Deformation Processing - Rolling

ver. 1
Overview

- Process
- Equipment
- Products
- Mechanical Analysis
- Defects
Process
Process
Process

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Ring Rolling
Equipment
Equipment
Products
Products

- Shapes
  - I-beams, railroad tracks
- Sections
  - door frames, gutters
- Flat plates
- Rings
- Screws
Products

• A greater volume of metal is rolled than processed by any other means.
Rolling Analysis

• Objectives

  – Find distribution of roll pressure
  – Calculate roll separation force ("rolling force") and torque
  – Processing Limits
  – Calculate rolling power
Flat Rolling Analysis

• Consider rolling of a flat plate in a 2-high rolling mill

Width of plate $w$ is large $\rightarrow$ plane strain

$V_0$ $h_b$ $V_f (> V_0)$

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Flat Rolling Analysis

- Friction plays a critical role in enabling rolling: cannot roll without friction; for rolling to occur
  \[ \mu \geq \tan \alpha \]
- Reversal of frictional forces at neutral plane (NN)
Flat Rolling Analysis

Stresses on Slab in Entry Zone

Stresses on Slab in Exit Zone

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Equilibrium

- Applying equilibrium in x (top entry, bottom exit)

\[
(\sigma_x + d\sigma_x) \cdot (h + dh) - 2 pR \cdot d\phi \cdot \sin \phi \pm 2 \mu pR \cdot d\phi \cdot \cos \phi - \sigma_x h = 0
\]

Simplifying and ignoring HOTs

\[
\frac{d(\sigma_x h)}{d\phi} = 2 pR \cdot (\sin \phi \mp \mu \cos \phi)
\]
Simplifying

• Since $\alpha << 1$, then $\sin \phi = \phi$, $\cos \phi = 1$

$$\frac{d(\sigma_x h)}{d\phi} = 2pR \cdot (\phi \mp \mu)$$

• Plane strain, von Mises

$$p - \sigma_x = 1.15 \cdot Y_{flow} \equiv Y'_{flow}$$
Differentiating

- Substituting

\[
\frac{d}{d\phi} \left[ (p - Y'_{\text{flow}}) \cdot h \right] = 2pR \cdot (\phi \mp \mu)
\]

- or

\[
\frac{d}{d\phi} \left[ Y'_{\text{flow}} \cdot \left( \frac{p}{Y'_{\text{flow}}} - 1 \right) \cdot h \right] = 2pR \cdot (\phi \mp \mu)
\]
Differentiating

\[ Y_{flow}' \cdot h \cdot \frac{d}{d\phi} \left( \frac{p}{Y_{flow}'} \right) + \left( \frac{p}{Y_{flow}'} - 1 \right) \cdot \frac{d}{d\phi} \left( Y_{flow}' \cdot h \right) = 2pR \cdot (\phi + \mu) \]

Rearranging, the variation \( Y_{flow}' \cdot h \) with respect to \( \phi \) is small compared to the variation \( p/ Y_{flow}' \) with respect to \( \phi \) so the second term is ignored.

\[ \frac{d}{d\phi} \left( \frac{p}{Y_{flow}'} \right) = \frac{2R}{h} (\phi + \mu) \]
Thickness

\[ h = h_f + 2R \cdot (1 - \cos \phi) \]

from the definition of a circular segment

or, after using a Taylor’s series expansion, for small \( \phi \)

\[ \cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \cdots \]

\[ h = h_f + R \cdot \phi^2 \]
Substituting and integrating

\[
\int \frac{d\left( \frac{p}{Y'_{flow}} \right)}{p / Y'_{flow}} = \int \frac{2R}{h_f + R \cdot \phi^2} (\phi - \mu) \, d\phi
\]

\[
\ln \frac{p}{Y'_f} = \ln \frac{h}{R} + 2\mu \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right) + \ln C
\]
Eliminating $ln()$

\[ p = C \cdot Y'_{flow} \cdot \frac{h}{R} \exp(\mp \mu H) \]

\[ H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right) \]
Entry region

- at $\phi = \alpha$, $H = H_b$,

$$p = C \cdot Y'_\text{flow} \cdot \frac{h}{R} \exp(-\mu H)$$

$$C = \frac{R}{h_b} \exp(\mu H_b) \quad p = Y'_\text{flow} \frac{h}{h_b} \exp(\mu[H_b - H])$$

$$p = (Y'_\text{flow} - \sigma_{xb}) \frac{h}{h_b} \exp(\mu[H_b - H]) \quad \text{With back tension} = (Y'_\text{flow} - \sigma_{xb})$$

$$H_b = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \frac{\alpha}{\sqrt{\frac{R}{h_f}}} \right)$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \frac{\phi}{\sqrt{\frac{R}{h_f}}} \right)$$
Exit region

at $\phi = 0$, $H = H_f = 0$,

$$C = \frac{R}{h_f}$$
$$p = \left( Y_{flow} \right) \frac{h}{h_f} \exp(\mu H)$$

$$p = \left( Y_{flow}' - \sigma_{xf} \right) \frac{h}{h_f} \exp(\mu H) \quad \text{With forward tension}$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$

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Effect of back and front tension

\[ \bar{Y} \]
\[ \bar{Y} - \sigma_{xb} \]
\[ \bar{Y} - \sigma_{xf} \]

maximum pressure

distance
Flat Rolling Analysis Results – without front and back tension

**Stresses on Slab in Entry Zone**

\[
p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_0} e^{\mu(H_0 - H)}
\]

\[
H = 2\sqrt{\frac{R}{h_f}} \tan^{-1}\left(\sqrt{\frac{R}{h_f}} \phi\right)
\]

**Entry Zone**

\[
H_0 = H \oplus \phi = \alpha
\]

**Stresses on Slab in Exit Zone**

\[
p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_f} e^{\mu H}
\]

**Exit Zone**
Average rolling pressure – per unit width

\[ p_{ave,entry} = -\frac{1}{R(\alpha - \phi_n)} \int_{\alpha}^{\phi_n} p_{entry} R d\phi; \quad p_{ave,exit} = \frac{1}{R\phi_n} \int_{0}^{\phi_n} p_{exit} R d\phi \]
Rolling force

\[ F = p_{\text{ave,entry}} \times \text{Area}_{\text{entry}} + p_{\text{ave,exit}} \times \text{Area}_{\text{exit}} \]
Force

• An alternative method

\[ F = \int_{\phi_n}^{\alpha} w \cdot p_{\text{entry}} \cdot R \cdot d\phi + \int_{0}^{\phi_n} w \cdot p_{\text{exit}} \cdot R \cdot d\phi \]

• again, very difficult to do.
Force - approximation

\[ F / \text{roller} = L \, w \, p_{\text{ave}} \]

\[ L \approx \sqrt{R \Delta h} \]

\[ \Delta h = h_b - h_f \]

\[ p_{\text{ave}} = f \left( \frac{h_{\text{ave}}}{L} \right) \]
Derivation of “L”

circular segment

\[ h = h_f + 2R \cdot (1 - \cos \phi) \]

Taylor’s expansion

\[ \cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \cdots \]

\[ h = h_f + R \cdot \phi^2 \]

\[ R \cdot \phi = L \]
Derivation of “L”

setting $h = h_b$ at $\phi = \alpha$, substituting, and rearranging

$$h_b - h_f = \Delta h = R \cdot \left( \frac{L}{R} \right)^2$$

or

$$L = \sqrt{R \cdot \Delta h}$$
Approximation based on forging plane strain – von Mises

\[ p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right) \]

average flow stress: due to shape of element
Small rolls or small reductions

\[ \Delta = \frac{h_{ave}}{L} \gg 1 \]

- friction is not significant \( (\mu \to 0) \)

\[ p_{ave} = 1.15 \cdot \overline{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right) \]

\[ p_{ave} = 1.15 \cdot \overline{Y}_{flow} \]
Large rolls or large reductions

\[ \Delta \equiv \frac{h_{\text{ave}}}{L} \ll 1 \]

- Friction is significant (forging approximation)

\[ P_{\text{ave}} = 1.15 \cdot \bar{Y}_{\text{flow}} \left( 1 + \frac{\mu L}{2h_{\text{ave}}} \right) \]
Force approximation: low friction

\[ \Delta \equiv \frac{h_{ave}}{L} \gg 1 \]

\[ \frac{F}{\text{roller}} = 1.15 \cdot Lw\bar{Y}_{\text{flow}} \]
Force approximation: high friction

$$\Delta \equiv \frac{h_{ave}}{L} \ll 1$$

$$\frac{F}{F_{roller}} = 1.15 \cdot L w \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$
Zero slip (neutral) point

• Entrance: material is pulled into the nip
  – roller is moving faster than material
• Exit: material is pulled back into nip
  – roller is moving slower than material
System equilibrium

- Frictional forces between roller and material must be in balance.
  - or material will be torn apart
- Hence, the zero point must be where the two pressure equations are equal.

\[
\frac{h_b}{h_f} = \frac{\exp(\mu H_b)}{\exp(2\mu H_n)} = \exp(\mu(H_b - 2H_n))
\]
Neutral point

\[ H_n = \frac{1}{2} \left( H_b - \frac{1}{\mu} \ln \frac{h_b}{h_f} \right) \]

\[ \phi_n = \sqrt{\frac{h_f}{R}} \tan \left( \frac{H_n \sqrt{\frac{h_f}{R}}}{2} \right) \]
Torque

\[ L \approx \sqrt{R\Delta h} \]

\[ \Delta h = h_b - h_f \]

\[ \sum F_y = 0 \]

\[ \therefore F_{roller} = p_{ave}A \]

\[ T = \int_{\phi_n}^{\alpha} w\mu pR^2 d\phi - \int_{0}^{\phi_n} w\mu pR^2 d\phi \]

\[ Torque / roller = r \cdot F_{roller} = \frac{L}{2} \cdot F_{roller} = \frac{F_{roller}L}{2} \]

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Power

Power / roller = Tω = F_{roller}Lω / 2

ω = 2πN
N = [rev/min]
Processing limits

- The material will be drawn into the nip if the horizontal component of the friction force ($F_f$) is larger, or at least equal to the opposing horizontal component of the normal force ($F_n$).

\[ F_f \cos \alpha \geq F_n \sin \alpha \]

\[ F_f = \mu \cdot F_n \]

\[ \tan \alpha = \mu \]

\[ \mu = \text{friction coefficient} \]
Processing limits

Also

\[\cos \alpha = \frac{R - \frac{\Delta h}{2}}{R} = 1 - \frac{\Delta h}{2R}\]

and \(\Delta h << R\)

\[\sin \alpha = \sqrt{1 - \cos^2 \alpha}\]

\[\sin \alpha = \sqrt{1 - \frac{\Delta h}{2R} - \left(\frac{\Delta h}{2R}\right)^2} \approx \sqrt{\frac{\Delta h}{R}}\]

\[\tan \alpha = \sqrt{\frac{\Delta h}{R - \Delta h}} \approx \sqrt{\frac{\Delta h}{R}}\]
Processing limits

So, approximately

\[(\tan \alpha)^2 = \mu^2 = \frac{\Delta h}{R}\]

Hence, maximum draft

\[\Delta h_{\text{max}} = \mu^2 R\]

Maximum angle of acceptance

\[\phi_{\text{max}} = \alpha = \tan^{-1} \mu\]
Max. reduction in thickness

\[
(\Delta h)_{\text{max}} = \mu^2 R
\]

Max. angle of acceptance

\[
\phi_{\text{max}} = \alpha = \tan^{-1} \frac{1}{\mu}
\]
Cold rolling
(below recrystallization point)
strain hardening, plane strain – von Mises

\[ 2\tau_{\text{flow}} = 1.15 \cdot \bar{Y}_{\text{flow}} = 1.15 \cdot \frac{K \varepsilon^n}{n + 1} \]

average flow stress:
due to shape of element
Hot rolling – (above recrystallization point) strain rate effect, plane strain - von Mises

- Average strain rate

\[ \frac{\dot{\varepsilon}}{\varepsilon} = \frac{V}{t} = \frac{V_R}{L} \ln \left( \frac{h_b}{h_f} \right) \]

- Average flow stress:

\[ 2\tau_{flow} = 1.15 \cdot \bar{Y}_{flow} = 1.15 \cdot C \cdot \dot{\varepsilon}^m \]

average flow stress: due to shape of element
Example 1.1

- Cold roll a 5% Sn-bronze
- Calculate force on roller
- Calculate power
- Plot pressure in nip (no back or forward tension)
Example 1.2

- $w = 10\ \text{mm}$
- $h_b = 2\ \text{mm}$
- height reduction = 30% ($h_f = 0.7\ h_b$)
  - $h_f = 1.4\ \text{mm}$
- $R = 75\ \text{mm}$
- $v_R = 0.8\ \text{m/s}$
- mineral oil lubricant ($\mu = 0.1$)
- $K = 720\ \text{MPa},\ n = 0.46$
Example 1.3

• Maximum draft:

\[ \Delta h_{\text{max}} = \mu^2 R \]
\[ = (0.1)^2 \cdot 75 = 0.75 \text{ mm} \]

\[ \Delta h_{\text{actual}} = h_b - h_f = 2 - 1.4 \]
\[ = 0.6 \text{ mm} \]
Example 1.4

- Maximum angle of acceptance

\[ \phi_{\text{max}} = \tan^{-1} \mu = \tan^{-1}(0.1) = 0.1 \text{ radian} \]

\[ \alpha = \sqrt{\frac{(h_b - h_f)}{R}} = \sqrt{\frac{(2 - 1.4)}{75}} \]

\[ = 0.089 \text{ rad} = 5.12^\circ \]
Example 1.5

- Roller force: \( F = L \cdot w \cdot p_{ave} \)
- \( L = (R \Delta h)^{0.5} = [75 \times (2-1.4)]^{0.5} \)
  \[ = 6.7 \text{ mm} \]
- \( w = 10 \text{ mm} \)
- \( h_{ave} = (h_b + h_f) / 2 = 1.7 \text{ mm} \)
  \( h_{ave} / L = 1.7 / 6.7 = 0.25 < 1 \)

\[ \therefore \text{ friction is important} \]

\[ F_{roller} = 1.15 \cdot L \cdot w \cdot Y_{flow} \left(1 + \frac{\mu L}{2h_{ave}}\right) \]
Example 1.6

\[ \varepsilon_f = \left| \ln \left( \frac{h_f}{h_b} \right) \right| = \left| \ln \left( \frac{1.4}{2} \right) \right| = 0.36 \]

\[ 2\tau_{flow} = 1.15 \cdot \bar{Y} = 1.15 \cdot \frac{K\varepsilon_f^n}{n+1} \]

\[ = 1.15 \cdot \frac{720 \cdot (0.36)^{0.46}}{1.46} = 354 \text{ MPa} \]
Example 1.7

\[
\frac{F}{\text{roller}} = 1.15 \cdot L \bar{w} \bar{Y}_{\text{flow}} \left(1 + \frac{\mu L}{2h_{\text{ave}}} \right) \\
= 6.7 \times 10^{-3} \cdot 10 \times 10^{-3} \cdot 354 \times 10^6 \\
\times \left(1 + \frac{0.1 \times 6.7}{2 \times 1.7} \right) \\
= 28,392 \ N = 3.2 \ tons
\]
Example 1.8

\[
\text{Power (kW) / roller} = T \times \omega = \frac{F \cdot L \cdot V_R}{2 \cdot R}
\]

\[
\text{Power (kW) / roll} = \frac{28,392 \cdot 6.7 \times 10^{-3} \cdot 0.8}{2 \cdot 0.075}

= 1.01 kW / roll = 1.35 hp
\]
Example 1.9

- Entrance

\[ p = \left( Y_{flow} - \sigma_{xb} \right) \frac{h}{h_b} \exp(\mu(H_b - H)) \]

- Exit

\[ p = \left( Y_{flow} - \sigma_{xf} \right) h \frac{1}{h_f} \exp(\mu(H)) \]
Example 1.10

\[ \phi = \sqrt{\frac{R}{h - h_f}} \]

\[ H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1}\left( \phi \sqrt{\frac{R}{h_f}} \right) \]
Example 1.11

Friction hill

Pressure / 2Tflow

Exit

Entrance

φ / φ_max
Rolling

Shear stress

Normal Stress

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EFFECT OF FINISH HOT-ROLLING ON THE STRIP SHAPE AND THE AUSTENITE GRAIN STRUCTURE.

Initial Grain Shape. Equixed, Strain-Free

Intermediate Grain Shape. Pancaked, Highly Deformed

Final Grain Shape. Equixed, Strain-Free
Widening of material

Side view

Top view
Residual stresses - due to frictional constraints

a) small rolls or small reduction (ignore friction)
b) large rolls or large reduction (include friction)
Defects

• a) wavy edges
  – roll deflection
• b) zipper cracks
  – low ductility
• c) edge cracks
  – barreling
• d) alligatoting
  – piping, inhomogeneity

(a)  
(b)  
(c)  
(d)  

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Roll deflection

Rolls can deflect under load

Rolls can be crowned
Roll deflection

Rolls can be stacked for stiffness
Method to reduce roll deflection
Summary

• Process
• Equipment
• Products
• Mechanical Analysis
• Defects