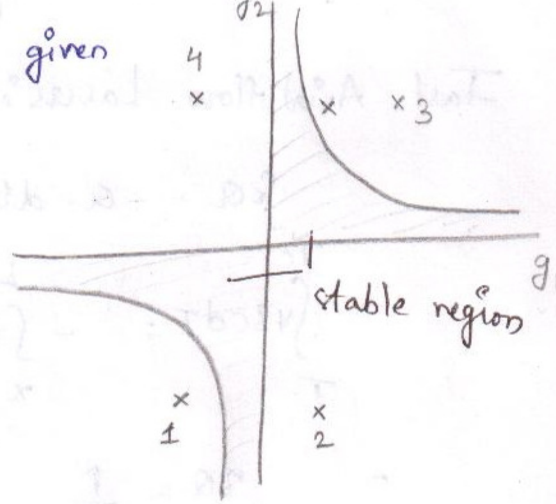
ME-677, Laser Surface Texturing, Homework-01

roll no: 08010049 Name:- Anusha Kolli

(2) 1, 2, 3, 4, 5 in the same order as given in the question



shaded region - stability region

(3) Slow flow lasers:-

Let Q be the rate of volumetric heat flow

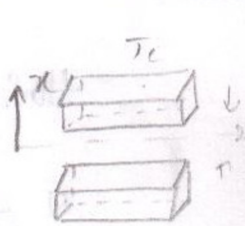
$\Rightarrow$ at steady state heat generated per unit length = heat removed per unit length

$$\Rightarrow Q\pi r^2 = -2\pi r k \frac{dT}{dr} \Rightarrow \int_T^{T_c} dT = \int_r^a -\frac{Qr}{2k}\,dr$$

$$T_{max}\,(r=0) = \frac{Qa^2}{4k} + T_c \qquad \eta \rightarrow \text{coupling factor}$$

$$\eta Q = \frac{P}{\pi a^2 L} \Rightarrow \boxed{P = \eta Q \pi a^2 L = 4\pi\eta k L\,(T_{max} - T_c)}$$

Slow flow wave guide cooling:-



lly to above method, $Q\cancel{A}x = -2k\cancel{A}\frac{dT}{dx}$

$$\Rightarrow T_{axis}\,(x=0) = \frac{Qg^2}{4k} + T_c$$

$$\eta Q = \frac{P}{A \times 2g} \Rightarrow \boxed{P = \frac{8A\eta k}{g}(T_{max} - T_c)}$$

Fast Axial flow Lasers:-

$$\delta Q = -Q \cdot dt$$

$$\int_T^{T_c} vSc\,dT = -\int_x^L Q\,dx$$

Q - rate of heat generation per unit volume

v - velocity

C - Heat Capacity

$$\Rightarrow \eta Q = \frac{P}{AL}$$

$$\Rightarrow \boxed{P = \eta g A v c (T_{max} - T_c)}$$

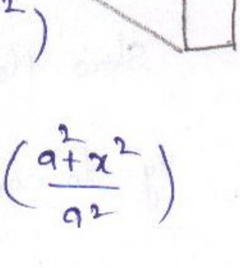
1. (a) $N(r) = N_0 e^{\left(-\frac{2r^2}{r_i^2}\right)}$ x & y vary from -a to a.

r_i - radius of inscribed cycle = a

$$r = \sqrt{x^2 + y^2} \Rightarrow N(r) = N_0 e^{-\frac{2(x^2+y^2)}{a^2}}$$

along the centre x & y remain constant

$\Rightarrow$ N(x, y) remains constant.



Along edges :-

Vertical edges :- $N(x,y) = N_0 e^{-2\left(\frac{a^2+y^2}{a^2}\right)}$

Horizontal edges :- $N(x,y) = N_0 e^{-2\left(\frac{a^2+x^2}{a^2}\right)}$

N - Areal density - molecules / cm^2

at centre $N(0,0) = N_0$

(a) Fractions of photons absorbed $= \dfrac{\text{Opaque area}}{\text{Total area}}$

Opaque area [annulus] $= \sigma N(r) \cdot dA \times dz$ $\qquad$ area of circle ↓

$$= \sigma N_0 e^{-\frac{2r^2}{r_i^2}} \cdot 2\pi r\,dr \cdot dz \qquad \text{Total area} = a^2 \times \pi$$

$$\Rightarrow \text{Fractions of photons absorbed} = \frac{\int_0^{r_i} \sigma N_0 e^{-\frac{2r^2}{r_i^2}} 2\pi r\,dr\,dz}{\pi a^2}$$

$$\frac{dI}{I} = -\frac{\sigma N_0 (e^2-1)\pi r_i^2\,dz}{2e^2 \times \pi a^2}$$

$$I(z) = I_0 \cdot e^{-[\sigma N_0 (e^2-1) r_i^2 / 2e^2 a^2]\,z}$$

where I_0 is $I(z)$ at $z=0$, $r_i \to$ radius of inscribed circle

(b) proceeding in a similar manner

$$\frac{dI}{I} = \frac{\int_{-b}^{b}\int_{-a}^{a} \sigma N_0 e^{-\frac{2y^2}{b^2}}\,dx\,dy\,dz}{\text{Total area}}$$

Since x & y vary from $-a$ to a $\quad dx = 2a$

Total area (area of rectangle) $= 2a \times 2b = 4ab$

$$\Rightarrow \frac{dI}{I} = \frac{2a}{4ab}\left[\int_{-b}^{b} \sigma N_0 e^{-2y^2/b^2}\,dy\right] dz$$

$$\sim \cancel{b}\; \underbrace{\frac{\sigma N_0}{2}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}(\sqrt{2})}_{-K \text{ (let)}}\,dz \quad -①$$

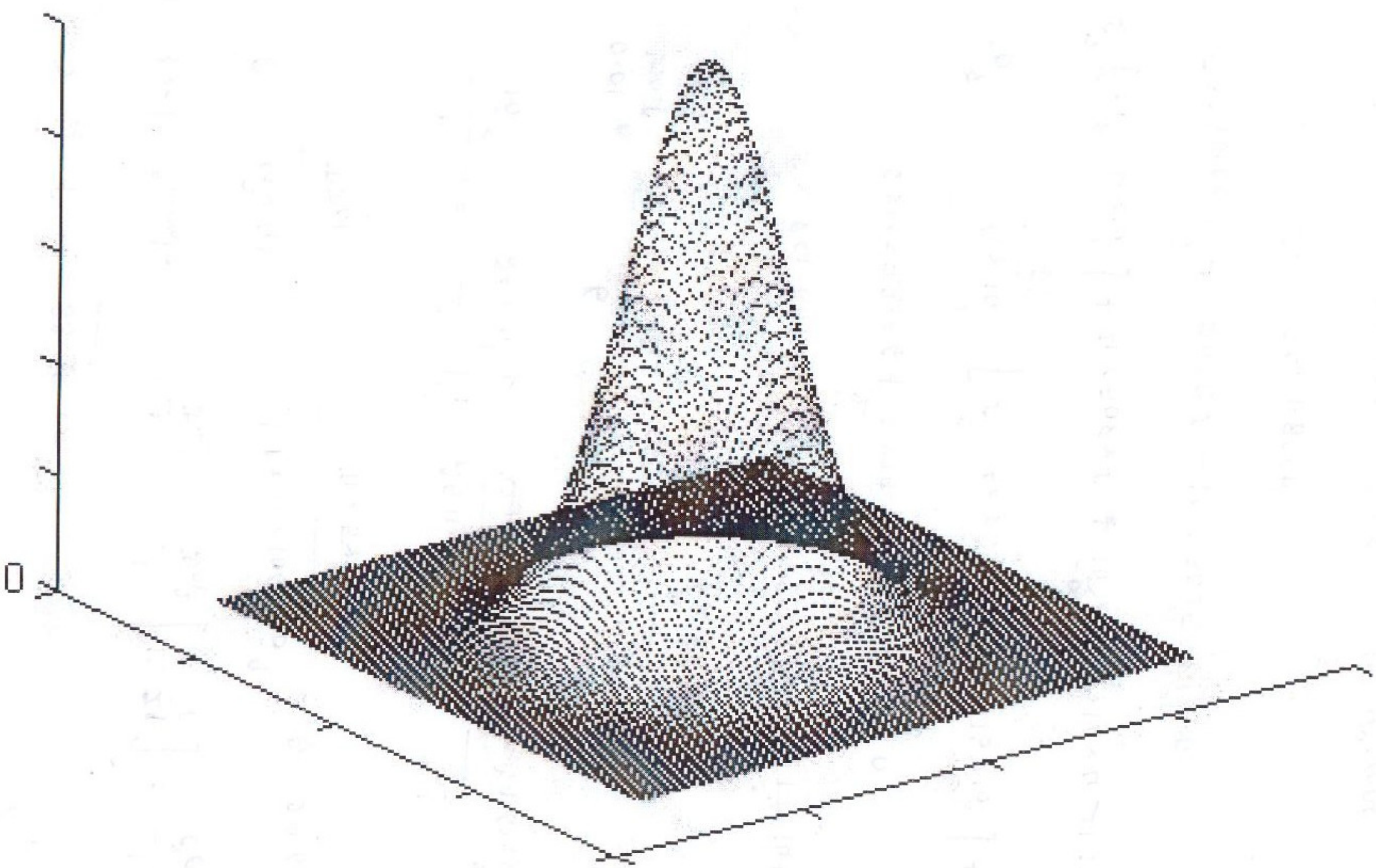
$$\frac{dI}{I} = -KZ$$

$$\Rightarrow \ln(I)\Big|_{I_0}^{I} = -KZ\Big|_{0}^{Z}$$

$$\Rightarrow \ln\left(\frac{I}{I_0}\right) = -KZ$$

$$\Rightarrow I_0 = e^{-KZ}$$ where K given by ①

1 (a) Gaussian Intensity $N = e^{-2(x^2+y^2)}$ all the constants are taken as unity



4 (a) using approximate formula:- $M = 1.1$

$$D_{02} = \frac{4f\lambda M^2}{\pi D_{01}} \Rightarrow 100\times\cancel{10^{-6}} = \frac{4\times \cancel{9}1060\times\cancel{10^{-9}}\times f\times 1.1}{\pi\times 5\times\cancel{10^{-3}}}$$

$$\Rightarrow \quad f = \underline{33.68 \text{ cm}}$$

using Exact formula:- $\frac{1}{D_{02}^2} = \frac{1}{D_{01}^2}\left[1-\frac{Z_1}{f}\right]^2 + \frac{1}{f^2\theta_1^2}$

$$\theta = \frac{M^2\times 4\lambda}{\pi D_{01}} = \frac{1.1\times 4\times 1060\times 10^{-9}}{\pi\times 5\times 10^{-3}} \Rightarrow \theta = 29.692\times 10^{-5}$$

$$\Rightarrow \frac{1}{10^{-8}} = \frac{1}{25\times 10^{-6}}\left[1-\frac{25.4\times 10^{-3}}{\cancel{33.68\times 10^{-2}}\, f}\right]^2 + \frac{1}{f^2\times(29.692)^2\times 10^{-10}}$$

$$\Rightarrow \cancel{\frac{0.01}{1000} = \frac{4\times 10^{-6}}{f^2}[f-0.0254]^2 + \frac{1}{f^2\, 881.61}}$$

$$\Rightarrow \cancel{f^2\times 881.61\times 10^4 = 1 + 3526.44f^2 - 179.14f + 2.275}$$

$$\Rightarrow \cancel{8812573.56f^2 + 179.14f - 3.275 = 0}$$

$$10^8 = \frac{4\times 10^4}{f^2}\left[f^2 - 50.8\times 10^{-3}f + 645.16\times 10^{-6}\right] + \frac{1.1343\times 10^7}{f^2}$$

$$\Rightarrow f^2\left[10^8 - 4\times 10^4\right] + 4\times 508\times f - 10^{-2}\times 645.16\times 4 - 1.1343\times 10^7 = 0$$

$$99960000f^2 + 2032f - 11343025.81 = 0.$$

$$\Rightarrow \quad f = 33.685\text{cm}.$$

$$\Rightarrow \quad \text{Error} = 33.685 - 33.68 = \underline{50 \text{ microns}}$$

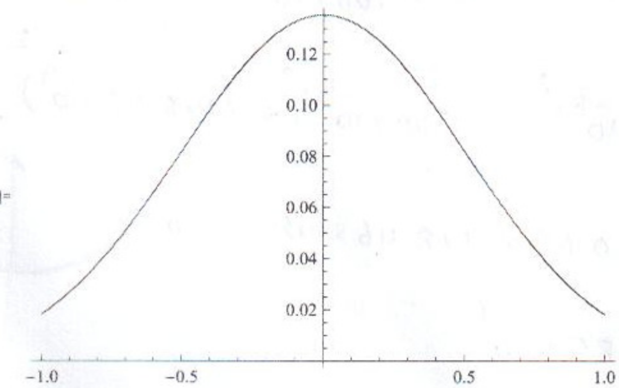
$N(x, y)$ variation along one of x/y edges (Symmetric w.r.t $x \& y$

1 (a)

08010049

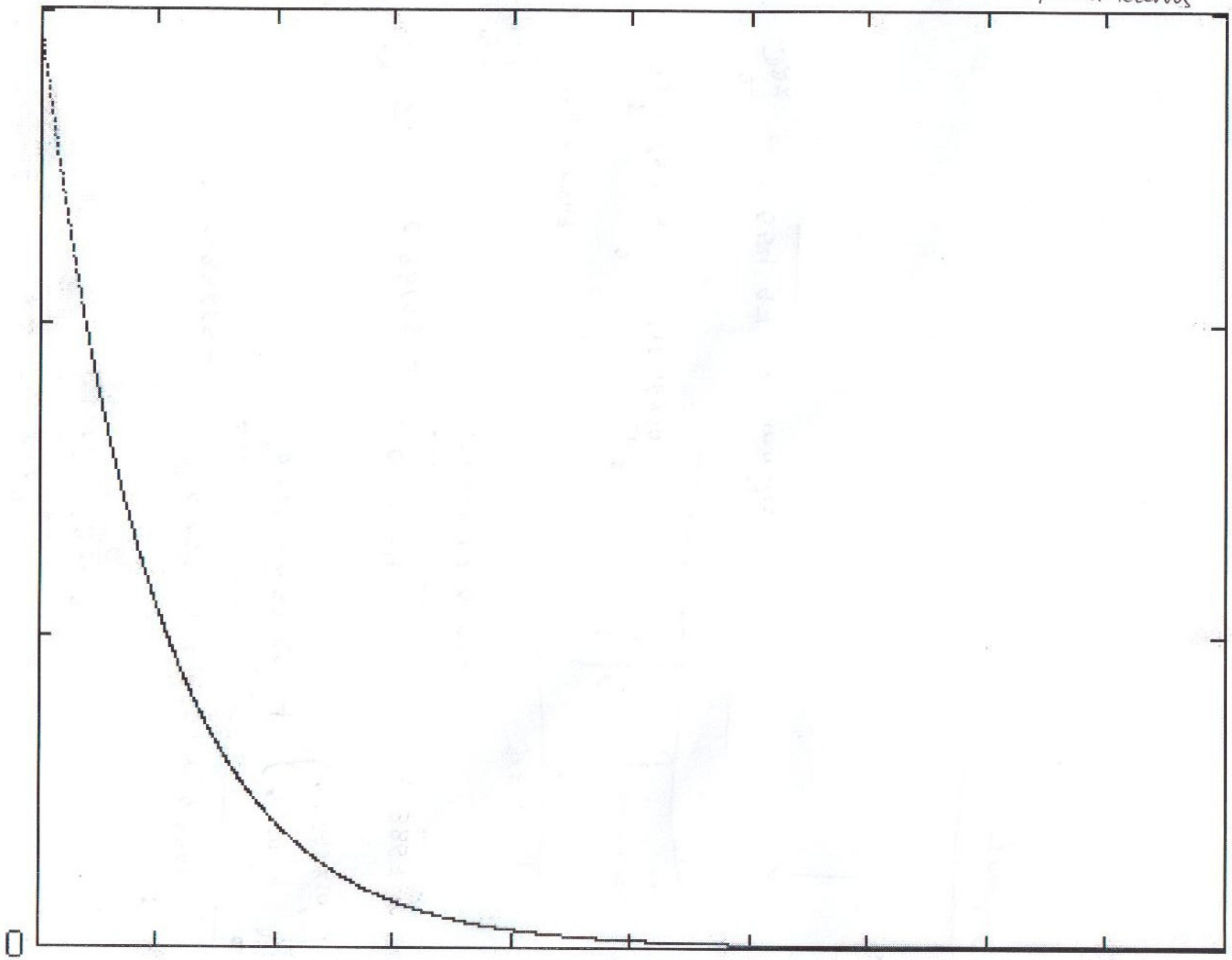
```
In[1]:= Plot[Exp[-2 * (1 + y * y)], {y, -1, 1}]
```

Out[1]=



(both for Gaussian & parabolic distribution $I(z) = e^{-z}$ along centre (since all other constants taken as unity /coefficients

(a)



Same for parabolic distribution along horizontal edge (all other overall constants are taken as unity

4 (b) $D_{2z}^2 = D_{02}^2 + z^2\theta^2$ $\quad \theta = \frac{4\lambda M^2}{\pi D_{02}} = 148.46\times10^{-4}$

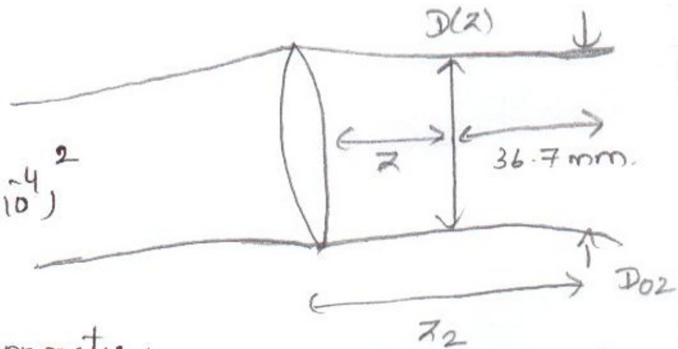
we need to shilat the origin to find actual z.

$$z_2 = f + \frac{(z_1 - f)f^2}{(z_1-f)^2 + \left(\frac{D_{01}}{\theta_1}\right)^2}$$

$$= 0.33685 + \frac{(0.0254 - 0.33685)\times(0.33685)^2}{(0.0254-0.33685)^2 * \left(\frac{5\times10^{-3}}{29.692\times10^{-5}}\right)^2}$$

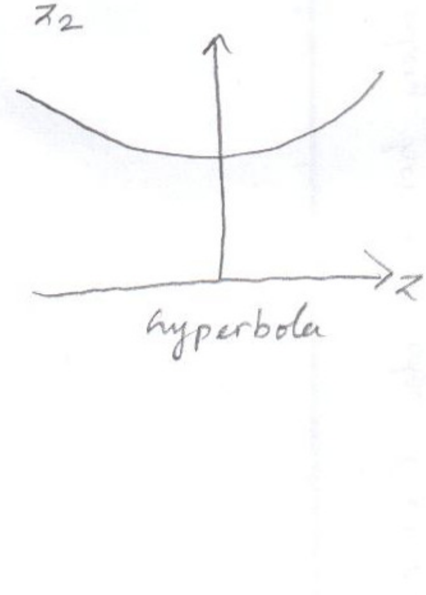
$$\Rightarrow z_2 = 0.33685 - \frac{0.03534}{283.57 + 0.097} = 0.3367 \text{ m}$$

$D_{2z}^2 \Big|$ at $z = 0.0367$



$$= 10^{-8} + (0.0367)^2 \times (148.46\times10^{-4})^2$$

$\Rightarrow D_{2z}^2 = 554.97$ micrometer



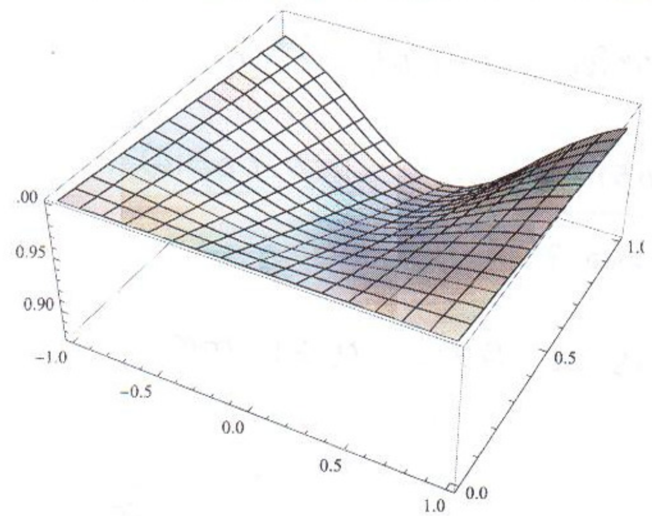
hyperbola

$\Phi(x, y, z)$ variation along one of x/y edges (symmetric w.r.t. y)

all constants are taken as unity 08010049

1) (a), (b)

```
Plot3D[Exp[-z * Exp[-2 * (1 + y * y)]], {y, -1, 1}, {z, 0, 1}]
```



(4) (c) Depth of focus is defined as depth for which the focal spot size changes by ±5%.

$$D_{2z}^2 = 10^{-8} + z^2 \times 10^{-8} \times 22040.3716$$

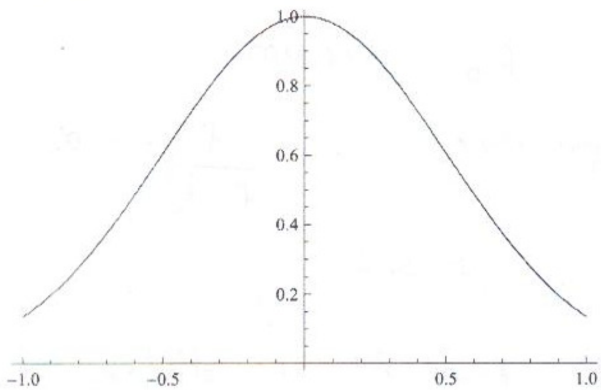
for +5% $D_{2z} = 1.05 D_2 = 1.05 \times 10^{-4}$ m.

$$\Rightarrow \quad z^2 = \frac{(1.05)^2 - 1}{22040.3716} = (0.00216)^2 \text{ m}$$

$\Rightarrow$ Depth of focus $= 2z = 4.32$ mm.

1. (b) $N(x,y)$ variation along vertical edge.

```
Plot[Exp[-2 * (y * y)], {y, -1, 1}]
```

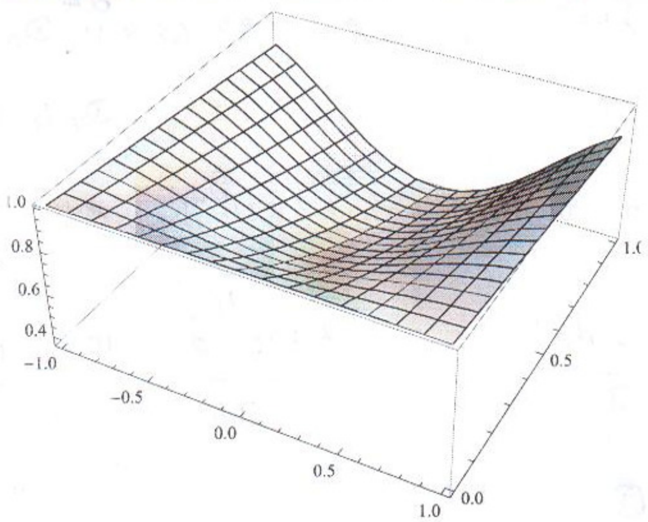


along horizontal edge N is constant

1. (b) $I(x, y, z)$ variation along vertical edge.

```
In[8]:= Plot3D[Exp[-z * Exp[-2 * (y * y)]], {y, -1, 1}, {z, 0, 1}]
```

Out[8]=



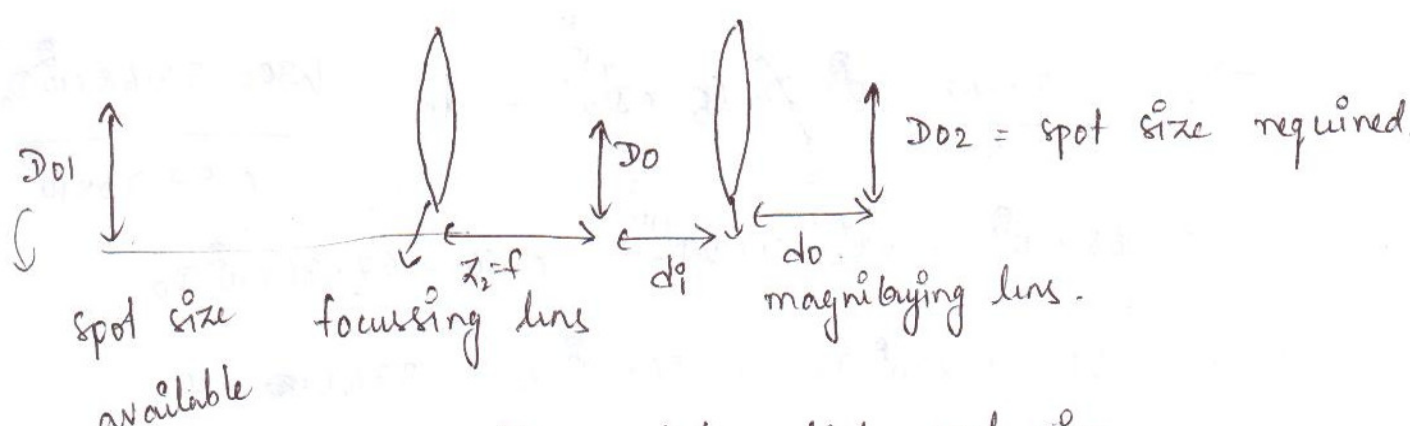
(4) (d) from part (a) taking all the parameters

$f \propto D_{02}$ $D_{02} : 500 \times 10^{-6}$ m (required)

$\Rightarrow f = 33.68 \times 5 = 168.4$ cm $= \underline{1.684 \text{ m}}$

but the space available is only 300 mm.

So we use one focussing lens and one magnifying lens.



$D_0 \rightarrow$ intermediate spot size.

assuming z_2 to approximately equal to focal length we want $f + d_i + d_o$ to be at max 300 mm.

$$\frac{d_i}{d_o} = \frac{D_{o2}}{D_o} \quad \text{— I}$$

$$D_{o2} = \frac{4f\lambda M^2}{\pi D_{o1}} \Rightarrow f = 33.68 \times 10^{2} D_o \text{ m}.$$

D_o in m.

$$\Rightarrow \quad d_i + d_o = (0.300 - 33.68 \times 10^{2} D_o) \text{ m.} \quad \text{— ①}$$

$$\text{I} \Rightarrow d_i = \frac{D_{o2}\, d_o}{D_o} = \frac{5\times 10^{-4}\, d_o}{D_o} \text{ m} = \frac{0.5\, d_o}{D_o} \text{ mm}.$$

Substitute d_i in ①

$$\Rightarrow \frac{10^{-3} \times 0.5\, d_o}{D_o} + d_o = 0.300 - 33.68 \times 10^{2} D_o$$

$$\Rightarrow (0.5 + D_o \times 10^{3})\, d_o = 0.300 D_o - 33.68 \times 10^{2} D_o^2 \quad \text{— ②}$$

So d_o & D_o should be such that they satisfy ② & $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

using same focal length

$$\frac{1}{f} = \frac{1}{33.68 \times 10^{2} D_o} = \frac{1}{d_o} - \frac{1}{d_i} = \frac{1}{d_o} - \frac{D_o \times 10^{4}}{5 d_o}$$

$$\Rightarrow \frac{33.68 \times 10^{2}}{5} \times D_o (5 - 2D_o \times 10^{4}) = d_o = \frac{(0.300 - 33.68 \times 10^{2} D_o) \times D_o}{0.5 + D_o \times 10^{3}}$$

$$\Rightarrow 33.68 \times 10^{2} (1 - 2D_o \times 10^{3})(1 + 2D_o \times 10^{3}) = 0.600 - 67.36 \times 10^{2} D_o$$

$$\Rightarrow 134.72 \times 10^{8} D_o^2 - 67.36 \times 10^{2} D_o - 33674 = 0.$$

$$\Rightarrow D_o = 500.2 \times 10^{-5} \text{ m}.$$

$f = 16.9$ cm $\quad d_o = 2.38$ cm $\quad d_i = 10.72$ cm

3) Comparing using Typical numbers:-

(3) Slow flow lasers:-

Comparing for fixed temperature rise

$$\frac{P}{L(T_{max}-T_c)} = 4\pi\eta K$$

Let the material be Ti alloy whose $K \sim 6$ W/mK

Let $\eta = 0.5$

$\Rightarrow$ Power per unit length and unit temperature rise

$$= 4\pi \times 0.5 \times 6 = \underline{37.7} \ \frac{W}{mK}$$

Slow flow wave guide:-

$$\frac{P}{T_{max}-T_c} = \frac{8A\eta K}{g}$$

Let $A = l \times b$ and b be of the order of $2g$.

(~~twice~~ same as the gap between the plates

$$\Rightarrow \frac{P}{L(T_{max}-T_c)} \sim 8 \times 2 \times \eta \times K = 48 \ \frac{W}{mK}$$

Fast Axial blow :

taking $b = 2g$ ~~mas~~ for slow wave guide lasers

typical velocity of laser $v \sim 5$ mm/s

& taking same Ti alloy $\rho = 4510\ kg/m^3$ & $C = 526.3\ J/kgK$

$$\Rightarrow \left(\frac{P}{l(T_{max}-T_{min})}\right) = \eta \rho \times 2g \times vC$$

let $2g \sim 1$ cm

$$= 0.5 \times 4510 \times 0.01 \times 0.005 \times 526.3$$

$$= 59.3\ W/mK$$

$\Rightarrow$ For given material & parameters, per temperature rise power per unit length of

Fast Axial blow laser > slow blow wave guide lasers > slow blow rod lasers.

5) $I(x,y) = \frac{2P}{A}\left(1-\frac{y^2}{b^2}\right)\left(1-\frac{x^2}{a^2}\right)$

$$\int_{-a}^{a}\left[\int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}} \frac{2P}{A}\left(1-\frac{y^2}{b^2}\right) dy\right] \times \left(1-\frac{x^2}{a^2}\right) dx$$

[using Mathematica]

$$= \int_{-a}^{a}\left[2b\left(1-\frac{x^2}{a^2}\right)^{0.5} - \frac{2b}{3}\left(1-\frac{x^2}{a^2}\right)^{1.5}\right]\left(1-\frac{x^2}{a^2}\right) dx$$

$$= 1.7017\ ab. = \iint I(x,y)\,dx\,dy$$

$$\iint I(x,y)\,x\,dx\,dy = 0.609524\ a^2 b.$$

lly $\iint I(x,y)\,y\,dx\,dy = 0.609524\ ab^2$

$$\bar{x} = \frac{\iint x I(x,y)\,dx\,dy}{\iint I(x,y)\,dx\,dy} = 0.3582a$$

lly $\bar{y} = 0.3582b.$

$$\iint (x-\bar{x})^2 I(x,y)\,dx\,dy = \cancel{0.0462819\, a^3 b} \quad 0.529227\, a^3 b$$

$$\text{Ily} \quad \iint (y-\bar{y})^2 I(x,y)\,dx\,dy = 0.529227\, b^3 a$$

$$\bar{\sigma}(x)^2 = \frac{\iint (x-\bar{x})^2 I(x,y)\,dx\,dy}{\iint I(x,y)\,dx\,dy} = 0.311\, a$$

$$\Rightarrow \quad \bar{\sigma}(x) = 0.5577\, a \quad \text{Ily} \quad \bar{\sigma}(y) = 0.5577\, b$$

$$\Rightarrow \quad D_{\sigma x} = 4\cdot\sigma_x = 2.2307\, a$$

$$D_{\sigma y} = 4\sigma_y = 2.2307\, b$$

$\Rightarrow$ beam radius in x & y direction is $1.1153\,a$ & $1.1153\,b$ respectively.