Forging Analysis - 1

ver. 1

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Overview

• Slab analysis
  – frictionless
  – with friction
  – Rectangular
  – Cylindrical
• Strain hardening and rate effects
• Flash
• Redundant work
Slab analysis assumptions

- Entire forging is plastic
  - no elasticity
- Material is perfectly plastic
  - strain hardening and strain rate effects later
- Friction coefficient ($\mu$) is constant
  - all sliding, to start
- Plane strain
  - no z-direction deformation
- In any thin slab, stresses are uniform
Open die forging analysis – rectangular part

unit depth into figure

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Expanding the $dx$ slice on LHS

- $p = \text{die pressure}$
- $\sigma_x, d\sigma_x$ from material on side
- $\tau_{\text{friction}} = \text{friction force} = \mu p$
Force balance in $x$-direction

\[ h d \sigma_x + 2 \tau_{\text{friction}} dx = 0 \]

\[ d \sigma_x = -\frac{2 \tau_{\text{friction}}}{h} dx \]

Mohr’s circle

\[ \sigma_x + p = 2k = \frac{2}{\sqrt{3}} \sigma_{\text{flow}} = 1.15 \cdot \sigma_{\text{flow}} \]

(distortion energy (von Mises) criterion, plane strain)

N.B. all done on a per unit depth basis

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Force balance

Differentiating, and substituting into Mohr’s circle equation

\[ d(2k) = d(\sigma_x + p) \quad \therefore \quad dp = -d\sigma_x \]

\[ d\sigma_x = -\frac{2\tau_{\text{friction}}}{h} \, dx \quad \therefore \quad dp = \left(\frac{2\tau_{\text{friction}}}{h}\right) dx \]

noting: \( \tau_{\text{friction}} = \mu p \)

\[ dp = \frac{2\mu}{h} \, p \, dx \quad \longrightarrow \quad \frac{dp}{p} = \frac{2\mu}{h} \, dx \]
Sliding region

\[ \int_{2k}^{p_x} \frac{dp}{p} = \int_{0}^{x} \frac{2\mu}{h} \, dx \]

- Noting: \(@ x = 0, \sigma_x = 2k = 1.15 \sigma_{\text{flow}}\)
Forging pressure – sliding region

\[ \ln p_x - \ln(2k) = 2\mu \frac{x}{h} \]

Sliding region result \((0 < x < x_k)\)

\[ \frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right) \]

\[ p_x = 1.15 \cdot \sigma_{flow} \cdot \exp\left(\frac{2\mu x}{h}\right) \]

N.B done on a per unit depth basis
Forging pressure – approximation

• Taking the first two terms of a Taylor’s series expansion for the exponential about 0, for $|x| \leq 1$

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$$

yields

$$\frac{p_x}{2k} = \left(1 + \frac{2\mu x}{h}\right)$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{2\mu x}{h}\right)$$
Average forging pressure – all sliding approximation

• using the Taylor’s series approximation

\[
\frac{P_{ave}}{2k} = \frac{\int_{0}^{w} \frac{p_x}{2k} \, dx}{\frac{w}{2}} = \frac{\int_{0}^{w} \left(1 + \frac{2\mu x}{h}\right) \, dx}{\frac{w}{2}} = \left(\frac{x + \frac{2\mu x^2}{2h}}{\frac{w}{2}}\right)_{0}^{w}
\]

\[
P_{ave} = \left(1 + \frac{\mu w}{2h}\right) \cdot \sigma_{flow}
\]

N.B done on a per unit depth basis

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Forging force – all sliding approximation

\[ F_{\text{forging}} = p_{\text{ave}} \cdot \text{width} \cdot \text{depth} \]

\[ F_{\text{forging}} = 1.15 \cdot \sigma_{\text{flow}} \cdot \left(1 + \frac{\mu w}{2h}\right) \cdot w \cdot \text{depth} \]
Slab - die interface

- Sliding if $\tau_f < \tau_{\text{flow}}$
- Sticking if $\tau_f \geq \tau_{\text{flow}}$
  - can’t have a force on a material greater than its flow (yield) stress
  - deformation occurs in a sub-layer just within the material with stress $\tau_{\text{flow}}$
Sliding / sticking transition

- Transition will occur at $x_k$
- using $k = \mu p$, in:

$$\frac{p_x}{2k} = \exp\left(\frac{2\mu x}{h}\right)$$

$$\frac{k}{2\mu k} = \exp\left(\frac{2\mu x_k}{h}\right)$$

- hence:

$$\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}$$
Sticking region

\[ dp = \frac{2\mu}{h} p \, dx \]

- Using \( p = \frac{k}{\mu} \)

\[ dp = \frac{2\mu}{h} \frac{k}{\mu} \, dx \]

\[ p_x - p_{x_k} = \frac{2k}{h} (x - x_k) \]
Sticking region

We know that

- at $x = x_k$, $p_{x_k} = k/\mu$

- and

$$\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}$$
Forging pressure - sticking region

Combining (for $x_k < x < w/2$)

$$\frac{p_x}{2k} = \frac{1}{2\mu} \left( 1 - \ln \left( \frac{1}{2\mu} \right) \right) + \frac{x}{h}$$

$$p_x = 1.15 \cdot \sigma_{flow} \cdot \left[ \frac{1}{2\mu} \left( 1 - \ln \left( \frac{1}{2\mu} \right) \right) + \frac{x}{h} \right]$$
Forging pressure – all sticking approximation

- If $x_k \ll w$, we can assume all sticking, and approximate the total forging force per unit depth (into the figure) by:
Forging pressure –
all sticking approximation

\[ p_{edge} = 2k \]

\[ \int_{p_x}^{x} dp = \int_{0}^{x} \frac{2k}{h} dx \]

\[ p_x - 2k = \frac{2k}{h}(x) \]

\[ \therefore \frac{p_x}{2k} = \left(1 + \frac{x}{h}\right) \]

\[ p_x = 1.15 \cdot \sigma_{flow} \cdot \left(1 + \frac{x}{h}\right) \]
Average forging pressure – all sticking approximation

\[ \frac{p_{ave}}{2k} = \frac{\int_0^{\frac{w}{2}} \frac{p_x}{2k} \, dx}{\frac{w}{2}} = \frac{\int_0^{\frac{w}{2}} \left( 1 + \frac{x}{h} \right) \, dx}{\frac{w}{2}} = \left. \left( x + \frac{x^2}{2h} \right) \right|_0^{\frac{w}{2}} \]

\[ \frac{p_{ave}}{2k} = \left( 1 + \frac{w}{4h} \right) \]

\[ p_{ave} = 1.15 \cdot \sigma_{flow} \cdot \left( 1 + \frac{w}{4h} \right) \]

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Forging force –
all sticking approximation

\[ F_{\text{forging}} = p_{\text{ave}} \cdot \text{width} \cdot \text{depth} \]

\[ F_{\text{forging}} = 1.15 \cdot \sigma_{\text{flow}} \cdot \left(1 + \frac{w}{4h}\right) \cdot w \cdot \text{depth} \]
Sticking and sliding

• If you have both sticking and sliding, and you can’t approximate by one or the other,
• Then you need to include both in your pressure and average pressure calculations.

\[ F_{\text{forging}} = F_{\text{sliding}} + F_{\text{sticking}} \]

\[ F_{\text{forging}} = (p_{\text{ave}} \cdot A)_{\text{sliding}} + (p_{\text{ave}} \cdot A)_{\text{sticking}} \]
Material Models

Strain hardening (cold – below recrystallization point)

\[ \sigma_{\text{flow}} = Y = K \varepsilon^n \]

Strain rate effect (hot – above recrystallization point)

\[ \sigma_{\text{flow}} = Y = C(\dot{\varepsilon})^m \]

\[ \dot{\varepsilon} = \frac{1}{h} \frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}} \]
Forging - Ex. 1-1

- Lead 1” x 1” x 36”
- $\sigma_y = 1,000$ psi
- $h_f = 0.25”$, $\mu = 0.25$
- Show effect of friction on total forging force.
- Use the slab method.
- Assume it doesn’t get wider in 36” direction.
- Assume cold forging.
Forging - Ex. 1-2

• At the end of forging:
  \( h_f = 0.25'' \), \( w_f = 4'' \) (conservation of mass)

• Sliding / sticking transition

\[
\frac{x_k}{h} = \frac{1}{2\mu} \ln \frac{1}{2\mu}
\]

\[
x_k = \frac{0.25}{2 \times 0.25} \ln \frac{1}{2 \times 0.25} = 0.347''
\]
Forging - Ex. 1-3

- Sliding region:

\[ p_x = 1.15 \cdot \sigma_{flow} \cdot \exp\left(\frac{2\mu x}{h_f}\right) \]

\[ = 1150 \cdot \exp(2x) \]
Forging - Ex. 1-4

• Sticking region

\[ p_x = 1.15 \cdot \sigma_{flow} \cdot \left\{ \frac{1}{2\mu} \left( 1 - \ln \left( \frac{1}{2\mu} \right) \right) + \frac{x}{h_f} \right\} \]

\[ = 1150 \cdot (0.6 + 4x) \]
Forging - Ex. 1-5

Forging pressure (psi)

Distance from forging edge (in)

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Forging - Ex. 1-6

• Friction hill
  – forging pressure must be large (8.7x) near the center of the forging to “push” the outer material away against friction
Forging - Ex. 1-7

• Determine the forging force from:

\[ \text{Force} = \iint p \cdot dA \]

• since we have plane strain

\[ \frac{F}{\text{unit depth}} = \int_{0}^{x} p_x \, dx \]
Forging - Ex. 1-8

• We must solve separately for the sliding and sticking regions

\[
F_{\text{forging}} = 2 \left( \int_{0}^{x_k} p_x \, dx \right) \cdot \text{depth}_{\text{sliding}} + 2 \left( \int_{x_k}^{w/2} p_x \, dx \right) \cdot \text{depth}_{\text{sticking}}
\]
Forging - Ex. 1-9

Sliding first

\[
\frac{p_{ave}}{1.15 \sigma_{flow}} \frac{unit\ depth}{x_k - 0} = \int_0^{x_k} \exp\left(\frac{2\mu x}{h}\right) dx = \frac{h}{2\mu} \exp\left(\frac{2\mu x}{h}\right)\bigg|_0^{x_k}
\]

\[
= \frac{h}{2\mu} \left[ \exp\left(\frac{2\mu x_k}{h}\right) - 1 \right] \over (x_k - 0)
\]
Forging - Ex. 1-12

Substituting values

sliding

\[
\frac{p_{ave}}{1.15\sigma_{flow} \text{ unit depth}} = \frac{0.25}{2 \times 0.25} \left[ \exp \left( \frac{2 \times 0.25 \times 0.347}{0.25} \right) - 1 \right] = 1.44
\]

sticking

\[
\frac{p_{ave}}{1.15\sigma_{flow} \text{ unit depth}} = \frac{1}{2 \times 0.25} \left[ 1 - \ln \left( \frac{1}{2 \times 0.25} \right) \left( \frac{4}{2} - 0.347 \right) + \frac{1}{2 \times 0.25} \left( \frac{4^2}{4} - 0.347^2 \right) \right] = 5.3
\]
Forging - Ex. 1-10

Sticking next

\[
\frac{p_{ave}}{1.15\sigma_{flow}} = \frac{1}{\text{unit depth}} \int_{x_k}^{w/2} \left( \frac{1}{2\mu} \left( 1 - \ln \frac{1}{2\mu} \right) + \frac{x}{h} \right) dx
\]

\[
= \left[ \frac{1}{2\mu} \left( 1 - \ln \frac{1}{2\mu} \right) \cdot x + \frac{x^2}{2h} \right]_{x_k}^{w/2}
\]

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Forging - Ex. 1-11

\[
\frac{p_{ave}}{1.15 \sigma_{flow}} = \frac{1}{2 \mu} \left( 1 - \ln \frac{1}{2 \mu} \right) \left( \frac{w}{2} - x_k \right) + \frac{1}{2h} \left( \frac{w^2}{4} - x_k^2 \right)
\]
Forging - Ex. 1-13

Now calculate the area/unit depth

\[ A_{sliding} = 0.347 \times 2 = 0.69 \]
\[ A_{sticking} = 4 - 2 \times 0.347 = 3.31 \]
Forging - Ex. 1-14

Now calculate the forces

\[ \frac{F}{\text{unit depth}} = 1.15 \sigma_{flow} \left( (p_{ave} \cdot A)_{sliding} + (p_{ave} \cdot A)_{sticking} \right) \]

\[ \frac{F}{\text{unit depth}} = 1150 \times \left( (1.44 \times 0.69) + (5.3 \times 3.31) \right) \]

\[ = 21,110 \text{ lb/ inch} \]
Forging - Ex. 1-15

Now, assume all sticking, so:

\[
\frac{F}{\text{unit length}} = 1.15 \sigma_{\text{flow}} \cdot w_f \cdot \left(1 + \frac{w_f}{4h_f}\right)
\]

\[
= 1150 \cdot 4 \cdot \left(1 + \frac{4}{4 \cdot 0.25}\right)
\]

\[= 23,000 \text{ lb/inch depth}\]
Forging - Ex. 1-16

or since the part is 36” deep:
F(both) = 759,960 lbs = 380 tons

\[ F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left( 1 + \frac{w}{4h} \right) \cdot w \cdot depth \]

F(all sticking) = 828,000 lbs = 414 tons

\[ F_{forging} = 1.15 \cdot \sigma_{flow} \cdot \left( 1 + \frac{\mu w}{2h} \right) \cdot w \cdot depth \]

F(all sliding) = 496,800 lbs = 225 tons

All sticking over-estimates actual value.
Forging – Effect of friction

- Effect of friction coefficient ($\mu$) – all sticking

<table>
<thead>
<tr>
<th>Friction coefficient</th>
<th>$F_{\text{max}}$ (lbf/in depth)</th>
<th>$x_k$</th>
<th>Stick/slide</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4600</td>
<td>2</td>
<td>slide</td>
</tr>
<tr>
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<td>11365</td>
<td>2</td>
<td>slide</td>
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<tr>
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<td>both</td>
</tr>
<tr>
<td>0.4</td>
<td>22868</td>
<td>0.070</td>
<td>both</td>
</tr>
<tr>
<td>0.5</td>
<td>23000</td>
<td>0</td>
<td>stick</td>
</tr>
</tbody>
</table>

- Friction is very important
Forging - Ex. 1-17

Forging force vs. stroke – all sticking

Forging force (lbf/in depth) vs. Stroke (in)

- mu=0
- mu=0.1
- mu=0.2
- mu=0.25
- mu=0.3
- mu=0.4
- mu=0.5

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Forging - Ex. 1-19
Maximum forging force vs. friction coefficient ($\mu$)
all sticking

![Graph showing maximum forging force vs. friction coefficient. The x-axis represents the friction coefficient ranging from 0 to 0.6, and the y-axis represents the max forging force in lbf/in (depth). The graph includes data points at various friction coefficients and corresponding max forging forces.](image-url)
Summary

• Slab analysis
  – frictionless
  – with friction
  – Rectangular
  – Cylindrical
• Strain hardening and rate effects
• Flash
• Redundant work