Forging Analysis - 2

ver. 1

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Overview

• Slab analysis
  – frictionless
  – with friction
  – Rectangular
  – Cylindrical
• Strain hardening and rate effects
• Flash
• Redundant work
Forging – cylindrical part sliding region
Equilibrium in r direction

\[ \sum dF_r = 0 = -\sigma_r \cdot h \cdot r \cdot d\theta - 2 \cdot \mu \cdot p \cdot r \cdot d\theta \cdot dr \]

\[ -2 \cdot \sigma_\theta \cdot h \cdot dr \cdot \frac{d\theta}{2} + (\sigma_r + d\sigma_r) \cdot (r + dr) \cdot h \cdot d\theta \]

N.B. \[ \sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2} \]

neglecting HOTs

\[ 2 \mu pr \cdot dr + h \sigma_\theta \cdot dr - h \sigma_r \cdot dr - hr \cdot d\sigma_r = 0 \]
Axisymmetric flow and yield

For axisymmetric flow

\[ \varepsilon_r = \frac{d}{r}; \quad \varepsilon_\theta = \frac{2\pi(r + dr) - 2\pi r}{2\pi r} = \frac{dr}{r} \]

\[ \varepsilon_r = \varepsilon_\theta; \quad \sigma_r = \sigma_\theta \]

By Tresca

\[ \sigma_r + p = \sigma_{flow} = 2k = 2\tau_{flow} \]

\[ d\sigma_r = -dp \]

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Stress in z direction

substituting

\[ 2 \mu \rho r \cdot dr + h \sigma_r \cdot dr - h \sigma_r \cdot dr + hr \cdot dp = 0 \]

or

\[ 2 \mu \rho r \cdot dr = -hr \cdot dp \]

rearranging

\[
\frac{dp}{p} = -\frac{2 \mu}{h} dr
\]
Forging pressure - sliding

\[ \int_{pr}^{2\tau_{\text{flow}}} \frac{dp}{p} = -\int_{r}^{R} \frac{2\mu}{h} \cdot dr \]

for \( r_k < r < R \)

\[ \frac{p_r}{2\tau_{\text{flow}}} = \exp \left[ \frac{2\mu}{h} (R - r) \right] \]
Average forging pressure – sliding

\[
\frac{p_{ave}}{2\tau_{\text{flow}}} = \frac{1}{\pi(R^2 - r_k^2)} \int_{r_k}^{R} \frac{p_r}{2\tau_{\text{flow}}} \cdot 2\pi r \cdot dr = \frac{2}{(R^2 - r_k^2)} \int_{r_k}^{R} \exp\left[\frac{2\mu}{h}(R - r)\right] rdr
\]

\[
\frac{p_{ave}}{2\tau_{\text{flow}}} = \frac{2}{(R^2 - r_k^2)} \left(\frac{h}{2\mu}\right)^2 \exp\left(\frac{2\mu R}{h}\right) \left\{ \exp\left(\frac{-2\mu r}{h}\right) \cdot \left(\frac{-2\mu r}{h} - 1\right) \right\} \bigg|_{r_k}^{R}
\]

\[
\frac{p_{ave}}{2\tau_{\text{flow}}} = \frac{2}{(R^2 - r_k^2)} \left(\frac{h}{2\mu}\right)^2 \exp\left(\frac{2\mu R}{h}\right) \left[ \exp\left(\frac{-2\mu R}{h}\right) \cdot \left(\frac{-2\mu R}{h} - 1\right) \right] - \left[ \exp\left(\frac{-2\mu r_k}{h}\right) \cdot \left(\frac{-2\mu r_k}{h} - 1\right) \right]
\]

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Average forging pressure – sliding

\[ \frac{p_{ave}}{2\tau_{flow}} = \frac{2}{(R^2 - r_k^2)} \left( \frac{h}{2\mu} \right)^2 \left[ \exp \left( \frac{2\mu(R - r_k)}{h} \right) \cdot \left( \frac{2\mu r_k}{h} + 1 \right) - \left( \frac{2\mu R}{h} \right) - 1 \right] \]
Forging force – sliding

\[ F_{\text{forging}} = p_{\text{ave}} \cdot A = p_{\text{ave}} \cdot \pi \cdot \left( R^2 - r_k^2 \right) \]

\[ F_{\text{forging}} = 4\tau_{\text{flow}} \cdot \left( \frac{h}{2\mu R} \right)^2 \left[ \exp\left( \frac{2\mu (R - r_k)}{h} \right) \cdot \left( \frac{2\mu r_k}{h} + 1 \right) - \left( \frac{2\mu R}{h} \right) - 1 \right] \cdot \pi \cdot \left( R^2 - r_k^2 \right) \]
Average forging pressure – all sliding approximation \((r_k = 0)\)

- Taking the first four terms of a Taylor’s series expansion for the exponential about 0 for \(|x| \leq 1\)

\[
\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}
\]

yields

\[
\frac{P_{ave}}{2\tau_{flow}} = \left[1 + \left(\frac{2\mu R}{3h}\right)\right]
\]

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Forging force – all sliding approximation

\[ F_{\text{forging}} = p_{\text{ave}} \cdot A = p_{\text{ave}} \cdot \pi \cdot R^2 \]

\[ F_{\text{forging}} = 2\tau_{\text{flow}} \cdot \left[ 1 + \left( \frac{2\mu R}{3h} \right) \right] \cdot \pi R^2 \]
Transition sticking / sliding

• Set $\tau_{\text{flow}} = \mu p$ and solve for $r_k$

\[
\frac{p}{2\tau_{\text{flow}}} = \exp\left[2\mu\left(\frac{R-r_k}{h}\right)\right] \quad \Rightarrow \quad \frac{p}{2\mu \cdot p} = \exp\left[2\mu\left(\frac{R-r_k}{h}\right)\right]
\]

\[
\ln\left(\frac{1}{2\mu}\right) = 2\mu\left(\frac{R-r_k}{h}\right) \quad \Rightarrow \quad r_k = R - \frac{h}{2\mu} \ln\left(\frac{1}{2\mu}\right)
\]
Forging pressure - sticking region

• Use the same method as for sliding
• Substitute $\mu p = \tau_{\text{flow}}$
• Assume Tresca yield criterion

$$2\mu pr \cdot dr = -hr \cdot dp$$

$$2\tau_{\text{flow}} r \cdot dr = -hr \cdot dp$$

$$dp = -\frac{2\tau_{\text{flow}}}{h} dr$$
Forging pressure - sticking region

\[
\int_{p_r}^{p_{r_k}} dp = -\int_{r_k}^{r} \frac{2\tau_{\text{flow}}}{h} dr
\]

\[
p_r - p_{r_k} = -\frac{2\tau_{\text{flow}}}{h} (r - r_k)
\]

\[
\frac{p_r - p_{r_k}}{2\tau_{\text{flow}}} = \frac{(r_k - r)}{h}
\]
Forging pressure - sticking region

$p_{r_k}$ determined from sliding equation

$$\frac{p_{r_k}}{2\tau_{\text{flow}}} = \exp\left[\frac{2\mu}{h}(R - r_k)\right]$$

for $0 < r < r_k$

$$\frac{p_r}{2\tau_{\text{flow}}} = \exp\left[\frac{2\mu}{h}(R - r_k)\right] + \frac{(r_k - r)}{h}$$
Average forging pressure - sticking

\[
\frac{p_{ave}}{2\tau_{flow}} = \frac{1}{\pi r_k^2} \int_{0}^{r_k} p_r \cdot 2\pi r \cdot dr = \frac{2}{r_k^2} \int_{0}^{r_k} \left( \exp \left[ \frac{2\mu}{h} (R - r_k) \right] + \frac{r_k - r}{h} \right) \cdot r dr
\]

\[
\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2} \int_{0}^{r_k} \left( r \cdot \exp \left[ \frac{2\mu}{h} (R - r_k) \right] + \frac{r_k \cdot r}{h} - \frac{1}{h} r^2 \right) \cdot dr
\]

\[
\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2} \left. \left( \frac{r^2}{2} \cdot \exp \left[ \frac{2\mu}{h} (R - r_k) \right] + \frac{r_k \cdot r^2}{2h} - \frac{r^3}{3h} \right) \right|_{0}^{r_k}
\]
Average forging pressure - sticking

\[
\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2} \left( \frac{r_k^2}{2} \cdot \exp \left[ \frac{2\mu}{h} \left( R - r_k \right) \right] + \frac{r_k^3}{2h} - \frac{r_k^3}{3h} \right)_{\kappa}
\]

\[
\frac{p_{ave}}{2\tau_{flow}} = \frac{2}{r_k^2} \left( \frac{r_k^2}{2} \cdot \exp \left[ \frac{2\mu}{h} \left( R - r_k \right) \right] + \frac{r_k^3}{2h} - \frac{r_k^3}{3h} \right)
\]

\[
\frac{p_{ave}}{2\tau_{flow}} = \exp \left[ \frac{2\mu}{h} \left( R - r_k \right) \right] + \frac{r_k}{3h}
\]
Forging force – sticking region

\[ F_{\text{forging}} = p_{\text{ave}} \cdot A = p_{\text{ave}} \cdot \pi \cdot r_k^2 \]

\[ F_{\text{forging}} = 2\tau_{\text{flow}} \cdot \left( \exp\left[ \frac{2\mu}{h} (R - r_k) \right] + \frac{r_k}{3h} \right) \cdot \pi \cdot r_k^2 \]
Sticking and sliding

• If you have both sticking and sliding, and you can’t approximate by one or the other,
• Then you need to include both in your pressure and average pressure calculations.

\[ F_{\text{forging}} = F_{\text{sliding}} + F_{\text{sticking}} \]

\[ F_{\text{forging}} = (p_{\text{ave}} \cdot A)_{\text{sliding}} + (p_{\text{ave}} \cdot A)_{\text{sticking}} \]
Strain hardening
(cold - below recrystallization point)

Tresca

\[ 2\tau_{\text{flow}} = Y = K\varepsilon^n \]
Strain rate effect
(hot – above recrystallization point)

\[ \dot{\varepsilon} = \frac{1}{h} \frac{dh}{dt} = \frac{v}{h} = \frac{\text{platen velocity}}{\text{instantaneous height}} \]

Tresca

\[ 2\tau_{\text{flow}} = Y = C(\dot{\varepsilon})^m \]
Flash for closed die forging (plane strain)

- Say we have a typical flash with thickness $h/20$ and length $w/4$
Average forging pressure

• in forging (Tresca)

\[ p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{\mu w}{2h}\right) \]

• in flash (Tresca)

\[ p_{ave} = 2\tau_{flow} \cdot \left(1 + \frac{5\mu w}{h}\right) \]
Flash

• Flash’s high deformation resistance results in filled mold
• Process wouldn’t work without friction
Deformation Work

In general, work done in bulk deformation processes has three components

Total work, \( W = W_{\text{ideal}} + W_{\text{friction}} + W_{\text{redundant}} \)

Work of ideal plastic deformation, \( W_{\text{ideal}} \)

\[
W_{\text{ideal}} = \left( \text{area under true stress-true strain curve} \right) \cdot \text{(volume)}
\]

\[
W_{\text{ideal}} = \left( \int_{0}^{\varepsilon_l} \sigma_t \, d\varepsilon_t \right) \cdot \text{(volume)}
\]

For a true stress-true strain curve:
\( \sigma_t = K \varepsilon_t^n \)

\( \bar{Y}_f = \text{Avg. flow stress} \)
Deformation Work

Friction between dies and workpiece causes inhomogeneous (non-uniform) deformation called barreling.
Deformation Work

Internal shearing of material requires redundant work to be expended
Redundant Zone
Closed/Impression Die Forging

• Analysis more complex due to large variation in strains in different parts of workpiece

• Approximate approaches
  – Divide forging into simple part shapes e.g. cylinders, slabs etc. that can be analyzed separately
  – Consider entire forging as a simplified shape
Closed/Impression Die Forging

Steps in latter analysis approach

- **Step 1**: calculate average height from volume $V$ and total projected area $A_t$ of part (including flash area)

  $$h_{avg} = \frac{V}{A_t} = \frac{V}{Lw}$$

- **Step 2**: $\varepsilon_{avg} = \text{avg. strain} = \ln\left(\frac{h_i}{h_{avg}}\right)$

  $\dot{\varepsilon}_{avg} = \text{avg. strain rate} = \frac{v}{h_{avg}}$
Closed/Impression Die Forging

• **Step 3**: calculate flow stress of material $Y_f$ for cold/hot working

• **Step 4**:  
  \[
  \text{Avg. forging load} = F_{avg} = K_p Y_f A_t
  \]

  $K_p$ = pressure multiplying factor  
  $= 3\sim5$ for simple shapes without flash  
  $= 5\sim8$ for simple shapes with flash  
  $= 8\sim12$ for complex shapes with flash
Other Analysis Methods

- Complex closed die forging simulated using finite element software

Source: http://nsmwww.eng.ohio-state.edu/html/f-flashlessforg.html

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Upper Bound Theorem

• Any estimate of the collapse load of a structure made by equating the internal rate of energy dissipation to the rate at which external forces do work in some assumed pattern of deformation will be $>\text{ or } =\text{ to the correct load.}$
Upper Bound Theorem Assumptions

• Isotropic and homogeneous
• Neglect strain hardening and strain rate
• Frictionless or constant shear stress condition exists at tool-work piece interface
• 2-D, plane strain with all deformation occurring by shear on a few planes. Elsewhere, material is rigid.
Upper Bound Theorem

\[ \frac{dW}{dt} = \sum_{i=1}^{n} kS_i V_i^* \]

- \( k = \) shear flow stress
- \( S_i = \) length of shear plane
- \( V_i^* = \) velocity of shear
Upper Bound Theorem

• Indentation of a plate (slip-line analysis)

\[ h = \infty \]
Work, shearing force

- Work is done by shearing along AB, BC, AC, and CD.
  - Lengths calculated from figure at right.
- Shearing force along any boundary, per unit length, \( w \), is \( k \) (shear yield stress) times the length of the boundary, \( L \).
Shearing velocities

\[ v_{BA} = v_o \sqrt{2} \]
\[ v_{BC} = v_o \]
\[ v_{CA} = v_o \sqrt{\frac{1}{2}} \]
\[ v_{DC} = v_o \sqrt{\frac{1}{2}} \]
Motions

This triangle moves down by $u$

This triangle moves up by $u/2$ and sideways by $u/2$

This triangle moves sideways by $u$
Total power delivered

\[ Lp v_o = 2k \left( \frac{L v_o \sqrt{2}}{\sqrt{2}} + L v_o + \frac{L v_o}{2} + \frac{L v_o}{2} \right) \]

- each term has been counted twice
  - due to symmetry
- Simplifying

\[ p = 6k \]
Total power delivered

\[ p = 6k \]

- using von Mises

\[ k = \frac{Y}{\sqrt{3}} = 0.577 \cdot Y \]

- hence

\[ p = \frac{6Y}{\sqrt{3}} = 3.46 \cdot Y \]
Exact solution

\[ p = 5.14 \quad k = 2.97 \quad Y \]

- Solution above
  \[ p = 6 \quad k = 3.46 \quad Y \]

- so we can see the effect of constraint
  - redundant work: higher pressure
Non-homogeneous deformation and Redundant work

• If the slab is thick or friction:
  – non-homogeneous deformation
  – redundant work

• If the slab is thin or unconstrained:
  (e.g., open die forging without friction)
  – no redundant work
Indenting at h/L = 1
Analysis - power delivered

\[ BC = CE = \frac{L}{\sqrt{2}} \]

\[ v_{BC} = v_{CE} = \sqrt{2} \times v_o \]

\[ \frac{2 p v_o L}{2} = 2v_o \times \sqrt{2} \times \frac{kL}{\sqrt{2}} \]

\[ p = 2k = 1.15 \text{ Y (plane strain result)} \]
Redundant work limit \((\Delta = \frac{h}{L})\)  
(plane strain)

- \(h/L < 1\): no redundant work  
  - \(p = 1.15 \, Y\)
- \(1 < h/L < 8.7\): some redundant work  
  - \(1.15 \, Y < p < 2.97 \, Y\)
- \(h/L > 8.7\): redundant work  
  - same as infinite plate  
  - \(p = 2.97 \, Y\)
Redundant work correction factor \((Q_r)\)

- Can be characterized by:
  \[ p = Q_r Y \]
  or
  \[ Q_r = \frac{p}{\sigma_y} = \frac{p}{2\tau_y} \text{ (by Tresca)} \]
- where \(Q_r\) = correction factor for redundant work
Redundant work factor
(Backofen) (frictionless)

\[ Q_r = \frac{p}{2\tau_y} \]

Fig. 7-1. The \( \Delta \)-dependence of yield pressure for the frictionless plane strain-indentation of a nonstrain-hardening material.

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Redundant work factor
(Kalpakjian) -
(friction)

FIGURE 6.9 Ratio of average die pressure to yield stress as a function of friction and aspect ratio of the specimen: (a) plane-strain compression; and (b) compression of a solid cylindrical specimen. Note that the yield stress in (b) is $Y$, and not $Y'$ as in plane-strain compression in (a).
Summary

• Slab analysis
  – frictionless
  – with friction
  – Rectangular
    – Cylindrical
• Strain hardening and rate effects
• Flash
• Redundant work