

Q1. We could approach this in two ways (a) local (b) average over the plate

Local approach :

$$\frac{dI(x, y, z)}{I} = \frac{\text{Photons absorbed an incremental area}}{\text{Photons incident in an incremental area}} = \frac{-\sigma N_0 e^{-\frac{2(x^2+y^2)}{a^2}} dx dy dz}{dx dy}$$

$$I(x, y, z) = e^{\sigma N_0 e^{-\frac{2(x^2+y^2)}{a^2}} z}$$

Average approach :

$$\frac{dI(z)}{I(z)} = \frac{-\left(\int_{-a}^a \int_{-a}^a \sigma N_0 e^{-\frac{2(x^2+y^2)}{a^2}} dx dy\right) dz}{a^2}$$

$$\int_{-a}^a \int_{-a}^a \sigma N_0 e^{-\frac{2(x^2+y^2)}{a^2}} dx dy$$

$$\frac{1}{2} a^2 \pi \sigma \text{Erf}\left[\sqrt{2}\right]^2 N_0$$

$$I(z) = I_0 e^{-\frac{1}{2} a^2 \pi \sigma \text{Erf}\left[\sqrt{2}\right]^2 N_0 z}$$

Q 2.

$$\lambda = 1.06 \cdot 10^{-3};$$

$$D1 = 5;$$

$$z1 = 25.4;$$

$$M2 = 1.1;$$

$$D2 = 0.1$$

$$0.1$$

$$\theta = \frac{4 \lambda M2}{3.1415 D1}$$

$$0.000296928$$

$$\text{Solve}\left[D2 == \sqrt{1 / \left( \frac{1}{D1^2} \left( 1 - \frac{z1}{f} \right)^2 + \frac{1}{f^2 \theta^2} \right)}, f\right]$$

{f → -336.86}, {f → 336.839}}

f = 336.839;

$$z2 = f + \frac{(z1 - f) f^2}{(z1 - f)^2 + \frac{D1^2}{\theta^2}}$$

z2 = 336.715

Depth of focus

In[10]:= D2 = 0.1;

$$\theta^2 = \frac{4 \lambda M2}{3.1415 D2};$$

Df = 1.05 D2;

M2 = 1.1;

$\lambda = 1.06 \cdot 10^{-3}$ ;

In[15]:= Solve[Df ==  $\sqrt{D2^2 + z2^2 \theta^2}$ , z2]

{{z2 → -2.15645527357368`}, {z2 → 2.15645527357368`}}

Depth of focus = 4.3 mm