosec } x/2 = 2.1657$, $\cot x/2 = 1.921$

$$M = E + d\,(1 + \text{cosec } x/2) - p/2 \cot x/2 = D - 0.64p + d\,(1 + 2.1657) - p/2\,(1.921)$$

$$= D + 3.1657d - 1.6005p$$

$$M = D + 3.1657d - 1.6p, \text{ where } D = \text{outside dia.}$$

(ii) *In case of metric threads* : Depth of thread = $0.6495p$.

so, $E = D - 0.6495p$, $x = 60°$, $\text{cosec } x/2 = 2$; $\cot x/2 = 1.732$

$$M = D - 0.6495\,p + d\,(1 + 2) - p/2\,(1.732) = D + 3d - (0.6495 + 0.866)\,p = D + 3d - 1.5155p.$$

We can measure the value of M practically and then compare with the theoretical values with the help of formulae derived above. After finding correct value of M and knowing d, E can be found out.

If the theoretical and practical values of M (*i.e.* measured over wires) differ, then this error is due to one or more of the quantities appearing in the formula.

Effect of lead angle on measurement by 3-wire method. If the lead angle is large (as with worms ; quick traversing lead screw, etc.) then error in measurement is about 0.0125 mm when lead angle is 4.5° for 60° single thread series.

For lead angles above 4.5° compensation for rake and compression must also be considered.

There is no recommendation for B.S.W. threads.

Rake Correction in U.S. Standard :

$$E = M + \frac{\cot x/2}{2n} - x\left(1 + \text{cosec}\,\frac{x}{2} + \frac{S^2}{2}\cos\frac{x}{2}\cot\frac{x}{2}\right)$$

where $x/2$ = half the included angle of threads, E = effective diameter, M = actually measured diameter over wires, n = number of threads/inch, d = diameter of wire, S = tangent of the helix angle in thread.

(Also refer Problem 13.5 at the end of this chapter for rake correction and compression correction).

13.4.6. Best Size Wire. This wire is of such diameter that it makes contact with the flanks of the thread on the effective diameter or pitch line. Actually effective diameter can be measured with any diameter wire which makes contact on the true flank of the thread, but the values so obtained will differ from those obtained with 'best size' wires if there is any error in angle or form of thread. It is recommended that for measuring the effective diameter, always the best size wire should be used and for this condition the wire touches the flank at mean diameter line within ± 1/5 of flank length (Refer Solved Problem 13.2). With best size wire, any error on the measured value of simple effective diameter due to error in thread form or angle is minimised.

It can be shown that size of best wire diameter = $d = \dfrac{p}{2\cos x/2}$

[Refer Solved Problem 13.1 at the end of this chapter]

With best size wire, P-value = d (cosec $x/2$ + 1) – d cos $x/2$ cot $x/2$

$$= d\left(\frac{1 + \sin x/2 - \cos^2 x/2}{\sin x/2}\right) = d\,(1 + \sin x/2) = \frac{p}{2}\cdot\frac{1 + \sin x/2}{\cos x/2}.$$

13.4.7. Measurement of Effective Diameter of Tapered Threads. The measurement of the effective diameter of taper threads is *not made perpendicular to the axis,* but at an angle depending on the taper. The measurement is made at a given point or distance from the end of the thread, and in the three wire method, the single wire is placed at this point. The other two wires are placed in the two opposite grooves and care must be taken to ensure that the micrometer or *measuring anvils make contact with each of the three wires.*

The formula for the effective diameter of the taper thread is :

$$E = (M - d)\sec h + \frac{\cot x/2}{2n} - d\,\text{cosec}\,x/2$$

where E = effective diameter, M = measurement over the wires, d = diameter of the wires, h = half the angle of taper, $x/2$ = half the included angle of the thread form, n = number of threads per inch.

8.5 PITCH ERRORS IN SCREW THREADS

If a screw thread is generated by a single point cutting tool its pitch depends on:

(*a*) the ratio of linear velocity of the tool and angular velocity of the work being correct;

(*b*) this ratio being constant.

If these conditions are not satisfied then pitch errors will occur, the type of error being determined by which of the above conditions is not satisfied. Whatever type of error is present the net result is to cause the total length of thread engaged to be too great or too small and this error in overall length of thread is called the *cumulative pitch error*. This, then, is the error which must be determined. It can be obtained either by:

(*a*) measuring individual thread to thread errors and adding them algebraically, i.e. with due regard to sign;

(*b*) measuring the total length of thread, from a datum, at each thread and subtracting from the nominal value.

8.51 Types of Pitch Error

8.511 Progressive Pitch Error

This error occurs when the tool–work velocity ratio is constant but incorrect. It may be caused through using an incorrect gear train, or an approximate gear train between work and tool lead screw as when producing a metric thread with an inch pitch lead screw when no translatory gear is available. More commonly, it is caused by pitch errors in the lead screw of the lathe or other generating machine.

If the pitch error per thread is δp then at any position along the thread the cumulative pitch error is $n\delta p$ where n is the number of threads considered. A graph of cumulative pitch error against length of thread is therefore a straight line [Fig. 8.12 (*a*)].

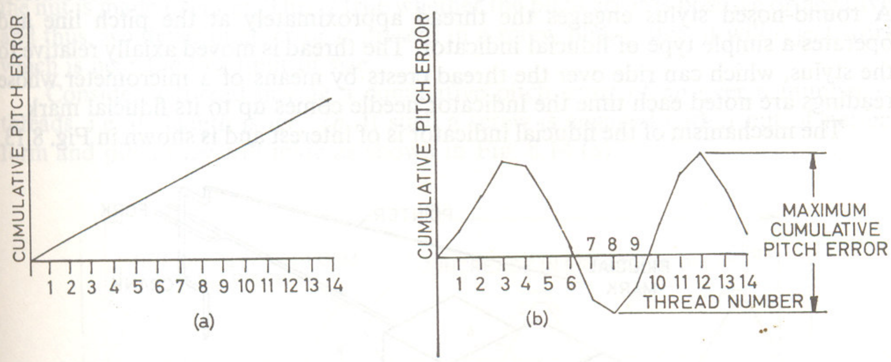


Fig. 8.12(*a*). Progressive pitch error. (*b*) Periodic pitch error.

8.512 *Periodic Pitch Error*

This type of error occurs when the tool–work velocity ratio is not constant. It may be caused by pitch errors in the gears connecting the work and lead screw or by an axial movement of the lead screw due to worn thrust faces. Such a movement would be superimposed on the normal tool motion to be reproduced on the work. It will be appreciated that errors due to these causes will be cyclic, i.e. the pitch will increase to a maximum, reduce through normal to a minimum and so on.

A graph of cumulative pitch error will thus be of approximately sinusoidal form as in Fig. 8.12 (*b*), and the maximum cumulative pitch error will be the total error between the greatest positive and negative peaks within the length of thread engaged.

8.513 *Thread Drunkenness*

A drunken thread is a particular case of a periodic pitch error recurring at intervals of one pitch. This means that the pitch measured parallel to the thread axis will always be correct, and all that is in fact happening is that the thread is not cut to a true helix. A development of the thread helix will be a curve and not a straight line. Such errors are extremely difficult to determine and except on large threads will not have any great effect.

8.52 Measurement of Pitch Error

Apart from drunken threads, pitch errors may be determined using a pitch measuring machine, the design of which originated at the National Physical Laboratory. A round-nosed stylus engages the thread approximately at the pitch line and operates a simple type of fiducial indicator. The thread is moved axially relative to the stylus, which can ride over the thread crests by means of a micrometer whose readings are noted each time the indicator needle comes up to its fiducial mark.

The mechanism of the fiducial indicator is of interest and is shown in Fig. 8.13.

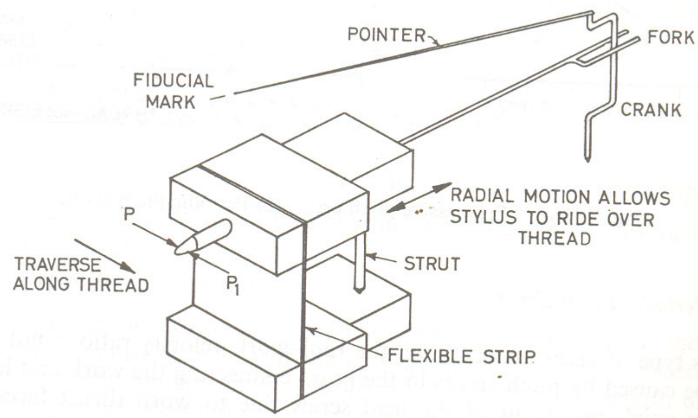


Fig. 8.13. Fiducial indicator used on pitch-measuring machine.

The stylus is mounted in a block supported by a thin metal strip and a strut. It may thus move back and forth over the threads, the strut and strip giving a parallel-type motion. If, however, the side pressures on the stylus, P and P_1, are unequal the strip twists and the block pivots about the strut. The forked arm causes the crank to rotate and with it the pointer. Thus the pointer will only be against the fiducial mark when the pressures P and P_1 caused by the stylus bearing on the thread flanks are the same in each thread.

Errors in the micrometer are reduced by a cam-type correction bar and, with care, accuracies of greater than 0·002 mm may be consistently achieved.

If thread to thread pitches are required then each micrometer reading is subtracted from the next. More usually cumulative pitch errors are required and can be obtained by simply noting the micrometer readings and subtracting them from the expected reading. This should normally be repeated with the thread turned through 180° in case the thread axis does not coincide with the axis of the centres on which it is mounted. The mean of the two readings, usually determined graphically, is then used as the pitch error.

8.53 Effects of Pitch Errors

If a thread has a pitch error it will only enter a nut of perfect form and pitch if the nut is made oversize. This is true whether the pitch error is positive or negative, and thus, whatever pitch error is present in a screw plug gauge, it will reject work which is near the low limit of size.

Consider a thread having a cumulative pitch error of δp over a number of threads, i.e. its length is $np + \delta p$. If such a screw is engaged with a nut of perfect form and pitch they will mate as shown in Fig. 8.14 (*a*).

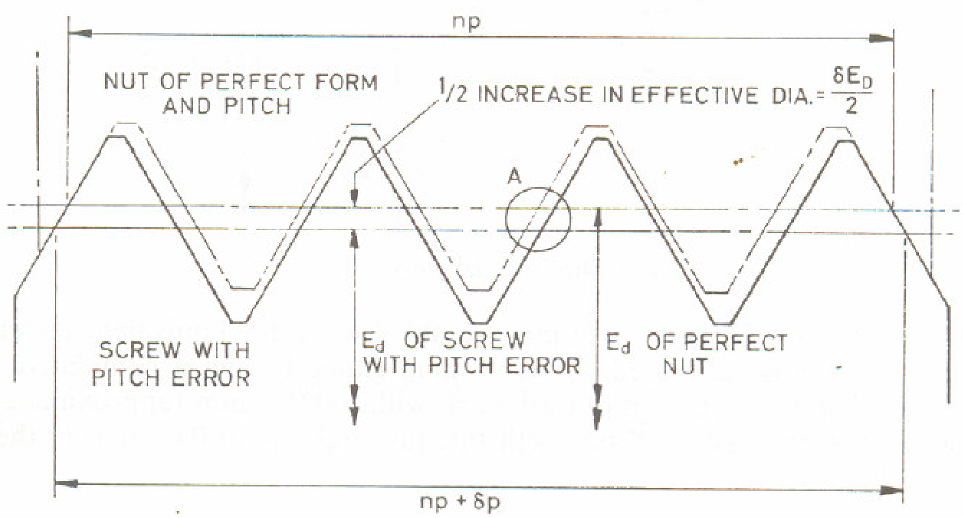


Fig. 8.14(*a*). Screw having cumulative pitch error δp in mesh with a nut of perfect form and pitch.

Consider an enlarged view of the thread flanks at A as in Fig. 8.14 (*b*). It is seen that

$$\tan \theta = \frac{\frac{\delta p}{2}}{\frac{\delta E_d}{2}} = \frac{\delta p}{\delta E_d}$$

$$\therefore \; \delta E_d = \delta p \cot \theta$$

where δp is the cumulative pitch error over the length of engagement and δE_d is the equivalent increase in effective diameter

The importance of this is emphasized when a Whitworth thread is considered in which the flank angle θ is $27\frac{1}{2}°$ and cotangent $27\frac{1}{2}° = 1{\cdot}920$.

$$\text{For Whitworth threads } \delta E_a = 1{\cdot}920\ \delta p$$
$$\text{For Metric threads } \delta E_a = 1{\cdot}732\ \delta p$$

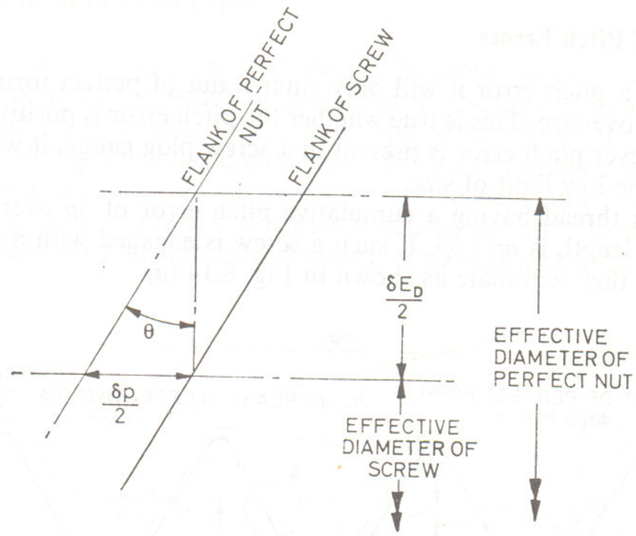


Fig. 8.14(b). Enlarged view at A.

The pitch error is therefore almost doubled when the equivalent increase in effective diameter is calculated. A screw plug gauge having a cumulative pitch error of 0·006 mm will thus reject all work within 0·012 mm (approximately) of the low limit in the case of Whitworth threads, and within 0·01 mm in the case of Metric threads.

Study Material for_Expt No. : Gear Measurement

Gear Measurement

7.1 INTRODUCTION

As technology has progressed from the Industrial Revolution to the present day, the need for closer control over the accuracy of systems used for transmitting the power made available has also progressed. Probably the most used means of transmitting power and multiplying torque is through the medium of gear trains. It is obvious that the strength of gear teeth has had to improve to meet increased loads, but this is a design problem which is not a primary concern of this book. However, it is also a requirement of a gear train that it shall have a constant velocity ratio. Variations in velocity ratio can cause a cyclic fluctuation of tooth loading which gives rise to (*a*) fatigue, leading to tooth failure; and (*b*) noise.

The noise problem is of interest if one considers the development of the automobile. Early automobiles had rudimentary exhaust silencers and the resulting engine noise caused most of the other mechanical noises to be overlooked. Efficient exhaust silencing made mechanical noises from the gear-box more apparent. This was silenced by the use of helical gears and closer control in their manufacture. The gear noise was reduced and carburettor intake noise became significant which, when reduced by efficient air cleaners and intake silencers, enabled rear axle 'whine' to make its presence felt. The use of spiral bevels and hypoid gears, again with closer manufacturing controls, reduced this and the valve timing gears again required attention. By this time, exhaust and intake silencers were improved and the whole cycle started again.

Thus a major item of development in the motor vehicle has been the development of efficient gears, and this only considers one commodity. If one considers this work applied to all of the mechanisms which rely on geared systems to transmit power, the importance of the subject of gear measurement becomes immediately apparent.

7.2 SCOPE

A few of the different types of gears required by modern industry have been mentioned above. Within the confines of this work it is proposed to deal only with involute gears of straight tooth (spur) and helical types. These constitute a large proportion of the gears in use today, bevel gears, spiral bevels, and hypoid gears being topics for works of a more specialist character. Cycloidal gears are used but little in modern engineering. Their main use is in horological work, which again the authors consider is outside the scope of this work.

The choice of the involute for the flank curve of gear teeth has two great advantages for general engineering.

(*a*) The velocity ratio of a pair of involute gears is constant, regardless of errors or variations in centre distance.

(*b*) An involute rack has straight teeth. This enables the complex involute form to be generated from a relatively simple cutter.

It is therefore necessary to consider the involute curve in some detail.

7.3 THE INVOLUTE CURVE

An involute is the locus of a point on a straight line which rolls around a circle without slipping. An alternative definition is: the locus of a point on a piece of string which is unwound from a stationary cylinder.

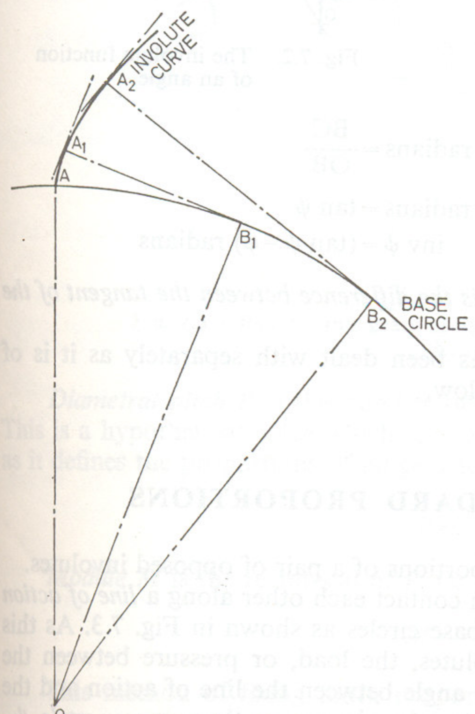


Fig. 7.1. The involute curve.

The curve is therefore as shown in Fig. 7.1.

From the figure it is seen that the length of the generator is equal to the arc length of the base circle from the point of tangency to the origin of the involute at A.

i.e. $A_1B_1 = \text{arc } AB_1$

$A_2B_2 = \text{arc } AB_2$ and so on.

Further, the tangent to the involute at any point, e.g. A_2, is perpendicular to the generator at that point.

Notice also that the shape of the involute depends entirely on the diameter of the base circle from which it is generated. As the base circle increases, so the curvature of the involute decreases, until the limit is reached for a base circle of infinite diameter, i.e. a straight line, when the involute is a straight line.

7.4 THE INVOLUTE FUNCTION

The involute function of an angle may be defined as the angle made by the radius to the origin of the involute and the radius to the intercept of the generator with the involute. This is the involute function of the angle between the radius to the point of tangency of the generator and the radius to the intercept of the generator and the involute.

This apparently complex statement is better described graphically in Fig. 7.2.

In Fig. 7.2:

AOC is the involute function of COB.

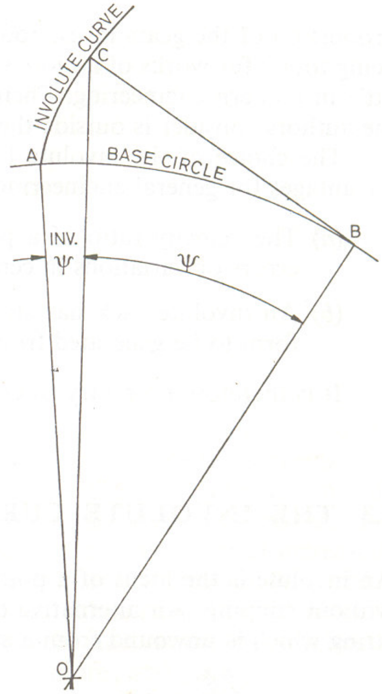


Fig. 7.2. The involute function of an angle.

From the diagram (7.2):

$$BC = \sqrt{OC^2 - OB^2}$$

$$\tan \psi = \frac{\sqrt{OC^2 - OB^2}}{OB}$$

But from Fig. 7.1:

$$\text{arc } AB = BC$$

$$\therefore \frac{AB}{OB} = \psi \text{ radians} + \text{inv } \psi \text{ radians} = \frac{BC}{OB}$$

$$\therefore (\psi + \text{inv } \psi) \text{ radians} = \tan \psi$$

$$\text{inv } \psi = (\tan \psi - \psi) \text{ radians}$$

i.e. *the involute function of an angle is the difference between the tangent of the angle and the angle in radians.*

This term of involute geometry has been dealt with separately as it is of particular importance in the work to follow.

7.5 DEFINITIONS AND STANDARD PROPORTIONS

A single tooth of a gear is made up of portions of a pair of opposed involutes.

The teeth of a pair of gears in mesh contact each other along a *line of action* which is the common tangent to their base circles as shown in Fig. 7.3. As this is the common generator to both involutes, the load, or pressure between the gears is transmitted along this line. The angle between the line of action and the common tangent to the pitch circles is therefore known as the *pressure angle*, ψ.

From Fig. 7.3:

$$\frac{OB}{OC} = \cos\psi = \frac{R_b}{R_p}$$

$$\therefore\ R_b = R_p \cos\psi$$

or $D_b = D\cos\psi$ where D_b = dia. of base circle

D = dia. of pitch circle

ψ = pressure angle

The standard values for pressure angle are $14\frac{1}{2}°$ and 20°, of which 20° is becoming the most used as it gives stronger teeth and allows gears of smaller numbers of teeth to be made, without interference with mating teeth.

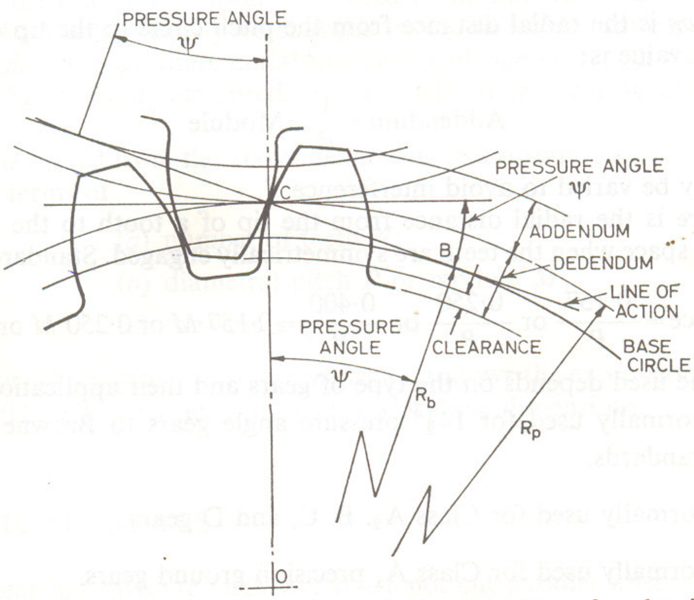


Fig. 7.3. Pair of spur gears in mesh, showing terms referred to in the text.

Diametral pitch P is the number of teeth per inch of pitch circle diameter. This is a hypothetical value which cannot be measured, but it is most important as it defines the proportions of all gear teeth.

$$P = \frac{N}{D}$$

Module M is the reciprocal of *P*, i.e.

$$M = \frac{D}{N}$$

This method of fixing tooth proportions is in common usage in countries using the metric system where *M* is made a whole number of millimetres.

Circular pitch CP is the arc distance measured around the pitch circle from the flank of one tooth to a similar flank in the next tooth.

$$\therefore CP = \frac{\pi D}{N} \text{ but } \frac{D}{N} = \frac{1}{P} = M$$

$$\therefore CP = \frac{\pi}{P} = \pi M$$

Base pitch P_b is the arc distance measured around the base circle from the origin of the involute on one tooth to the origin of a similar involute on the next tooth.

$$P_b = CP \cos \psi = \pi M \cos \psi$$

Addendum is the radial distance from the pitch circle to the tip of the tooth. The nominal value is:

$$\text{Addendum} = \frac{1}{P} = \text{Module}$$

This may be varied to avoid interference.

Clearance is the radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Standard values are:

$$\text{Clearance} = \frac{0{\cdot}157}{P} \text{ or } \frac{0{\cdot}250}{P} \text{ or } \frac{0{\cdot}400}{P} = 0{\cdot}157\,M \text{ or } 0{\cdot}250\,M \text{ or } 0{\cdot}400\,M$$

The value used depends on the type of gears and their application.

$0{\cdot}157\,M$ is normally used for $14\frac{1}{2}°$ pressure angle gears to Browne and Sharpe standards.

$0{\cdot}250\,M$ is normally used for Class A_2, B, C, and D gears.

$0{\cdot}400\,M$ is normally used for Class A_1 precision ground gears.

Dedendum is the radial distance from the pitch circle to the bottom of the tooth space.

$$\text{Dedendum} = \text{Addendum} + \text{Clearance}$$

$$= \frac{1}{P} + \frac{0{\cdot}157}{P} = \frac{1{\cdot}157}{P} = 1{\cdot}157\,M$$

$$\text{or} = \frac{1}{P} + \frac{0{\cdot}250}{P} = \frac{1{\cdot}250}{P} = 1{\cdot}250\,M$$

$$\text{or} = \frac{1}{P} + \frac{0{\cdot}400}{P} = \frac{1{\cdot}400}{P} = 1{\cdot}400\,M$$

Blank diameter. The diameter of the blank is equal to the pitch circle diameter plus two addenda:

$$\text{Blank diameter} = D + 2M$$

$$\text{but } D = NM$$

$$\therefore \text{ Blank diameter} = NM + 2M = (N+2) \times \text{Module or } \frac{(N+2)}{P}$$

Tooth thickness is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

$$\text{Nominally, tooth thickness} = \tfrac{1}{2} CP$$

$$= \frac{\pi}{2DP} \text{ or } \pi \times \frac{\text{Module}}{2}$$

In fact the thickness is usually reduced by an amount to allow for a certain amount of backlash and may be changed owing to addendum correction.

Backlash is the circumferential movement of one gear of a mating pair, the other gear being fixed, measured at the pitch circle, bearing clearances being eliminated.

It will be noted from the above definitions that a spur gear can be completely specified in terms of

(*a*) number of teeth N;

(*b*) diametral pitch P or module M;

(*c*) pressure angle ψ.

In the work on gear measurement which follows the expressions derived will, where possible, all be reduced to functions of these dimensions.

15.3. Terminology of Gear Tooth

A gear tooth is formed by portions of a pair of opposed involutes. Most of the terms used in connection with gear teeth are explained in Fig. 15.2.

Base Circle. It is the circle from which involute form is generated. Only the base circle on a gear is fixed and unalterable.

Pitch Circle. It is an imaginary circle most useful in calculations. It may be noted that an infinite number of pitch circles can be chosen, each associated with its own pressure angle.

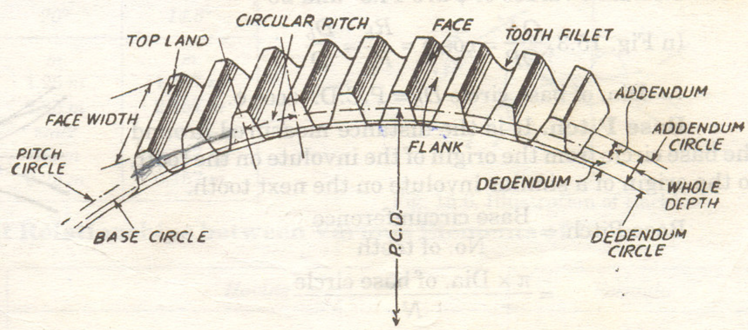


Fig. 15.2

Pitch Circle Diameter (P.C.D.). It is the diameter of a circle which by pure rolling action would produce the same motion as the toothed gear wheel. This is the most important diameter in gears.

Module. It is defined as the length of the pitch circle diameter per tooth. Thus if P.C.D. of gear be D and number of teeth N, then module $(m) = D/N$. It is generally expressed in mm.

Diametral Pitch. It is expressed as the number of teeth per inch of the P.C.D.

Circular Pitch (C.P.). It is the arc distance measured around the pitch circle from the flank of one tooth to a similar flank in the next tooth. $\therefore$ C.P. $= \pi D/N = \pi m$

Addendum. This is the radial distance from the pitch circle to the tip of the tooth. Its value is equal to one module.

Clearance. This is the radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Its standard value is $0.157\ m$.

Dedendum. This is the radial distance from the pitch circle to the bottom of the tooth space.

$$\text{Dedendum} = \text{Addendum} + \text{Clearance} = m + 0.157\ m = 1.153\ m.$$

Blank Diameter. This is the diameter of the blank from which gear is out. It is equal to P.C.D. plus twice the addenda.

$$\text{Blank diameter} = \text{P.C.D.} + 2m = mN + 2m = m\,(N + 2).$$

Tooth Thickness. This is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

Normally tooth thickness = C.P./2 $= \pi m/2$

But thickness is usually reduced by certain amount to allow for some amount of backlash and also owing to addendum correction.

Face of Tooth. It is that part of the tooth surface which is above the pitch surface.

Flank of Tooth. It is that part of the tooth surface which is lying below the pitch surface.

Line of Action and Pressure Angle. The teeth of a pair of gears in mesh, contact each other along the common tangent to their base circles as shown in Fig. 15.3. This path is referred to as line of action. As this is the common generator to both the involutes, the load or pressure between the gears is transmitted along this line. The angle between the line of action and the common tangent to the pitch circles is therefore known as pressure angle ϕ. The standard values of ϕ are 14.5° and 20°.

In Fig. 15.3, $\dfrac{OA}{OP} = \cos\phi = \dfrac{R_b}{R_p} = \dfrac{D_b}{D}$

$\therefore$ Dia. of base circle D_b = P.C.D. $\times \cos\phi$.

Base Pitch. It is the distance measured around the base circle from the origin of the involute on the tooth to the origin of a similar involute on the next tooth.

$$\text{Base Pitch} = \frac{\text{Base circumference}}{\text{No. of teeth}} = \frac{\pi \times \text{Dia. of base circle}}{N} = \frac{\pi \times D\cos\phi}{N} = \pi m \cos\phi$$

Involute Function. It is found from the fundamental principle of the involute, that it is the locus of the end of a thread (imaginary) unwound from the base circle.

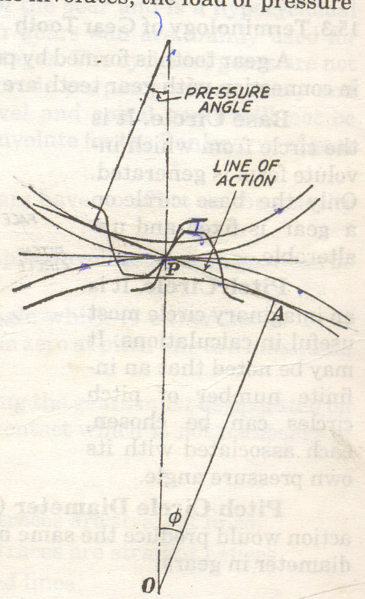


Fig. 15.3

Mathematically its value is Involute function $\delta = \tan\phi - \phi$

where ϕ is the pressure angle.

The relationship between the involute function and the pressure angle can be derived as follows :

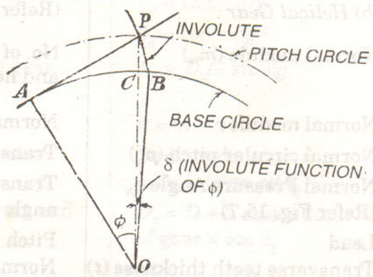


Fig. 15.4

In Fig. 15.4, OA = base circle radius = R_b

OP = pitch circle radius = R_p

and BP = involute profile of gear tooth.

AP is tangent to base circle at A,

$\hat{AOC} = \phi$ = pressure angle

Now $OA = OP\cos\phi$, or $R_b = R_p\cos\phi$

$\hat{COB}$ = Involute function of ϕ.

By definition of involute, length AP = arc AB

and $$\tan\phi = \frac{AP}{OA} = \frac{AP}{R_b} = \frac{\text{arc } AB}{R_b}, \text{ Also } \phi + \delta = \frac{\text{arc } AB}{R_b}$$

$\therefore$ $\phi + \delta = \tan\phi$, or $\delta = \tan\phi - \phi$.

Helix Angle : It is the acute angle between the tangent to the helix and axis of the cylinder on which teeth are cut.

Lead Angle : It is the acute angle between the tangent to the helix and plane perpendicular to the axis of cylinder (Refer Fig. 15.5).

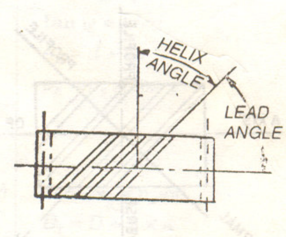


Fig. 15.5. Illustration of Helix and lead angle.

Back Lash : The distance through which a gear can be rotated to bring its non-working flank in contact with the teeth of mating gear. (Refer Fig. 15.6).

Basic Tooth Proportions for Involute Spur Gears

	Pressure Angles	
	20°	*14.5°*
Addendum	m	m
Dedendum	$1.25\,m$	$1.157\,m$
Teeth Depth	$2.25\,m$	$2.157\,m$
Circular teeth thickness	$\pi m/2$	$pm/2$
Fillet radius	$0.3\,m$	$0.157\,m$
Clearance	$0.25\,m$	$0.157\,m$

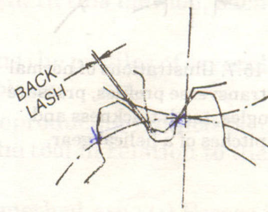


Fig. 15.6. Illustration of Backlash.

Some Important Relationships between Various Elements of Gears :

To find	*Having*	*Formula*
(*a*) *Spur Gears*		
Module (m)	No. of teeth (N) and pitch diameter (D)	$m = D/N$
Module	Circular pitch (p)	$m = p/\pi$
Outside diameter (D_o)	Pitch diameter and Module	$D_o = D + 2m$
Base circular diameter (D_b)	Pitch diameter and pressure angle	$D_b = D\cos\phi$

will be depicting the compound errors *i.e.*, all errors like eccentricity and tooth form errors etc., which occur together and the trace will be as shown in Fig. 15.8.

The machine could also be used to carry out more complex tests by suitable modification in its operation, *e.g.*, by locking the movable carriage at the running centre distance of the gears, and by fixing the master gear, the backlash can be determined by setting a dial gauge at the pitch line of the production gear. It is also possible to check the gears for smooth running at this setting and this is very essential for gears. This is judged by the noise produced.

For these tests, if master gear is not available, then any two mating gears are mounted on the spindle and they are tested twice at relative angular positions of 180° to each other so that any compensating errors in one angular position in gears are also revealed.

15.7. Measurement of Individual Elements

15.7.1. Measurement of tooth thickness. The permissible error or the tolerance on thickness of tooth is the variation of actual thickness of tooth from its theoretical value. The tooth thickness is generally measured at pitch circle and is therefore, the pitch line thickness of tooth. It may be mentioned that the tooth thickness is defined as the length of an arc, which is difficult to measure directly. In most of the cases, it is sufficient to measure the chordal thickness *i.e.*, the chord joining the intersection of the tooth profile with the pitch circle. Also the difference between chordal tooth thickness and circular tooth thickness is very small for gear of small pitch. The thickness measurement is the most important measurement because most of the gears manufactured may not undergo checking of all other parameters, but thickness measurement is a must for all gears. There are various methods of measuring the gear tooth thickness.

(*i*) Measurement of tooth thickness by gear tooth vernier calliper. (*ii*) Constant chord method. (*iii*) Base tangent method. (*iv*) Measurement by dimension over pins.

The tooth thickness can be very conveniently measured by a gear tooth vernier. Since the gear tooth thickness varies from the tip of the base circle of the tooth, the instrument must be capable of measuring the tooth thickness at a specified position on the tooth. Further this is possible only when there is some arrangement to fix that position where the measurement is to be taken. The tooth thickness is generally measured at pitch circle and is, therefore, referred to as pitch-line thickness of tooth. The gear tooth vernier has two vernier scales and they are set for the width (w) of the tooth and the depth (d) from the top, at which w occurs.

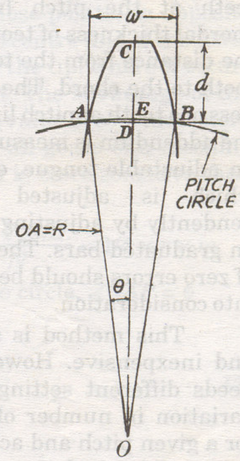


Fig. 15.14

Considering one gear tooth, the theoretical values of w and d can be found out which may be verified by the instrument. In Fig. 15.14, it may be noted that w is a chord ADB, but tooth thickness is specified as an arc distance AEB. Also the distance d adjusted on instrument is slightly greater than the addendum CE, w is therefore called chordal thickness and d is called the chordal addendum.

In Fig. 15.14, $w = AB = 2AD$

Now, $A\hat{O}D = \theta = 360°/4N$, where N is the number of teeth,

$w = 2AD = 2 \times AO \sin\theta = 2R \sin 360/4N$ (N = pitch circle radius)

$$\text{module } m = \frac{\text{P.C.D.}}{\text{No. of teeth}} = \frac{2R}{N}, \therefore \quad R = \frac{N.m.}{2}$$

$$\therefore \qquad w = 2\frac{Nm}{2}\sin\left(\frac{360}{4N}\right) = N.m.\ \sin\left(\frac{90}{N}\right) \qquad ...(1)$$

Also from Fig. 15.14, $d = OC - OD$

But $\quad OC = OE + \text{addendum} = R + m = (Nm/2) + m$

and $\quad OD = R\cos\theta = \frac{Nm}{2}\cos\left(\frac{90}{N}\right)$

$$\therefore \qquad d = \frac{Nm}{2} + m - \frac{Nm}{2}\cos\left(\frac{90}{N}\right) = \frac{Nm}{2}\left[1 + \frac{2}{N} - \cos\left(\frac{90}{N}\right)\right] \qquad ...(2)$$

Any error in the outside diameter of the gear must be allowed for when measuring tooth thickness.

In the case of helical gears, the above expressions have to be modified to take into account the change in curvature along the pitch line. The virtual number of teeth Nv for helical gear $= N/\cos^3\alpha$ (α = helix angle)

Hence in Eqs. (1) and (2), N can be replaced by $N/\cos^3\alpha$ and m by m_n (normal module).

$$\therefore \quad w = \frac{Nm_n}{\cos^3\alpha}\sin\left(\frac{90}{N}\cos^3\alpha\right), \text{ and } d = \frac{Nm_n}{\cos^3\alpha}\left[1 + \frac{2\cos^3\alpha}{N} - \cos\left(\frac{90}{N}\cos^2\alpha\right)\right]$$

These formulae apply when backlash is ignored. On mating gears having equal tooth thickness and without addendum modifications, the circular tooth thickness equals half the circular pitch minus half the backlash.

Gear Tooth Calliper.

(Refer Fig. 15.15). It is used to measure the thickness of gear teeth at the pitch line or chordal thickness of teeth and the distance from the top of a tooth to the chord. The thickness of a tooth at pitch line and the addendum is measured by an adjustable tongue, each of which is adjusted independently by adjusting screw on graduated bars. The effect of zero errors should be taken into consideration.

- This method is simple and inexpensive. However it needs different setting for a variation in number of teeth for a given pitch and accuracy is limited by the least count of instrument. Since the wear during use is concentrated on the two jaws, the calliper has to be calibrated at regular intervals to maintain the accuracy of measurement.

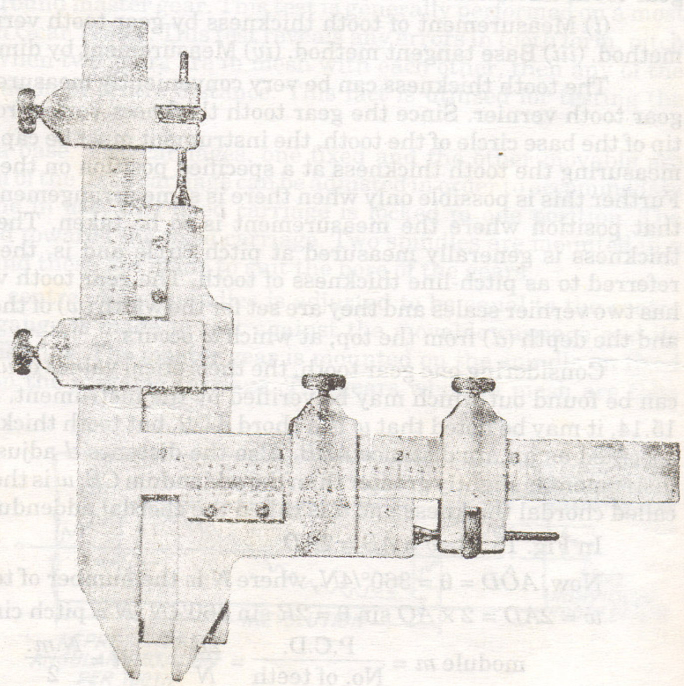


Fig. 15.15. Gear Tooth Vernier Calliper.

15.7.2. Constant Chord Method. In the above method, it is seen that both the chordal thickness and chordal addendum are dependent upon the number of teeth. Hence for measuring a large number of gears for set, each having different number of teeth would involve separate calculations. Thus the procedure becomes laborious and time-consuming one.

The constant chord method does away with these difficulties. Constant chord of a gear is measured where the tooth flanks touch the flanks of the basic rack. The teeth of the rack are straight and inclined to their centre lines at the pressure angle as shown in Fig. 15.16.

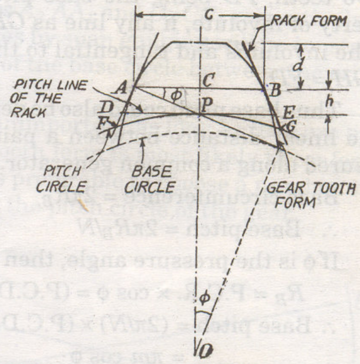


Fig. 15.16

Also the pitch line of the rack is tangential to the pitch circle of the gear and, by definition, the tooth thickness of the rack along this line is equal to the arc tooth thickness of the gear round its pitch circle. Now, since the gear tooth and rack space are in contact in the symmetrical position at the points of contact of the flanks, the chord is constant at this position irrespective of the gear of the system in mesh with the rack. This is the property utilised in the constant chord method of the gear measurement.

The measurement of tooth thickness at constant chord simplified the problem for all number of teeth. If an involute tooth is considered symmetrically in close mesh with a basic rack form, then it will be observed that regardless of the number of teeth for a given size of tooth (same module), the contact always occurs at two fixed point A and B. AB is known as constant chord. The constant chord is defined as the chord joining those points, on opposite faces of the tooth, which make contact with the mating teeth when the centre line of the tooth lies on the line of the gear centres. The value of AB and its depth from the tip, where it occurs can be calculated mathematically and then verified by an instrument. The advantage of the constant chord method is that for all number of teeth (of same module) value of constant chord is same. In other words, the value of constant chord is constant for all gears of a meshing system. Secondly it readily lends itself to a form of comparator which is more sensitive than the gear tooth vernier.

In Fig. 15.16, $PD = PF = \text{arc } PF = 1/4 \times \text{circular pitch} = \frac{1}{4} \times \frac{\pi \times \text{P.C.D.}}{N} = 1/4 \times \pi \times m$

Since line AP is the line of action, *i.e.* it is tangential to the base circle, $\angle CAP = \phi$

$\therefore$ In right angled ΔAPD, $AP = PD \cos \phi = (\pi/4) m \cos \phi$

In triangle PAC, $AC = AP \cos \phi = (\pi/4) m \cos^2 \phi$

$$c = \text{constant chord} = 2AC = (\pi/2)\, m \cos^2 \phi \qquad \text{...(3)}$$

where ϕ is the pressure angle (from Fig. 15.16)

For helical gear, constant chord $= (\pi/2)\, m_n \cos^2 \phi_n$

where m_n = normal module *i.e.* module of cutter used and ϕ_n = normal pressure angle.

Now $PC = AP \sin \phi = (\pi/4)\, m \cos \phi \sin \phi$

$$\therefore \quad d = \text{addendum} - PC = m - \frac{\pi}{4} m \cos \phi \sin \phi = m \left(1 - \frac{\pi}{4} \cos \phi \sin \phi \right) \qquad \text{...(4)}$$

$$\left[\text{For helical gear, } d = m_n \left(1 - \frac{\pi}{4} \cos \phi_n \sin \phi_n \right) \right]$$

15.7.2. Constant Chord Method.

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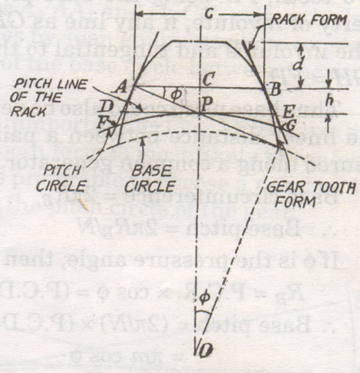

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$$\left[\text{For helical gear, } d = m_n \left(1 - \frac{\pi}{4} \cos \phi_n \sin \phi_n\right)\right]$$

(i) the measurements do not depend on two vernier readings, each being function of the
er.

(ii) the measurement is not made with an edge of the measuring jaw with the face.

Consider a straight generator (edge) ABC being rolled back and forth along a base circle . 15.19). Its ends thus sweep out opposed involutes A_2AA_1 and C_2CC_1 respectively. Thus measurements made across these opposed *involutes by span gauging will be constant (i.e.* $= A_1C_1 = A_2C_2 = A_0C_0$) and equal to the arc length of the base circle between the origins of lutes.

Further the position of the measuring faces is unimportant as long as they are parallel l on an opposed pair of the true involutes. As the tooth form is most likely to conform to a e involute at the pitch point of the gear, it is always preferable to choose a number of teeth ch that the measurement is made approximately at the pitch circle of the gear.

The value of the distance between two opposed involutes, or the dimension over parallel faces is equal to the distance round the base circle between the points where the corresponding tooth flanks cut i.e., ABC in Fig. 15.19. It can be derived mathematically as follows :

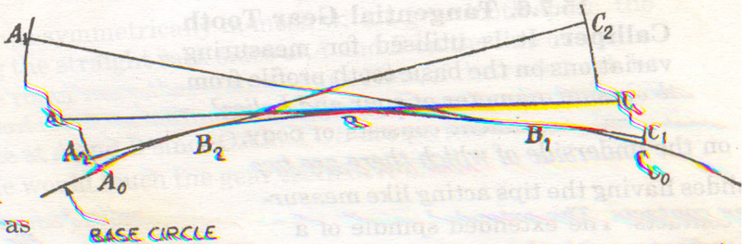

Fig. 15.19. Generation of pair of involutes by a common generato

The angle between the points *A* and *C* on the pitch circle where the flanks of the opp involute teeth of the gear cut this circle can be easily calculated.

Let us say that the gear has got '*N*' number of teeth and AC on pitch circle corres to 'S' number of teeth. (Fig. 15.20) ; $\therefore$ Distance $AC = (S - 1/2)$ pitches

$\therefore$ Angle subtended by $AC = (S - 1/2) \times 2\pi/N$ *radians.*

Angles of arcs BE and BD

Involute *function of pressure angle* $= \delta = \tan\phi - \phi$

$$\therefore \text{ Angle of arc } BD = \left(S - \frac{1}{2}\right) \times \frac{2\pi}{N} + 2(\tan\phi - \phi)$$

$$\therefore \quad BD = \text{Angle of arc } BD \times R_b$$

$$= \left[\left(S - \frac{1}{2}\right) \times \frac{2\pi}{N} + 2(\tan\phi - \phi)\right] \times R_p \cos\phi \quad [\text{because } R_b = R_p \cos\phi]$$

$$= \frac{mN}{2}\cos\phi\left[\left(S - \frac{1}{2}\right)\frac{2\pi}{N} + 2(\tan\phi - \phi)\right]\left[\text{because } R_p = \frac{mN}{2}\right]$$

$$= Nm\cos\phi\left[\frac{\pi S}{N} - \frac{\pi}{2N} + \tan\phi - \phi\right]$$

As already defined, length of arc BD = distance between two opposed involutes ar it is

$$= Nm\cos\phi\left[\tan\phi - \phi - \frac{\pi}{2N} + \frac{\pi S}{N}\right]$$

It may be noted that when backlash allowance is specified normal to the tooth this must be simply subtracted from this derived value.

Tables are also available which directly give this value for the given values of S, N and m.

This distance is first calculated and then set in the 'David Brown' tangent comparator (Fig. 15.21) with the help of slip gauges. The instrument essentially consists of a fixed anvil and a movable anvil. There is a micrometer on the moving anvil side and this has a very limited movement on either side of the setting. The distance is adjusted by setting the fixed anvil at desired place with the help of looking ring and setting tubes.

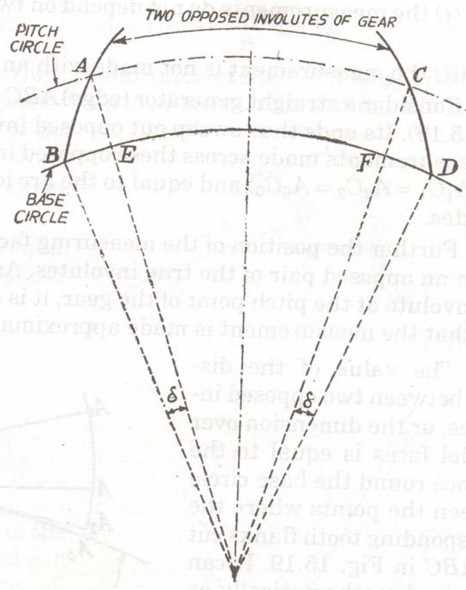

Fig. 15.20

15.7.6. Tangential Gear Tooth Calliper. It is utilised for measuring variations on the basic tooth profile from the outside diameter of spur and helical gears. The instrument consists of body, on the underside of which there are two slides having the tips acting like measuring contacts. The extended spindle of a dial indicator with the contact point A passes between the two tips along the vertical axis of symmetry of the instrument. The measuring tips are spread apart or brought together simultaneously and symmetrically in reference to the central axis by a screw which has a right-hand and a left-hand thread. The contact faces of the measuring tips are flat and arranged at angles of 14.5° or 20° with the central axis. The calliper is set up by means of a cylindrical master gauge of proper diameter based on the module of the gear being checked. After adjusting the tips by the screw, these are locked in position by locking nuts. The properly set up instrument is applied to the gear tooth and the dial indicator reading shows how much the position of the basic tooth profile deviates in reference to the outside diameter of the gear.

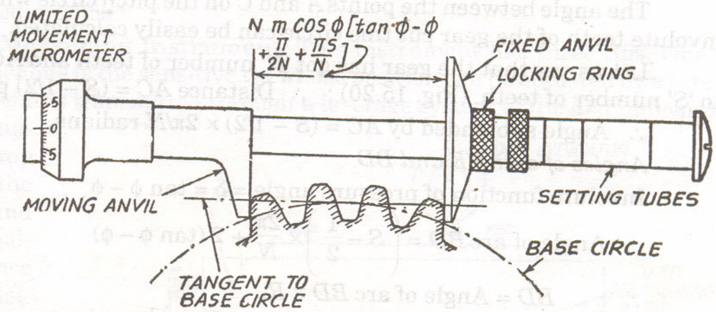

Fig. 15.21. 'David Brown' Base Tangent Comparator.