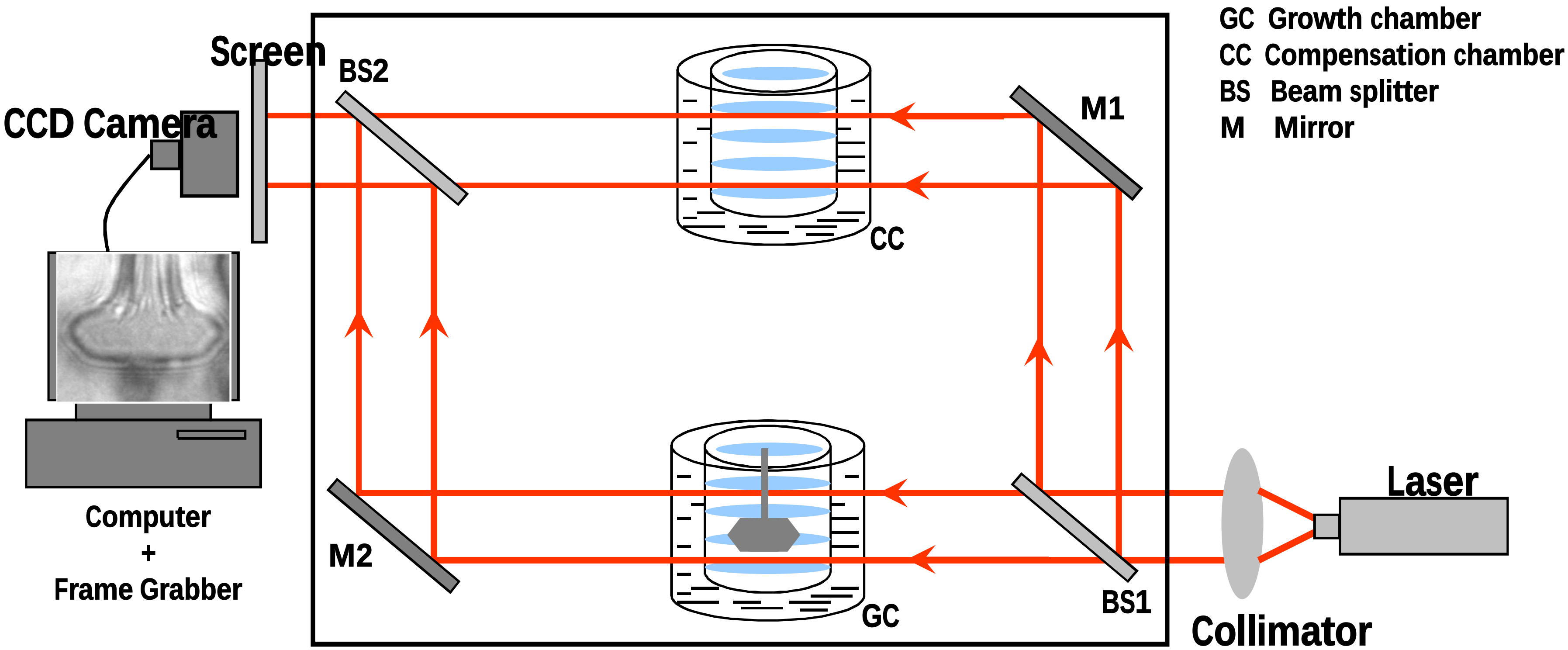
Mach-Zender Interferometer



INTERFERENCE

- Light waves interfere with each other much like mechanical waves do.
- All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

Conditions for Interference

- For sustained interference between two sources of light to be observed, there are two conditions which must be met
  - The sources must be ***coherent***
    - They must maintain a constant phase with respect to each other
  - The waves must have ***identical wavelengths***

Producing Coherent Sources

- Light from a monochromatic source is allowed to pass through a narrow slit.
- The light from the single slit is allowed to fall on a screen containing two narrow slits.
- The first slit is needed to insure the light comes from a tiny region of the source which is coherent
- ***Old method***.

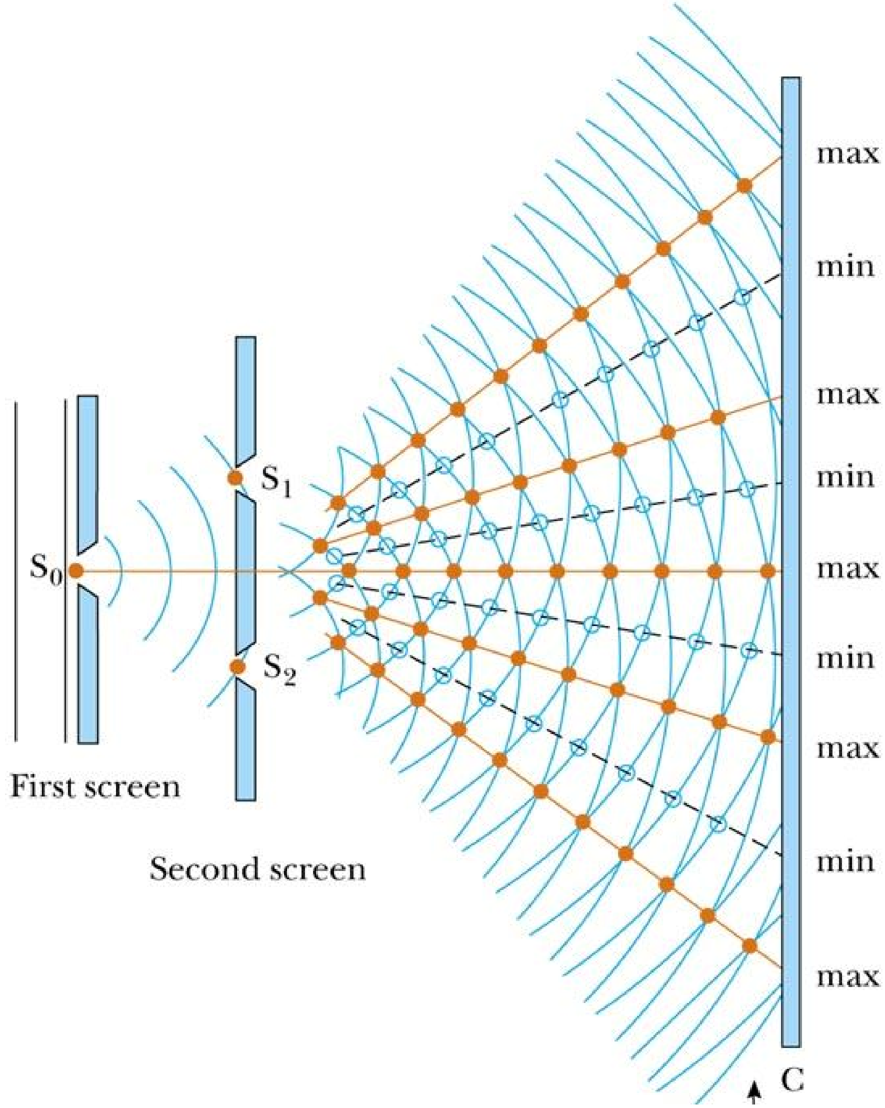
- Currently, it is much more common to use a **laser** as a coherent source.
- The laser produces an ***intense, coherent, monochromatic beam*** over a width of several millimeters.
- The laser light can be used to illuminate multiple slits directly.

Young's Double Slit Experiment

- Thomas Young first demonstrated interference in light waves from two sources in 1801.
- Light is incident on a screen with a narrow slit, $S_{o.}$
- The light waves emerging from this slit arrive at a second screen that contains two narrow, parallel slits, S_1 and S_2

Young's Double Slit Experiment, Diagram

- The narrow slits, S_1 and S_2 act as sources of waves
- The waves emerging from the slits originate from the same wave front and therefore are always in phase

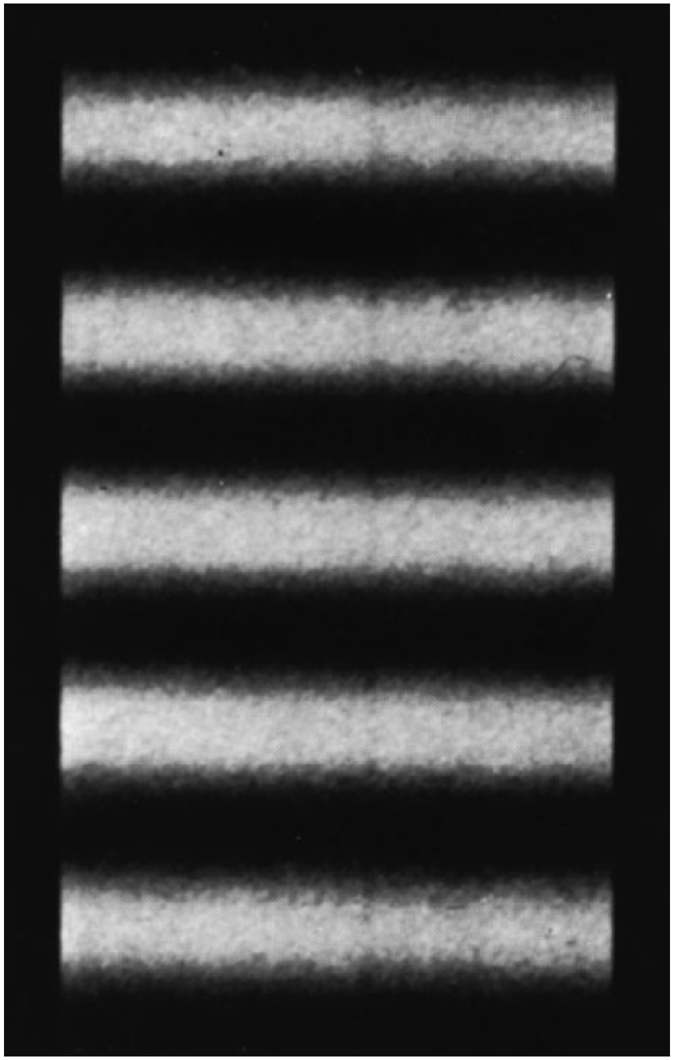


Resulting Interference Pattern

- **The light from the two slits form a visible pattern on a screen.**
- **The pattern consists of a series of bright and dark parallel bands called fringes.**
- ***Constructive interference* occurs where a bright fringe appears.**
- ***Destructive interference* results in a dark fringe.**

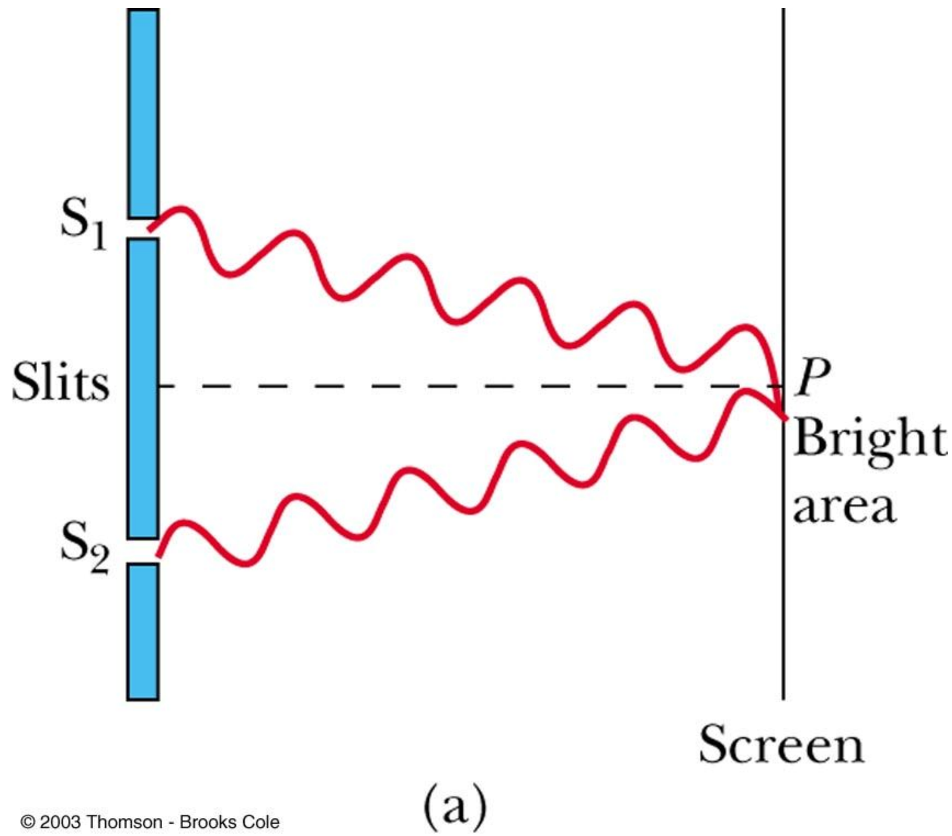
Fringe Pattern

- The fringe pattern formed from a Young's Double Slit Experiment would look like this
- The bright areas represent constructive interference
- The dark areas represent destructive interference



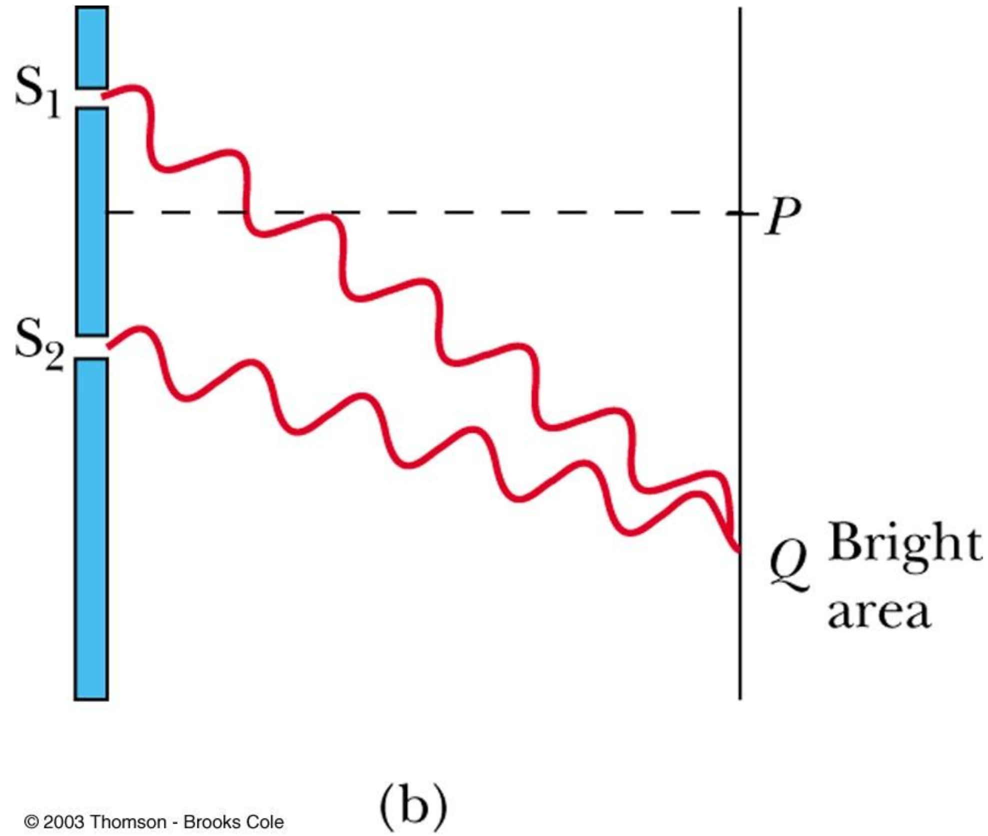
Interference Patterns

- Constructive interference occurs at the center point
- The two waves travel the same distance
  - Therefore, they arrive in phase



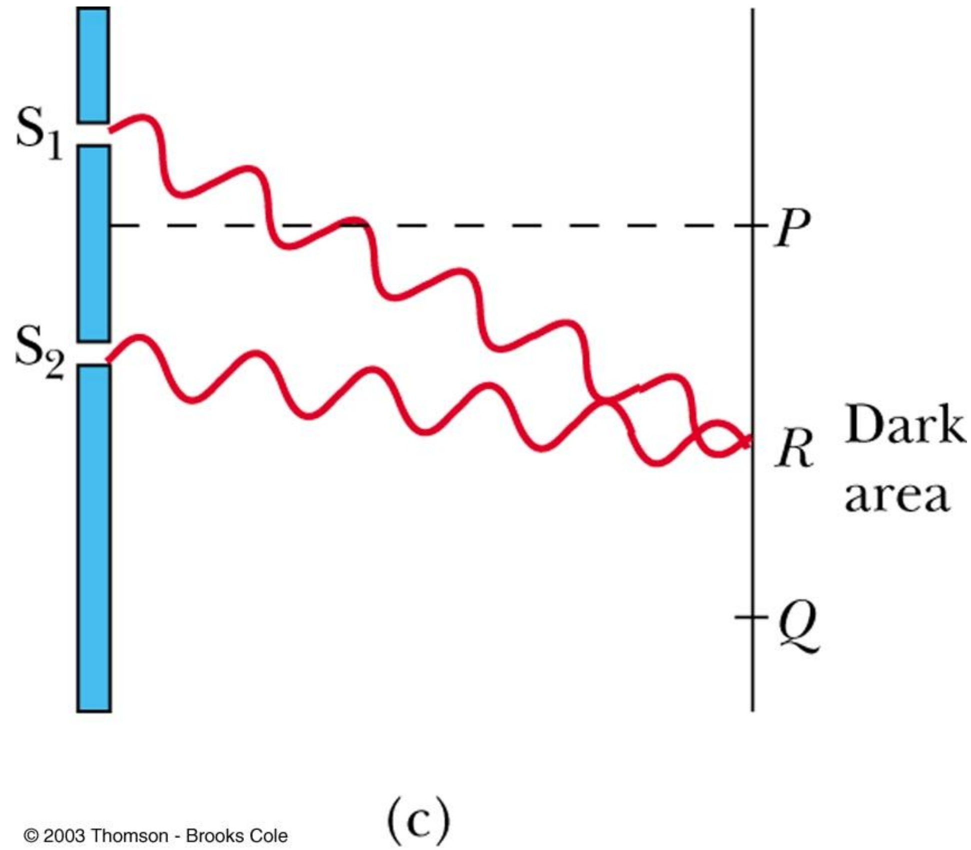
Interference Patterns - 2

- The upper wave has to travel farther than the lower wave
- The upper wave travels one wavelength farther
  - Therefore, the waves arrive in phase
- A bright fringe occurs

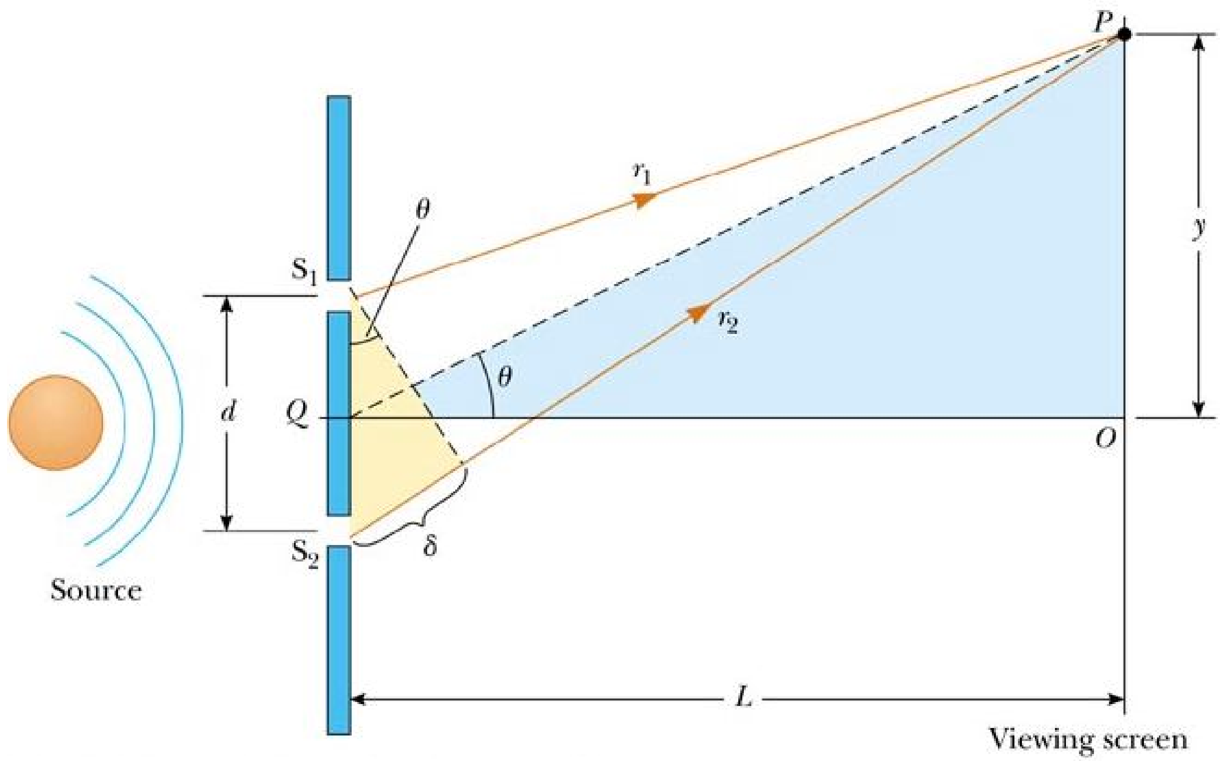


Interference Patterns - 3

- The upper wave travels **one-half of a wavelength** farther than the lower wave
- The trough of the bottom wave overlaps the crest of the upper wave
- This is **destructive interference**
  - A **dark fringe** occurs



Interference Equations



- The path difference, δ, is found from the tan triangle
- $\delta = r_2 - r_1 = d \sin \theta$
  - This assumes the paths are parallel
  - Not exactly parallel, but a very good approximation since L is much greater than d

- For a bright fringe, produced by constructive interference, the path difference must be either zero or some integral multiple of the wavelength

- $\delta = d \sin \theta_{bright} = m \lambda$

  - $m = 0, \pm 1, \pm 2, \ldots$
  - m is called the *order number*
    - When m = 0, it is the zeroth order maximum
    - When m = ±1, it is called the first order maximum

- When destructive interference occurs, a dark fringe is observed.

- This needs a path difference of an odd half wavelength.

- $\delta = d \sin \theta_{dark} = (m + \frac{1}{2}) \lambda$
  - $m = 0, \pm 1, \pm 2, \ldots$

Interference Equations, Final

- For bright fringes

$$y_{bright} = \frac{\lambda L}{d} m \qquad m = 0, \pm 1, \pm 2 \ldots$$

- For dark fringes

$$y_{dark} = \frac{\lambda L}{d} \left(m + \frac{1}{2} \right) \qquad m = 0, \pm 1, \pm 2 \ldots$$

- **Consider the interaction between two waves that travel along the same axis.**
  - **The waves are assumed to be coherent.**
  - **Wavelengths and wave velocities are identical.**

$$E_1 = A_1 \cos\left[\frac{2\pi}{\lambda}(z - vt)\right]$$

$$E_2 = A_2 \cos\left[\frac{2\pi}{\lambda}(z - vt - r)\right]$$

The extra parameter "*r*" in the second wave is introduced because that wave lags behind the other one, i.e. the two are out of phase.

Assume that the two amplitudes are equal = *A*

The combination of the two waves is E_s

$$E_s = A\left\{\cos\left[\frac{2\pi}{\lambda}(z - vt)\right] + \cos\left[\frac{2\pi}{\lambda}(z - vt - r)\right]\right\}$$

Using the identity $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$E_s = 2A\cos\frac{\pi r}{\lambda}\cos\left[\frac{2\pi}{\lambda}\left(z - vt - \frac{r}{2}\right)\right]$$

Amplitude of the resultant wave

The amplitude depends on the degree to which the two waves are out-of-phase.

The irradiance is the square of this amplitude,

$$I_s = 4A^2 \cos^2 \frac{\pi r}{\lambda}$$

$$\frac{r}{\lambda} = n, \text{ where } n = 0,1,2,...$$

Maximum irradiance

Constructive interference

$$\frac{r}{\lambda} = \frac{1}{2}, \frac{3}{2} ...$$

Minimum irradiance

Destructive interference

(Dark fringes to be formed)